25. NUCLEAR PHYSICS AND RADIOACTIVITY

NUCLEAR PHYSICS

1. INTRODUCTION

Nuclear physics is the field of physics that studies the constituents and interactions of atomic nuclei. Nuclear physics is the field of physics that studies the constituents and interactions of atomic nuclei. Nuclear physics is the field of physics that studies the constituents and interactions of atomic nuclei.

2. PROPERTIES OF ATOMIC NUCLEUS

Nuclear physics is the field of physics that studies the constituents and interactions of atomic nuclei. The most commonly known applications of nuclear physics are nuclear power generation and nuclear weapons technology, but the research has provided application in many fields, including those in nuclear medicine and magnetic resonance imaging, ion implantation in materials engineering and radiocarbon dating in geology and archaeology. The field of particle physics evolved out of nuclear physics and is typically taught in close association with nuclear physics.

Properties: Atomic nuclei have following properties:

2.1 Composition

All nuclei contain protons and neutrons except ordinary hydrogen atom which has only single proton. Proton has change $+e$ and neutron is neutral.

Mass no. of nuclei $\left(A\right)=Z+N$

Where $Z=$no. of protons in the nucleus; $N=$ no. of neutrons

Symbolically atomic nuclei is represented as ${}_{Z}^{A}X$

Illustration 1: How many electrons, protons, and neutrons are there in nucleus of atomic number 11 and mass number 24? **(JEE MAIN)**

Sol: The atomic number Z of atom represents the number of protons present in the nucleus. The number of electrons in an atom are same as the number of protons. The Atomic mass number A is sum of proton number Z and neutron number N.

Number of protons in nucleus = Atomic number = 11

Number of electrons = Number of protons = 11

Number of neutrons = Mass number A – atomic number Z N = 24 – 11 = 13

2.2 Mass

Nuclear mass has been measured accurately by using mass spectrometer. It is convenient to express mass in terms of amu which is defined as $\frac{1}{12}$ the mass of carbon isotope $^{12}_{6}C$

$1\text{amu} = 1.66 \times 10^{-27}\text{kg}$

According to Einstein's equation $E = mc^2$ 1amu can be expressed as energy

Energy equivalence of 1 amu $= \dfrac{1.66 \times 10^{-27} \times \left(3 \times 10^8\right)^2}{1.6 \times 10^{-19}} \text{eV} = 931\text{MeV}$

2.3 Nuclear Radius

The nuclear radius (R) is considered to be one of the basic quantities that any model must predict. For stable nuclei (not halo nuclei or other unstable distorted nuclei) the nuclear radius is roughly proportional to the cube root of the mass number (A) of the nucleus, and particularly in nuclei containing many nucleons, as they arrange in more spherical configurations:

The stable nucleus has approximately a constant density and therefore the nuclear radius R can be approximated by the following formula, $R = r_0 A^{1/3}$

Where A=Atomic mass number (the number of protons Z, plus the number of neutrons N) and $r_0 = 1.25\text{fm} = 1.25 \times 10^{-15}\text{m}$.

Illustration 2: The ratio of the radii of the nuclei $^{27}_{13}Al$ and $^{125}_{52}Te$ is approximately**.** **(JEE MAIN)**

Sol: The radius of the atomic nuclei is directly proportional to the cube root of atomic mass number.

$$R_{Al} / R_{Te} = \frac{(27)^{1/3}}{(125)^{1/3}} = \frac{3}{5} = \frac{6}{10}$$

Illustration 3: The radius of the $_{30}Zn^{64}$ nucleus is nearly (in fm) **(JEE MAIN)**

Sol: The radius of any atomic nucleus is given by $R = R_0 A^{1/3}$ where Ro = 1.2×10^{-15} is the Fermi radius.

$$R = R_0 A^{1/3} = 1.2 \times 10^{-15} \times (64)^{1/3} = 1.2 \times 10^{-15} \times 4 = 4.8\text{fm} \quad A \propto R^3$$

$A =$ Nucleon number or mass number

Any element X with mass number A and charge number Z can be represented by $^{A}_{Z}X$ or $^{A}_{Z}X$.

Number of neutron $= A - Z$ Mass number $= A = P + N$

1 amu $= \frac{1}{12}$th Mass of 12gm of $^{12}_{6}C$ atom.

2.4 Nuclear Density

Nuclear density is the density of the nucleus of an atom. The nuclear density for an atom with radius R and molar mass A(mass number) is $n = \dfrac{A}{\frac{4}{3}\pi R^3}$

Typical nucleus can be approximately calculated from the size of the nucleus, which itself can be approximated based on the number of protons and neutrons in it. The radius of a typical nucleus, in terms of number of nucleons, is $R = A^{1/3} r_0$ where A is the mass number and r_0 is 1.25 fm, with deviations of 0.2 fm from this value.

Illustration 4: Nuclear radius of ${}^{16}_{8}O$ is 3×10^{-15} m. Find the density of nuclear matter. **(JEE MAIN)**

Sol: Considering the nucleus of the oxygen as a sphere of the uniform density ρ, the density can be given as $\rho = \frac{M}{V}$ where M is the atomic mass number (convert it from amu to kg) of the oxygen and V is the volume of the sphere.

Use $\rho = \text{mass / volume} = \dfrac{1.66\times10^{-27}\times16}{(4/3)\pi\left(3\times10^{-15}\right)^2} = 2.35\times10^{17}\,\text{kgm}^{-3}$

2.5 Nuclear Spin and Magnetism

Many nuclides have an intrinsic nuclear angular momentum or spin and an associated intrinsic nuclear magnetic moment. Although nuclear angular momenta are roughly of the same magnitude as the angular momenta of atomic electrons, nuclear magnetic moments are much smaller than typical atomic magnetic moments.

2.6 Types of Nuclei

(a) Isotopes: Nuclei having same atomic number Z but different mass no. are called isotopes.

Ex. ${}^{1}_{1}H, {}^{2}_{1}H, {}^{3}_{1}H$

(b) Isobars: Nuclei having same mass number A but different atomic number Z are called isobars.

Ex. ${}^{14}_{6}C$ and ${}^{14}_{7}N$.

(c) Isotones: Nuclei having same number of neutrons are called isotones Ex. ${}^{3}_{1}H, {}^{4}_{2}He$.

3. NUCLEAR STABILITY AND RADIOACTIVITY

Nuclear Stability means that nucleus is stable meaning that it does not spontaneously emit any kind of radioactivity (radiation). On the other hand, if the nucleus is unstable (not stable), it has the tendency of emitting some kind of radiation, i.e., it is radioactive. Therefore the radioactivity is associated with unstable nucleus:

Stable nucleus -> non-radioactive, Unstable nucleus –> radioactive

PLANCESS CONCEPTS

Keep in mind that less stable means more radioactive and more stable means less radioactive.

We want to know why there is radioactivity. What makes the nucleus a stable one? There are no concrete theories to explain this but there are only general observations based on the available stable isotopes. It appears that neutron to proton (n/p) ratio is the dominant factor in nuclear stability. This ratio is close to 1 for atoms of elements with low atomic number and increase as the atomic number increases. Then how do we predict the nuclear stability? One of the simplest ways of predicting the nuclear stability is based on whether nucleus contains odd/even number of protons and neutrons:

Protons	Neutrons	Number of stable Nuclides	Stability
Odd	Odd	4	
Odd	Even	50	least stable
Even	Odd	57	↓
Even	Even	167	most stable

- Nuclides containing odd numbers of both protons and neutrons are the least stable means more radioactive.

PLANCESS CONCEPTS

- Nuclides containing even numbers of both proton and neutrons are most stable means less radioactive.
- Nuclides contain odd number of protons and even numbers of neutrons are less stable than nuclides containing even numbers of protons and odd numbers of neutrons.

In general, nuclear stability is greater for nuclides containing even numbers of protons and neutrons or both.

Yashwanth Sandupatla (JEE 2012, AIR 821)

4. NUCLEAR FORCE

The force that controls the motions of atomic electrons is the familiar electromagnetic force. To bind the nucleus together, however, there must be a strong attractive nuclear force of a totally different kind, strong enough to overcome the repulsive force between the (positively charged) nuclear protons and to bind both protons and neutrons into the tiny nuclear volume. The nuclear force must also be of short range because its influence does not extend very far beyond the nuclear "surface". Its range is of the order of 2fm. The present view is that the nuclear force that binds neutrons and protons in the nucleus is not a fundamental force of nature but is a secondary, or "spillover", effect of the strong force that binds quarks together to form neutrons and protons. In much the same way, the attractive force between certain neutral molecules is a spillover effect of the Coulomb electric force that acts within each molecule to bind it together. This strong force is independent of the charge. This means that the strong force of proton-proton, neutron-neutron, proton-neutron interactions is the same, apart from the additional repulsive Coulomb force for the proton-proton interaction. It is customary to talk of the potential energy when we talk of nuclear forces. Here, the potential energy of interaction of a proton and a neutron is shown in Fig 25.1.

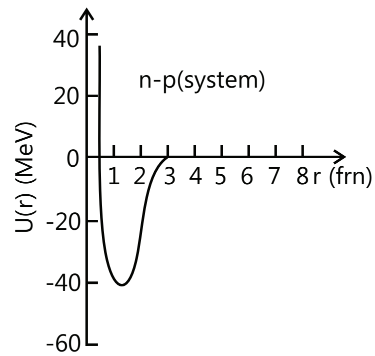


Figure 25.1

4.1 Properties of Nuclear Forces

(a) These forces are attractive by nature. At very short distance s (< 0.7 fm) these become repulsive.

(b) The nuclear force is short range force. It means that it exist only when particles are very-very close to each other. In nucleus the separation between particles is 10^{-15} m or I Fermi. At this infinitesimal small separation, the nuclear force becomes 100 times stronger than the repulsive than the electric forces between the nucleons. In the short range force, the force between the particles rapidly decreases. Thus the nuclear force only exists in the nucleus.

(c) These forces do not obey inverse square law.

(d) Nuclear forces are not central forces. It means that these forces do not depend upon the center of one particle to another particle.

(e) Strong nuclear forces are the strongest force in nature. In the given range of distance, the nuclear forces are 10^{38} times stronger than the gravitational forces.

5. MASS DEFECT

It has been observed that actual mass of the nucleus (determined by mass spectrometer of high resolving power) is always less than the sum of masses of proton and neutrons in Free State.

$\Delta m = \left[Zm_p + \left(A - Z\right)m_n \right] - M$, where M_p is mass of proton; m_n is mass neutron; M is mass of nucleus

Illustration 5: Consider the decay of radium (A=226) atom into an alpha particle and radon (A=222). Then, what is the mass defect of the reaction.

Mass of radium -226 atom = 226.0256u; Mass of radon -222 atom = 222.0715u and Mass if helium – 4 atom = 4.0026u **(JEE MAIN)**

Sol: Mass defect is the difference in masses of parent and daughter nuclei. Mass defect is given by $\Delta m = M(Ra^{226}) - M(Rn^{222}) - M(\alpha)$

Mass defect $\Delta m = M(Ra^{226}) - M(Rn^{222}) - M(\alpha) = 226.0256 - 222.0175 - 4.00026 = 0.0053u$

6. BINDING ENERGY

It is defined as energy released during formation nucleus as a result of disappearance of mass i.e., mass defect.

Binding energy = $(\Delta m)c^2$; Binding energy per nucleon= $\frac{\left(\Delta m\right)c^2}{A}$

Illustration 6: If mass equivalent to one mass of proton is completely converted into energy then determine the energy produced? **(JEE MAIN)**

Sol: When one proton is converted into its equivalent energy, the energy released during this conversion is given by $E = mc^2$

$$E = mc^2 = \left(1.66 \times 10^{-27}\right)\left(3 \times 10^8\right)^2 J = 1.49 \times 10^{-10}\ J = \frac{1.49 \times 10^{-10}}{1.6 \times 10^{-13}}\ MeV = 931.49\ MeV \quad \because 1amu = 931.49MeV$$

Variation of B.E. per nucleon with mass no. A

If the average binding energy per nucleon is calculated for all nuclides and the results are plotted against A, the mass number, a graph shown in Fig. 25.2 is obtained.

It is observed from the graph that binding energy per nucleon (except for He^4, C^{12} and O^{16}) rises first sharply and reaches a maximum value 8.8 MeV in the neighborhood of $A = 50$. The curve falls very slowly after $A = 50$ and reaches at 8.4 MeV at about $A = 140$. For higher mass number, the energy decreases to about 7.6 MeV.

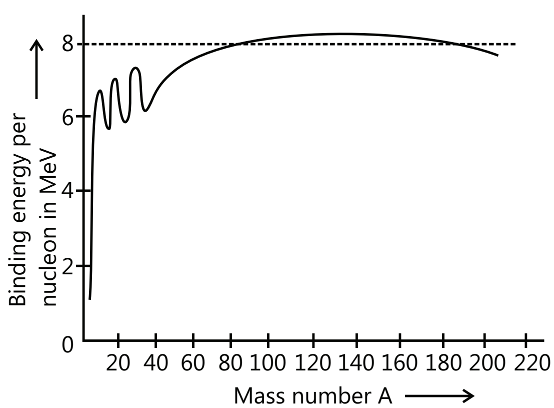


Figure 25.2

Illustration 7: Binding energy per nucleon of an α-particle from the following data:

Mass of the helium nucleus = 4.001265amu; Mass of proton = 1.007277amu

Mass of neutron = 1.00866amu; (1amu=931.4812MeV) **(JEE MAIN)**

Sol: The binding energy is given by $B.E. = \Delta m \times c^2$ J = $\Delta m \times 931.5$ Mev

Mass of two protons = $2 \times 1.007277 = 2.014554amu$

Mass of two neutron = $2 \times 1.008666 = 2.017332amu$

Total initial mass of two proton and neutrons = 2.014554 + 2.017332 = 4.031886amu

Mass defect $\Delta m = 4.031816 - 4.001265, \Delta m = 0.030621amu$

$\therefore$ Binding energy of α particle $= 0.030621 \times 931.4812 = 28.5221$MeV

Binding energy of nucleon = 28.5221/4=7.10525Mev

PLANCESS CONCEPTS

The energy differences in allowed energy levels of a nucleus are generally large of the order of MeVs. Hence, it is difficult to excite the nucleus by usual method of supplying energy as heat.

GV Abhinav (JEE 2012, AIR 329)

7. NEUTRON TO PROTON RATIO

According to Pauli Exclusion Principle, each quantum state can contain at most two protons or two neutrons that too with opposite spin. Hence nuclear forces favor pairing of two protons and two neutrons together. In lighter nuclei nuclear forces are dominant over coulomb repulsion and hence number of protons and number of neutrons are nearly the same. In heavier nuclei the case is different, the interaction between nucleon pairs through nuclear forces is not that effective and Coulomb repulsion dominates. Stability is achieved by having more neutrons as they are neutral and don't participate in Coulomb repulsion. That is why N/Z increases with atomic number for stable nuclides. The heaviest stable nuclide is $^{209}_{83}\text{Bi}$. Bismuth in fact is of radioactive nature but the decay rate is so less that it can be considered stable.

PLANCESS CONCEPTS

Having too many neutrons do not account for higher stability as many of these neutrons won't have pairing with protons. It will in fact decrease the stability.

The fact that the binding energy curve "drops" at both high and low mass numbers has very important practical consequences.

Anurag Saraf (JEE 2011, AIR 226)

RADIOACTIVITY

1. INTRODUCTION

The phenomenon of spontaneous disintegration of nuclei of unstable atoms is defined as radioactivity. Generally it is exhibited by atoms with A>192 and Z>82. It was discovered by Henry Becqurel. Lead isotope is the stable end product of any natural radioactive series Radio activity is a nuclear process and not an atomic process Radioactivity is not associated with the electron configuration of the atom.

Becquerel, in 1896, discovered accidently that uranium salt crystals emit an invisible radiation which affected a photographic plate even though it was properly covered. Further investigations by Marie and Pierre Curie and other workers showed that many other substances also emitted similar radiations. This property of spontaneous emission of radiation is called radioactivity. Subsequent works, notably of Rutherford, suggested that radioactivity was, in fact, due to decay or disintegration of unstable nuclei.

Emission of α particles: During α-particle emission atomic no. reduces by 2 while mass no. reduces by 4 i.e. ${}_{Z}^{A}x \rightarrow {}_{Z-2}^{A-4}y + {}_{2}^{4}He$

Emission of β-particle: When Nuclei has excess neutrons, it emits β-particle to bring n/p ratio into stable region. A neutron gets converted into proton and β-particle, therefore atomic mass remains constant white atomic number increases by 1.

Y-Radiation: After emission of α or β particle nuclei are left in excited state, Nucleus comes to stable state by emitting electromagnetic radiation known as γ radiation. There is no change in A or Z during this process, α and β emission don't take place simultaneously while γ radiation can emit along with any of them.

1.1 Properties of Alpha, Beta and Gamma Rays

The comparison of the properties of α , β and γ rays are shown below in the table:

Properties	**α-rays**	**β-rays**	**γ-rays**
Nature photons	Helium nucleus	Fast moving electrons	Electromagnetic waves
Nature of charge	Positive	Negative	No change
Magnitude of charge	3.2×10^{-19} coulomb	1.6×10^{-19} coulomb	Zero
Mass	6.6×10^{-27} kg	3.1×10^{-31} kg	Rest mass zero
Velocity	Between 1.4×10^{7} m/sec to 2.2×10^{7} m/sec	1% to 99% velocity of light	3×10^{8} m/sec.
Effect of electric & magnetic fields	Deflected	Deflected	Not deflected
Range	2.7to 8.62 cm in air or 1/100 mm of Al	5mm of Al or 1mm of lead	30 cm of iron
Penetrating power	Minimum	100 times of α-rays	1000 times of α-rays
Ionising power	Maximum	Lesser	Minimum

1.2 Natural Radioactivity

Natural Radioactivity is the spontaneous disintegration of an unstable atomic nucleus and the emission of particles or electromagnetic radiation. All naturally occurring elements with atomic numbers greater than 83 as well as some isotopes of lighter elements show natural radioactivity.

1.3 Artificial Radioactivity

Radioactivity produced in a substance by bombardment with high-speed particles (as protons or neutrons), also called as induced radioactivity.

1.4 Parent and Daughter Nuclei

Nucleus which decays in a radioactive decay is called parent nucleus. This parent nucleus transforms to an atom with a nucleus in a different state, or to a different nucleus containing different numbers of protons and neutrons. Either of these products is named the daughter nucleus.

1.5 Law of Radioactive Disintegration

(a) Radioactivity is a process in which nuclei of certain elements undergo spontaneous disintegration without excitation by any external means.

(b) The radioactivity results the emission of powerful radiations known as Alpha (α), Beta (β) and Gamma (γ) rays.

(c) Radioactivity is a nuclear phenomenon i.e. it is not depend upon no. of electrons present in outer shell.

It was studied by Rutherford and Soddy in 1902. The disintegration of nuclei is purely statistical which means all nuclei take different time to disintegrate and are independent for radioactive decay. Rate of disintegration is directly proportional to no. of not decayed nuclei present at that time, i.e. $-\frac{dN}{dt} \propto N = \lambda N$ ... (i)

Where λ is disintegration or decay constant.

Integrating equation (i)

$\log_e N = -\lambda t + C$

$\because$ at, $t = 0$, $N = N_0 \Rightarrow C = \log_e N$

$\Rightarrow \log_e N = -\lambda t + \log_e N_0$ or $N = N_0 e^{-\lambda t}$ ... (ii)

Equation (ii) shows that no. of nuclei of given radioactive substance decreases exponentially with time. It also shows that decays occurs rapidly initially and rate of decay decreases with time.

Half-life $(T_{1/2})$: The time in which half of radioactive substance decays is known as half-life.

or $t = T_{1/2}$, $N = \frac{N_0}{2}$; $\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$; $\Rightarrow T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$ ... (iii)

If $t = nT_{1/2}$ where n is integer, equation (ii) reduces to $N = N_0 (1/2)^n$

Illustration 8: A count-rate meter is used to measure the activity of a given sample. At one instant the meter shows 4750 counts per minute. Five minutes later it shows 2700 counts per minute. Find:

(a) Decay constant (b) the half-life of the sample. **(JEE MAIN)**

Sol: The decay constant of radioactive element is given by $\lambda = \frac{\log_e 10}{t} \log \frac{N_0}{N_t}$ where N0 is the number of radioactive nuclei at t=0 and Nt is the number of radioactive nuclei at time t. The half-life of the radioactive element is $t_{1/2} = \frac{0.693}{\lambda}$.

Initial activity, $A_0 = dN/dt$ at $t = 0$

Final activity, $A_t = dN/dt$ at $t = t$

$$\left.\frac{dN}{dt}\right|_{t=0} = IN_0 \quad \& \quad \left.\frac{dN}{dt}\right|_{t=5} = IN_t; \quad \frac{4750}{2700} = \frac{N_0}{N_t}$$

Using $\lambda t = 2.303 \log\frac{N_0}{N_t}$; $\lambda(5) = 2.303 \log\frac{4750}{2700}$; $\lambda = \frac{2.303}{5}\log\frac{4750}{2700} = 0.1129\ \text{min}^{-1}$

$t_{1/2} = \frac{0.693}{0.1129} = 6.14$ min

Mean life (τ): Mean life of radioactive substance is defined as sum of life times of all radioactive nuclei divided by total no. of nuclei.

or $\tau = \frac{\int t\,dN}{\int dN} = \frac{\int t dN}{N_0}$ or $\tau = \frac{1}{\lambda}$ ... (iv)

if $t = \tau$; $N = N_0 e^{-\lambda(1/\lambda)} = 0.37 N_0$

i.e., In mean life radioactive substance decays by nearly 63%.

From (iii) and (iv) $T_{1/2} = 0.693\ \tau$ ... (v)

Illustration 9: The mean lives of a radioactive substance are 1620 and 405 years for α emission and β emission respectively. Find out the time during which three fourth of a sample will decay if it its decaying both the α emission and β emission simultaneously. **(JEE ADVANCED)**

Sol: When substance decays by α and β emission simultaneously, the average rate of λ_{av} disintegration is given by $\lambda_{av} = \lambda_\alpha + \lambda_\beta$; Where λ_α and λ_β are disintegration constant for α emission and β emission respectively. The average time of the disintegration is given by $\lambda_{av} t_{av} = 2.303 \log\frac{N_o}{N_t}$ where N_o is the number of atoms present at time t=0 s. And N_t is the number of disintegration atoms present at the time t s.

Mean life is given by: $\tau_m = 1/\lambda$; $\lambda_{av} = \lambda_\alpha + \lambda_\beta$; $\frac{1}{\tau_{av}} = \frac{1}{\tau_\alpha} + \frac{1}{\tau_\beta} = \frac{1}{1620} + \frac{1}{405} = 3.08\times10^{-3}$

$\Rightarrow \lambda_{av} t = 2.303 \log\frac{100}{25}$; $(3.08\times10^{-3})\ t = 2.303\ \log\frac{100}{25}$

$\Rightarrow t = 2.303\times\frac{1}{3.08\times10^{-3}}\log 4 = 499.24$ years

Activity of a Radioactive Isotope: The activity of a radioactive substance (or radioisotope) means the rate of decay per second or the number of nuclei disintegrating per second. It is generally denoted by A. $\Rightarrow A = \frac{dN}{dt}$

If a time t=0 sec, the activity of a radioactive substance is A_o and after time t=t sec it is observed to be A_t, then:

$A_o = \left|\frac{dN}{dt}\right|_{t=0} = \lambda N_0$ $A_t = \left|\frac{dN}{dt}\right|_{t=t} = \lambda N_t$

Units of Rate of Decay or Activity: A number of units have been used to express the activity of a radioactive sample. The more commonly used ones are the following:

(a) Curie (Ci): The activity of a radioactive sample is said to be one curie when 3.7×10^{10} decays take place per second. Thus $1\text{Ci} \equiv 3.7\times10^{10}$ decays / s

This is the approximate activity of 1 g of radium. In practice, the smaller units milli curie and micro curie are used. $1\,\text{mCi} \equiv 3.7\times10^{7}$ decays / s; $1\,\mu\ \text{Ci} \equiv 3.7\times10^{4}$ decays / s

(b) Becquerel (Bq): The Si unit of activity is called the Becquerel and it represents 1 decay per second. Thus 1 Bq = 1 decay/s We thus have $1\text{Ci} \equiv 3.7\times10^{10}$ Bq

(c) Rutherford (Rd): Another unit for activity is Rutherford and it represents 10^6 decays per second. $1\text{Rd} = 10^6 \text{decays / s}$

Illustration 10: Radioisotopes of phosphorus P^{32} and P^{38} are mixed in the ratio 2:1 of atoms. The activity of the sample is 2 m Ci. Find the activity of the sample after 30 days, $t_{1/2}$ of P^{32} is 14 days and, $t_{1/2}$ of P^{38} is 25 days.

(JEE ADVANCED)

Sol: When the radio isotopes are mixed in the proportion 2:1, the compound activity of mixture over time t is given by $A_t = A_{1t} + A_{2t}$. The activity A of any radioactive substance with half-life τ is defined as $A = \lambda N = \frac{0.693 \times N}{\tau}$.

Let A_o be the initial activity of the sample,

Let A_{1o} be initial activity of isotope 1 and A_{2o} be the initial activity of sample 2

$A_o = A_{1o} + A_{2o}$

Similarly for final activity (Activity after time t), $A_t = A_{1t} + A_{2t}$ $\qquad A_t = A_{1o} e^{-\lambda_1 t} + A_{2o} e^{-\lambda_2 t}$

Now in the given equation A_o = 2 m Ci $\Rightarrow A_0 = A_{1o} + A_{2o} = 2 \text{ m Ci}$ ... (i)

Initial ratio of atoms of isotopes=2:1

We know from definition of activity, $A = \lambda N$ here λ is the decay constant and N is number of radioactive nuclei present at time instant t s.

$\frac{A_{1o}}{A_{2o}} = \frac{N_{1o}}{N_{2o}} \times \frac{T_2}{T_1}$ where T represents half-life; $\frac{A_{1o}}{A_{2o}} = \frac{2}{1} \times \frac{25}{14} = \frac{25}{7}$ ... (ii)

On solving equation (i) and (ii), we get, $A_{1o} = 25/16$ and $A_{2o} = 7/16$; $\quad A_t = A_{1o} e^{-\lambda_1 t} + A_{2o} e^{-\lambda_2 t}$

How to solve expression like this? For example, consider the first exponential term $\exp\left(-\frac{0.693 \times 30}{14}\right) = e^{-1.485}$

Let $y = e^{-1.485}$ Therefore, $\ln y = -1.485$; $\quad \log y = -(1.485/2.303)$ $y = \text{antilog}\,(-1.485/2.303)$

So, from above calculations you can derive a general result i.e. $e^{-x} = \text{antilog}\left|\frac{-x}{2.303}\right|$

$$A_t = \frac{25}{16} \times 0.2265 + \frac{7}{16} \times 0.4353 = 0.5444 \text{ Ci}.$$

Important Formulae

(a) $N = N_o e^{-\lambda t}$

(b) $A = A_o e^{-\lambda t}$

(c) $M = M_o e^{-\lambda t}$

(d) $\lambda = \frac{2.3027 \log_{10}\left(\frac{N_o}{N}\right)}{t}$

(e) $\lambda = \frac{2.3027 \log_{10}\left(\frac{A_o}{A}\right)}{t}$

(f) $\lambda = \frac{2.3027 \log_{10}\left(\frac{M_o}{M}\right)}{t}$

(g) $\lambda = \lambda_\alpha + \lambda_\beta$

(h) $\tau = \frac{\tau_\alpha \tau_\beta}{\tau_\alpha + \tau_\beta}$ (when two particles decay simultaneously)

(i) $N = \frac{N_o}{2^n} + \frac{N_o}{2^{\left(\frac{T}{T_{1/2}}\right)}}$

(j) $A = \frac{A_o}{2^{\left(\frac{T}{T_{1/2}}\right)}}$

(k) $M = \frac{M_o}{2^{\left(\frac{T}{T_{1/2}}\right)}}$

2. ALPHA DECAY

In alpha decay, the unstable nucleus emits an alpha particle reducing its proton number Z as well as its neutron N by 2. The alpha decay process may be represented as

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He.$$

As the proton number Z is changed, the element itself is changed and hence the chemical symbol of the residual nucleus is different from that of the original nucleus. The nucleus before the decay is called the parent nucleus and resulting after the decay is called the daughter nucleus. An example of alpha decay is $^{212}_{83}Bi \rightarrow {}^{208}_{81}Tl + {}^{4}_{2}He$.

(a) Characteristics of α-decay:

(i) The spectrum of α-particles is a discrete line spectrum.

(ii) Spectrum of α-particles has fine structure i.e. every spectral line consists of a number of fine lines.

(iii) The α- emitting nuclei have discrete energy levels i.e., energy levels in nuclei are analogous to discrete energy levels in atoms.

(iv) α-decay is explained on the basis of tunnel effect.

(v) Geiger-Muller law- $\log_e \lambda = A + B\log_e R$ For radioactive series B is same whereas A is different

(b) Size of the nucleus decreases by α emission

3. BETA DECAY

Beta Decay: Beta decay is a process in which either a neutron is converted into a proton or a proton is converted into a neutron. Thus, the ratio N/Z is altered in beta decay. If a nucleus is formed with more number of neutrons than needed for stability, a neutron will convert itself into a proton to move towards stability. Similarly, if a nucleus is formed with more number of protons than needed for stability, a proton will convert itself into a neutron. Such transformations take place because of weak forces operating within a neutron or a proton. When a neutron is converted into a proton, an electron and a new particle named antineutrino are created and emitted from the nucleus $n \rightarrow p + e + \bar{v}$

3.1 Characteristics β-Decay

(a) The energy spectrum of β-particles is continuous i.e. β-particles of all energies up to a certain maximum are emitted.

(b) The number of such β-particles is maximum whose energy is equal to the maximum probable energy i.e. at $E = E_{mp}$, N_B =maximum.

(c) There is a characteristic maximum value of energy in the spectrum of β-particles which is known as the end point energy (E_0).

(d) In β-decay process, a neutron is converted into proton or proton is converted into neutron.

$$^{1}_{0}n = {}^{1}_{1}p + {}^{0}_{-1}e \ \ \beta \text{ particle } {}^{1}_{1}p = {}^{1}_{0}n + {}^{0}_{1}e \ \ (\beta^{+}\text{particle})$$

(e) The energy of β-particles emitted by the same radioactive material may be same or different.

(f) The number of β-particles with energy $E = E_0$ (end point energy) is zero.

4. GAMMA DECAY

In gamma decay, a nucleus changes from a higher energy state to a lower energy state through the emission of electromagnetic radiation (photons). The number of protons (and neutrons) in the nucleus does not change in this process, so the parent and daughter atoms are the same chemical element. In the gamma decay of a nucleus, the emitted photon and recoiling nucleus each have a well-defined energy after the decay. The characteristic energy is divided between only two particles. The process is similar to that in a hydrogen atom when an electron jumps from a higher energy orbit to a lower energy orbit emitting a photon.

4.1 Characteristics γ-Decay

(a) The spectrum of γ -rays is a discrete line spectrum.

(b) Whenever α or β -particles is emitted by a nucleus then the daughter nucleus is left in the excited state. It suddenly makes transition in the ground state thereby emitting γ -rays.

(c) Knowledge about nuclear energy levels is obtained by γ -spectrum.

(d) γ -rays interact with matter as a consequence of which the phenomena of photoelectric effect, Compton Effect and pair production happen. (At low energy photoelectric effect and at high energy pair-production is effective).

5. RADIOACTIVE SERIES

(a) Elements beyond Bismuth are all radioactive in nature. These radioactive elements disintegrate to give new elements which further disintegrate to form other elements and so on. The process is continued till a non-radioactive end product is reached.

(b) The whole chain of such elements starting from the parent radioactive elements to the end non-radioactive element is called “radioactive series or a family.”

$(4n+1)$ is artificial series & 4n, $(4n+2)$, $(4n+3)$ are natural series.

S.No.	Series	Name of the series	Initial element	Final element	Nature of series	No of α & β particles emitted
1.	4n+2	Uranium series	$^{238}_{92}U$	$^{206}_{82}pb$	Natural	$8\alpha, 6\beta$
2.	4n+3	Actinium series	$^{235}_{92}U$	$^{207}_{82}pb$	Natural	$7\alpha, 4\beta$
3.	4n	Thorium series	$^{232}_{90}Th$	$^{208}_{82}Pb$	Natural	$6\alpha, 4\beta$
4.	4n+1	Neptunium series	$^{237}_{93}Np$	$^{209}_{83}Bi$	Artificial	$7\alpha, 4\beta$

5.1 Thorium Series

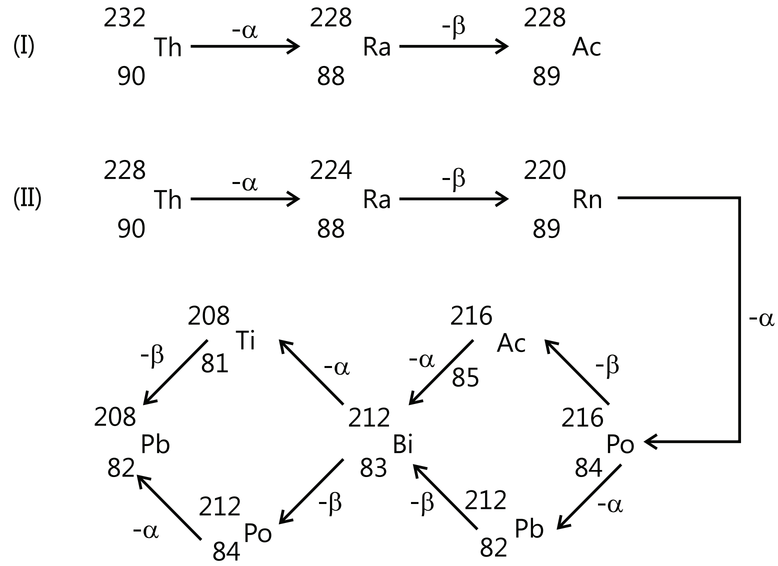


Figure 25.3

5.2 Uranium Series

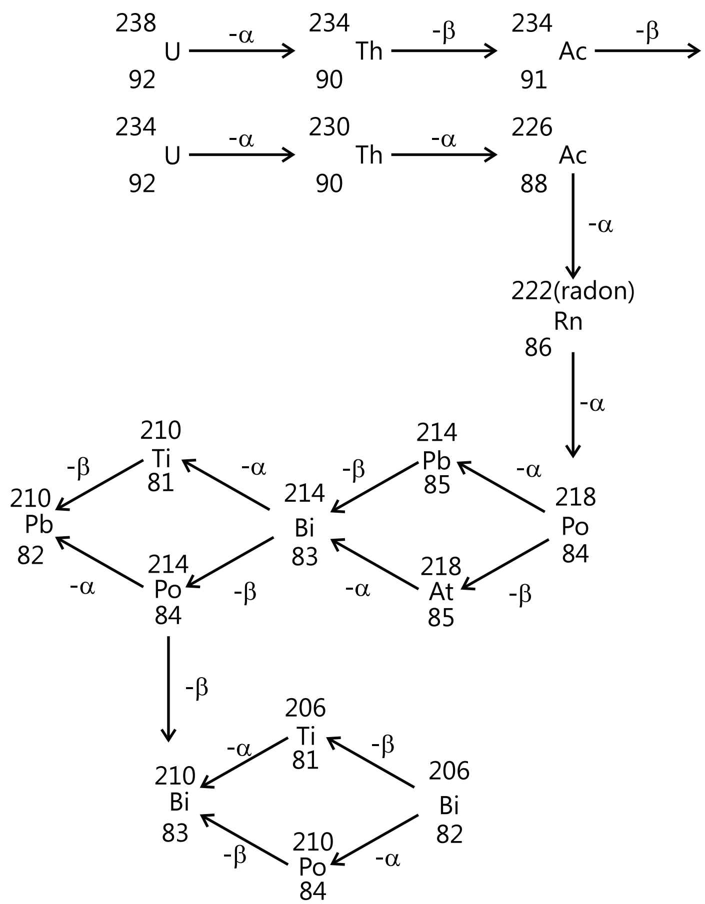


Figure 25.4

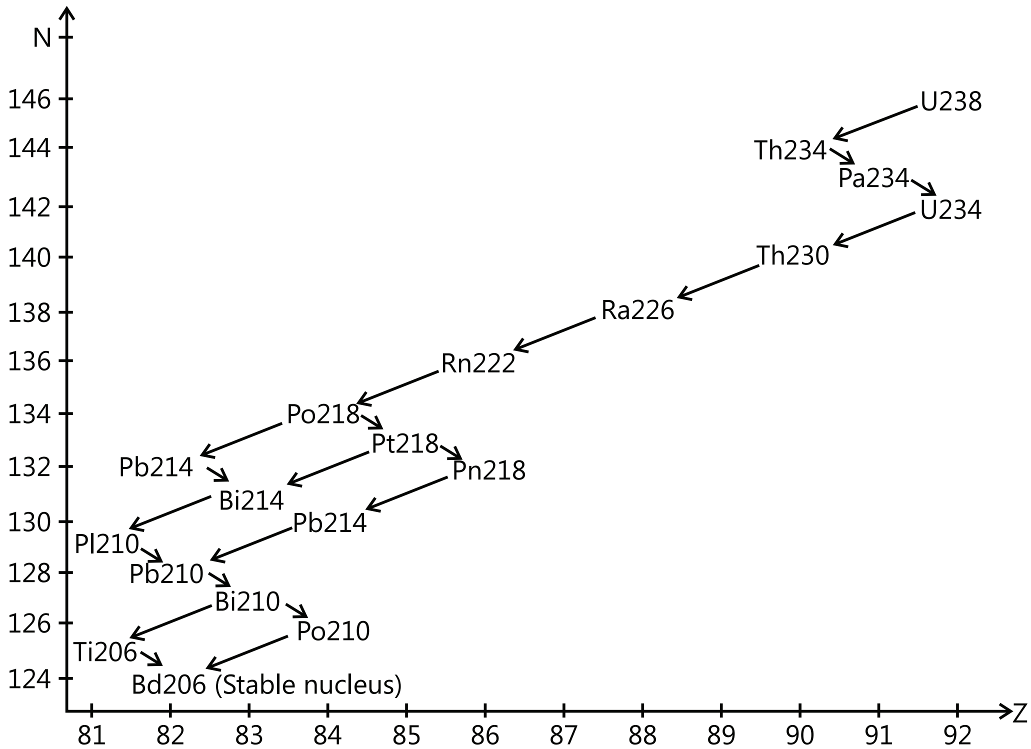


Figure 25.5: Uranium series

The mass number of each element in the series is equal to 4n+3. Where n is a positive integer.

5.3 Actinum Series

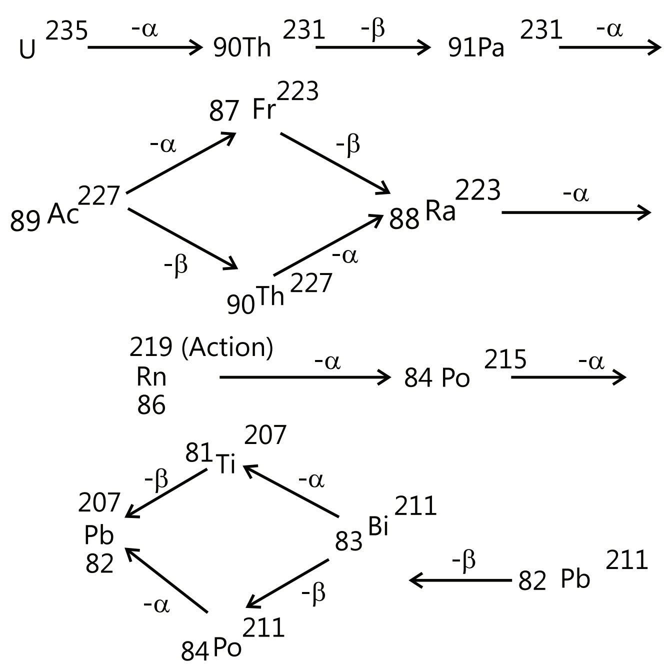


Figure 25.6

PLANCESS CONCEPTS

(a) In all series one element of zero group is present (atomic no=86) in gaseous state which is called emanation.

(b) In all series last product is an isotopes of lead ^{208}Pb, ^{206}Pb, ^{207}Pb respectively.

Pb is found in nature as a mixture of these three isotopes.

(c) The (4n+1) series (Neptunium series):-

- Except the last member all other members of this series have been obtained by artificial means.
- The series does not contain gaseous emanation.
- The last member of the series is an isotope of Bi and not an isotope of Pb.

Vijay Senapathi (JEE 2011, AIR 71)

6. ELECTRON CAPTURE

Electron capture is a process in which a proton-rich nuclide absorbs an inner atomic electron, thereby changing a nuclear proton to a neutron and simultaneously causing the emission of an electron neutrino. Various photon emissions follow, as the energy of the atom falls to the ground state of the new nuclide.

Electron capture is the primary decay mode for isotopes with a relative superabundance of protons in the nucleus, but with insufficient energy difference between the isotope and its prospective daughter (the isobar with one less positive charge) for the nuclide to decay by emitting a positron. Electron capture is an alternate decay mode for radioactive isotopes with sufficient energy to decay by positron emission. It is sometimes called inverse beta decay, through this term can also refer to the interaction of an electron anti-neutron with a proton.

A free proton cannot normally be changed to a free neutron by this process the proton and neutron must be part of a larger nucleus. In the process of electron capture, one of the orbital electrons, usually from the K or L electron shell (K-electron capture, also K-capture, or L-electron capture, L-capture) is captured by a proton in the nucleus forming a neutron and emitting an electron neutrino.

6.1 Calculation of Number of Alpha and Beta Particles Emitted

Consider the following general reaction. $\quad {}^{m}_{n}X \rightarrow {}^{m'}_{n'}Y + a\,{}^{4}_{2}\alpha + b\,{}^{0}_{-1}\beta$

Then, $m = m' + 4a + 0b$ (ii) $n = n' + 2a - b$

Solve for a and b

Where a is the number of ${}^{4}_{2}He$ emitted and b is the number of ${}^{0}_{-1}\beta$ emitted ${}^{A}_{Z}X \rightarrow {}^{A_1}_{Z_1}Y + x\,{}^{4}_{2}\alpha + y\beta$

x : no of α-particles emitted y : not of β-particles emitted

$X_Z^A \rightarrow Y_{Z^1}^{A^1} + xHe_2^4 + ye_{-1}^0;\quad A = A^1 + 4x;\qquad x = \frac{A - A^1}{4}$

$Z = Z^1 + 2x - y \qquad y = Z^1 - Z + 2x \quad y = \left(\frac{A - A^1}{2}\right) - \left(Z - Z^1\right)$

eg: $U_{92}^{238} \rightarrow Pb_{82}^{206} + xHe_2^4 + ye_{-1}^0;\qquad x = \left(\frac{A - A^1}{4}\right) = \left(\frac{238 - 206}{4}\right) = 8\alpha - \text{particles}$

$$y = \left(\frac{A - A^1}{2}\right) - \left(Z - Z^1\right) = \left(\frac{238 - 206}{2}\right) - \left(92 - 82\right) = 16 - 10 = 6\beta - \text{particles}$$

7. GROUP DISPLACEMENT LAW

Law is given by Fajan, Soddy and Russel.

The law is basically given for the position of daughter elements in periodic table.

(a) α-particle emission: When an alpha particle emits the position of daughter element is two places to the left in the periodic table from parent element.

Example: $^{214}_{84}Po \rightarrow {}^{210}_{82}Pb \qquad {}^{213}_{83}Bi \rightarrow {}^{209}_{81}Tl$

16	14	15	13
(VIA)	(IVA)	(VA)	(IIIA)

(b) β-particle emission: When an β-particle emits the position of daughter element is one place right in the periodic table from parent element.

Examples: $^{27}_{12}Mg \rightarrow {}^{27}_{13}Al + {}^{0}_{-1}e \qquad {}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e$

(IIA)	(IIIA)	(IV A)	(VA)

8. RADIOACTIVE ISOTOPES

The isotopes of elements which spontaneously decay by emitting radioactivity radiations are defined as radioactive isotopes.

They are two types.

(a) Natural radioactive isotopes (b) Artificial radioactive isotopes

(b) Natural radioactive isotopes: Those radioactive isotopes which exist naturally are known as natural radioactive isotopes. e.g. Th^{232}, Pu^{240} etc.

(c) Artificial radioactive isotopes: Those isotopes, which are prepared artificially by bombarding fundamental particles like α, β, γ, p, n etc. no matter, are known as artificial isotopes.

8.1 Uses of Radioactive Isotopes

(a) In Medicine:

- **(i)** For testing blood chromium-51
- **(ii)** For testing blood circulation-Sodium-24
- **(iii)** For detecting brain tumor-Radio mercury-203
- **(iv)** For detecting fault in thyroid gland-Radio iodine-131
- **(v)** For cancer-Cobalt-60
- **(vi)** For blood-Gold_189
- **(vii)** For skin diseases-Phosphorous-31

(b) In Archaeology:

- **(i)** For determining age of archaeological sample (Carbon dating) - C^{14}

(ii) For determining age of meteorites - K^{40}

(iii) For determining age of earth and isotopes

(c) In Agriculture:

(i) For protecting potato crop from earthworm-Cobalt-60

(ii) For artificial rains AgI

(iii) As fertilizers-Phosphorous-32

(d) As Tracers:

Very small quantity of radio isotopes present in mixture is known as tracer. Tracer technique is used for studying biochemical reactions in trees and animals.

(e) In Industries:

(i) For detecting leakage in oil or water pipe lines.

(ii) For testing machine parts.

(f) In Research:

(i) In the study of carbon-nitrogen cycle.

(ii) For determining the age of planets.

8.2 Radioactive Dating

Radioactive dating also called carbon dating is used to estimate the age of organic samples. The technique is based on the β-activity of the ratio-isotope C^{14}. $\quad {}^{14}_{6}C \rightarrow {}^{14}_{7}N + \beta^- + \bar{\nu}$

High energy particles from outer space, called cosmic rays, induce nuclear reactions in the upper atmosphere and create carbon-14. The carbon dioxide molecule of the earth's atmosphere has a constant ratio ($\approx 1.3 \times 10^{-12}$) of C^{14} and C^{12} isotope. All living organisms also show the same the same ratio as they continuously exchange CO_2 with their surroundings. However, after its death, an organism can no longer absorb CO_2 and the ratio C^{14}/C^{12} decrease due to the β-decay of C^{14} Thus by measuring the β-activity per unit mass, it is possible to estimate the age of a material.

Using such techniques samples of wood, sample of wood, charcoal, bone, etc., have been identified to have lived from 1000 to 25000 years ago.

9. PROPERTIES AND USES OF NUCLEAR RADIATION

9.1 Alpha Ray

(a) It is a stream of alpha particles, each particle containing two protons and two neutrons. An alpha particle is nothing but a helium nucleus.

(b) Being made of positively charged particles, alpha ray can be deflected by an electric field as well as by a magnetic field.

(c) Its penetrating power is low. Even in air, its intensity falls down to very small values within a few centimeters.

(d) Alpha rays coming from radioactive materials travel at large speeds of the order of $10^6\ ms^{-1}$

(e) All the alpha particles coming from a particular decay scheme have the same energy.

(f) Alpha ray produces scintillation (flashes of light) when it strikes certain fluorescent materials, such as barium platinocyanide.

(g) It causes ionization in gases.

9.2 Beta Ray

(a) It is a stream of electrons coming from the nuclei. Thus, the properties of beta ray, cathode ray, thermions, photoelectrons, etc., are all identical except for their origin. Beta particles are created at the time of nuclear transformation, whereas, in cathode ray, thermions, etc., the electrons are already present and get ejected.

(b) Being made of negatively charged particles, beta ray can be deflected by an electric field as well as by a magnetic field.

(c) Its penetrating power is greater than that of alpha ray. Typically, it can travel several meters in air before its intensity drops to small values.

(d) The ionizing power is less than that of alpha rays.

(e) Beta ray also produces scintillation in fluorescent materials, but the scintillation is weak.

(f) The energy of the beta particles coming from the same decay scheme are not equal. This is because the available energy is shared by antineutrinos. The energy of beta particles thus varies between zero and a maximum.

9.3 Beta-Plus Ray

Beta-plus ray has all the properties of beta ray, except that it is made of positively charged particles.

9.4 Gamma Ray

(a) Gamma ray is an electromagnetic radiation of short wavelength. Its wavelength is, in general, smaller than X-rays. Many of its properties are the same as those of X-rays.

(b) Being charge less, it is not deflected by electric or magnetic field.

(c) It has the least ionizing power and the largest penetrating power among different types of nuclear radiation.

(d) All the photons coming from a particular gamma decay scheme have the same energy.

(e) Being an electromagnetic wave, gamma ray travels in vacuum with the velocity c.

10. NUCLEAR FISSION

It was first observed by German Scientist Otto Hahn and Fritz Strassmann in 1983 in nuclear fission heavy nucleus splits into two smaller nuclei with liberation of energy. When uranium with Z=92 is bombarded with neutrons, it splits into two fragments namely barium (Z=56) and krypton (Z=36) and a large amount of energy is released which appears due to decrease in the mass. The reaction is represented as ${}^{238}_{92}U + {}^{1}_{0}n \rightarrow {}^{138}_{56}Ba + {}^{88}_{36}Kr + 3{}^{1}_{0}n + \text{energy}$

The disintegration process in which heavy nucleus after capturing a neutron splits up into nuclei of nearly equal mass is called nuclear fission.

Energy released in nuclear fission: The amount of energy released in nuclear fission may be obtained by the method of mass defect. For example consider the fission of U^{235} (Z=92) into Ba^{141} (Z=56) and Kr^{92} (Z=36) by slow neutrons. The reaction is given by

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n + Q$$

Let us estimate the actual masses before and after the fission reactions.

Actual mass before fission reaction

Mass of uranium nucleus 235.124 a. m. u

Mass of neutron 1.009 a. m. u

∴ Total mass 236.133 a. m. u

Actual mass after the fission reaction

Mass of barium nucleus 140.958 a. m. u

Mass of krypton nucleus 91.926 a. m. u

Mass of three neutrons 3.026 a. m. u

∴ Total mass 235.910 a. m. u

Now mass decrease during nuclear reaction = 236.133-235.910=0.223 a. m. u.

∴ Corresponding energy release $=0.223\times 931$ =200 MeV

If we calculate the energy produced by one gm of uranium its comes out to be

2.28×10^4 k. w. h. = 22.8 M watt.

This shows that 1 kg of U^{236} would give power of 1 M watt for more than two years.

PLANCESS CONCEPTS

The drooping of the binding energy curve at high mass numbers tells us that nucleons are more tightly bounded when they are assembled into two middle-mass nuclei rather than a single high-mass nucleus. In other words, energy can be released in the nuclear fission.

Shrikant Nagori (JEE 2009, AIR 30)

10.1 Chain Reaction

A nuclear chain reaction occurs when one nuclear reaction causes an average of one or more nuclear reactions, thus leading to a self-propagation series of these reactions. The specific nuclear chain reaction releases several million times more energy per reaction that any chemical reaction.

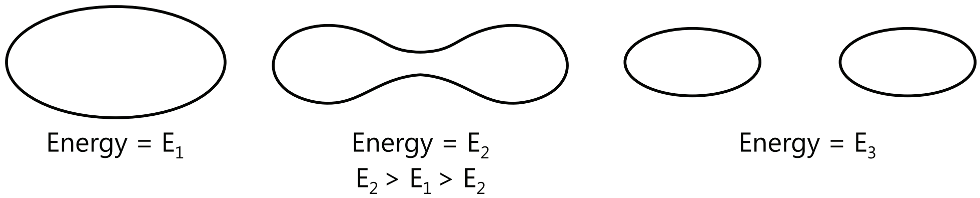


Figure 25.7

10.1.1 Fission Chain Reaction

Fission chain reaction occurs because of interactions between neutrons and fissile isotopes (such as ^{235}U). The chain reaction requires both the release of neutrons from fissile isotopes undergoing nuclear fission and the subsequent absorption of some of these neutrons in nuclear fission, a few

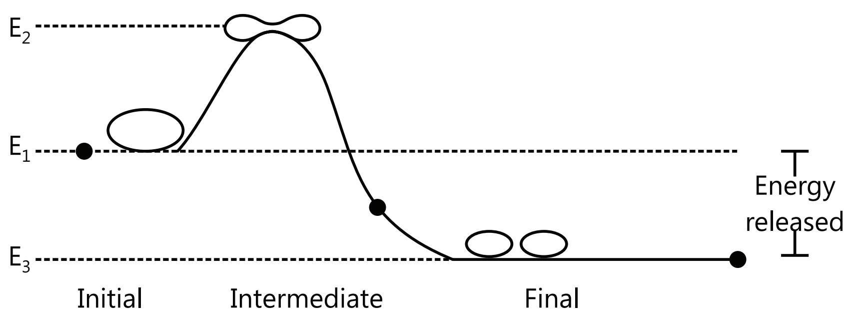


Figure 25.8

neutrons (the exact number depends on several factors) are ejected from the reaction. These free neutrons will then interact with the surrounding medium, and if more fissile fuel is present, some may be absorbed and cause more fission. Thus, the cycle repeats to give a reaction that is self-sustaining.

10.2 Nuclear Reactor

A nuclear reactor is a device to initiate and control a sustained nuclear chain reaction. Nuclear reactors are used at nuclear power plants for electricity generation and in propulsion of ships. Heat from nuclear fission is passed to a working fluid (water or gas), which runs through turbines. These either drive a ship's propellers or turn electrical generators. Nuclear generated steam in principle can be used for industrial process heat or for district heating. Let us see the working of a typical Uranium nuclear reactor. The volume in the core is filled with low-Z material like, D_2O graphite, beryllium etc. This material is called moderator.

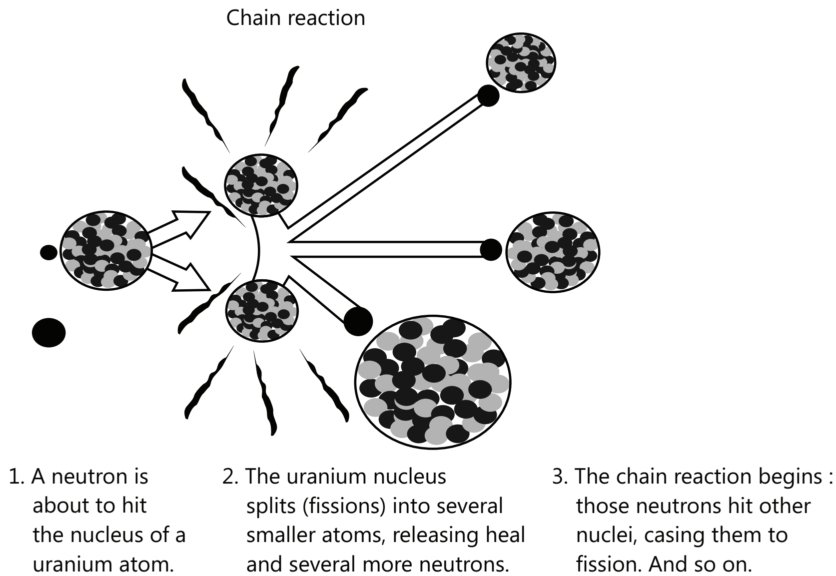


Figure 25.9

When fission takes place in a uranium rod, most of the fast neutrons produced escape from the rod and enter into the moderator. These neutrons make collisions with the particles of the moderator and thus slow down. About 25 collisions with deuteron (present in heavy water) or 100 collisions with carbon or beryllium are sufficient to slow down a neutron from 2 MeV to thermal energies. The distances between the rods are adjusted in such a way that a neutron coming from one rod is generally slowed down to thermal energies before entering the other rod. This eliminates the possibility of a neutron being absorbed by U^{238} in 1-100 eV regions. The geometry of the core is such that out of the average 2.5 neutrons produced per fission, 1 neutron is used to trigger the next fission and the remaining are lost without triggering any fission. The reaction is then sustained at a constant rate. If the rate of the loss of neutrons is decreased further, the fission rate will keep on increasing which may lead to explosion. If the rate of loss of neutrons is increased, the rate of fission will keep on decreasing and ultimately the chain reaction will stop. The finer control of fission rate is made by the control rods which are made of cadmium and are inserted up to a certain depth in the moderator. Cadmium is a very good neutron absorber. If the stage is set for stable chain reaction and the cadmium rods are pushed into the moderator, the reactor will be shut off. Pulling the cadmium rods out will start the reactor.

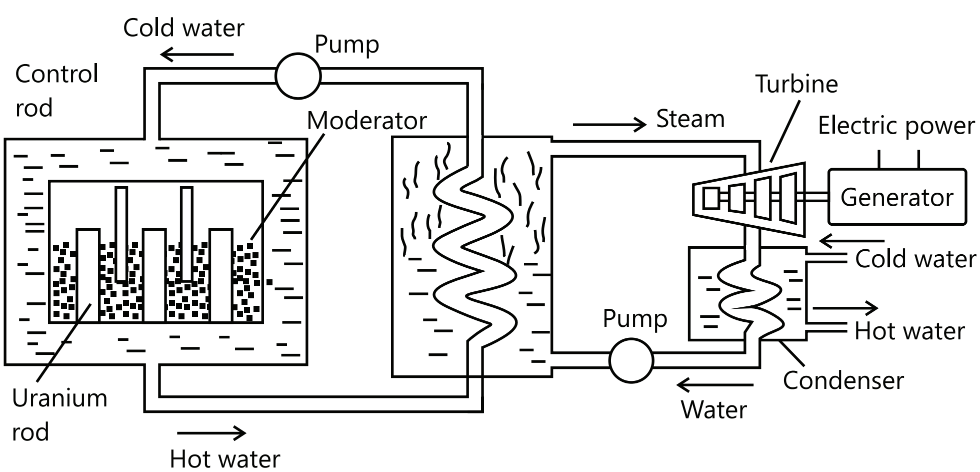


Figure 25.10

10.3 Uranium Fission Reactor

The most attractive bid, from a practical point of view, to achieve energy from nuclear fission is to use $^{236}_{92}U$ as the fission material. This nuclide is highly fissionable and hence is not found in nature. Natural uranium contains about 99.3% of $^{238}_{92}U$ and 0.7% of $^{235}_{92}U$. The technique is to hit a uranium sample by slow-moving neutrons (kinetic energy≈0.04eV, also called thermal neutrons). A $^{235}_{92}U$ nucleus has large probability of absorbing a slow neutron and forming $^{236}_{92}U$ nucleus. This nucleus then fissions into two parts. A variety of combinations of the middle-weight nuclei may be formed due to the fission. For example, one may have $^{236}_{92}U \rightarrow {}^{137}_{53}I + {}^{97}_{39}Y + 2n$,

And a number of the other combination $^{236}_{92}U \rightarrow {}^{140}_{56}Ba + {}^{94}_{36}Kr + 2n$

10.4 Breeder Reactors

Although fission generates large amount of energy and the world is heavily depending on fission for its energy requirement, uranium resources are also limited. The following Table shows that fission can easily take place with ^{240}Pu besides ^{236}U. But ^{239}Pu is not a naturally occurring isotope. However, ^{238}U can capture a neutron to produce ^{239}Pu which can be used as a nuclear fuel.

$$^{238}_{92}U + n \rightarrow {}^{239}_{92}U \xrightarrow{\beta} {}^{239}_{93}Np \xrightarrow{\beta} {}^{239}_{94}Pu$$

Suppose, used uranium rods, which contain only ^{238}U, are kept in or around a uranium-reactor core. Also suppose, the geometry is such that out of the average 2.5 neutrons produced in fission, one neutron is absorbed by a ^{238}U nucleus in these rods resulting in ^{239}Pu. Then we produce as much nucleus in these rods resulting in ^{239}Pu. Then we produce as much nuclear fuel in the form of ^{239}Pu as we consume in the form of ^{235}U. If more than one neutron can be absorbed by these ^{238}U rods per fission then we produce more fuel than what we consume. Thus, apart from nuclear energy, these reactors give us fresh nuclear fuel which often exceeds the nuclear fuel used. Such a reactor is called a breeder reactor.

11. NUCLEAR FUSION

Binding energy vs. Mass Number a graph shows that when lighter nuclei with A<20 combine to form bigger nuclei binding energy per nucleon increases. The total binding energy of the product is less than total binding energy of reactants resulting in release of energy. This process of combining of two lighter nuclei into bigger one is known as nuclear fusion. i.e.

$$^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{4}_{2}He + 24\,MeV$$

The following points deserve particular attention concerning nuclear fusion.

(a) The energy 21.62 MeV released in one fusion event is much smaller than about 200 MeV released in one fission event. This does not mean that fusion is a weaker source of energy than fission. If we compare the energy released per unit mass, we find that one fusion event is accompanied by a release of $\frac{21.62\ MeV}{6\,amu}$ or 3.6 MeV Per amu as against $\frac{200\ MeV}{235}$ or 0.85 MeV per amu released in one fission.

(b) For fusion, positively charged nuclei have to come very close to each other. This requires a very high energy to be provided to the fusing nuclei to enable them to overcome the strong electrostatic repulsion between them. Calculations show that the necessary energy can be provided by raising the temperature to about 10^8

K. Such high temperature can be produced by first inducing a fission event. A fusion reaction is therefore also called a thermonuclear reaction. This is the basic hydrogen bomb.

(c) Unlike the highly radioactive fission fragments, the end product of the fusion of hydrogen nuclei is safe, non-radioactive helium.

(d) Unfortunately a sustained and controllable fusion reactor that can deliver a net power output is not yet a reality. A great deal of effort is currently under way to resolve various difficulties in the development of a successful device. Nevertheless controlled fusion is regarded as the ultimate energy source because of the abundant availability of its main fuel: water.

PLANCESS CONCEPTS

The drooping of the binding energy curve at low mass number tells us that energy will be released if two nuclei of small mass numbers combine to form a single middle-mass nucleus. This is nuclear fusion.

Ankit Rathore (JEE Advanced 2013, AIR 158)

Illustration 11: In the nuclear fusion reaction: ${}_1^2H + {}_2^4He \Rightarrow 2\,{}_1^3H$ in a nuclear reactor, of 200 MW rating. If the energy from above reaction is used with a 25% efficiency in the reactor, how many grams of deuterium will be needed per day? (The masses of ${}_1^3H$ and ${}_2^4He$ are 2.0141 and 4.0026 amu respectively) **(JEE ADVANCED)**

Sol: The energy absorbed during the nuclear fusion reaction is calculated using Q value equation i.e. $Q = -mc^2 = -m \times (931)$ MeV. The number of deuterium atoms required during this reaction is obtained by

$$N = \frac{\text{Power Required}}{\text{Efficiency} \times (\text{energy released from one fusion reaction})}.$$

Let us first calculate the Q value of nuclear function, $Q = -mc^2 = -m \times (931)$ MeV

$Q = (2 \times 2.0141 - 4.0026) \times 931\text{MeV} = 23.834\text{MeV} = 23.834 \times 10^6$ eV. Now efficiency of reactor is 25%

So effective energy used $= 0.25 \times 23.834 \times 10^6 \times 1.6 \times 10^{-19}\text{J} = 9.534 \times 10^{-13}\text{J}$

Now $9.534 \times 10^{-13}\text{J}$ energy is released by fusion of 2 deuterium.

Requirement is $200\text{MW} = 200 \times 10^6$ J/s per second $= 200 \times 10^6 \times 86400$ J/s for 1 days.

No. of deuterium nuclei required $= \dfrac{200 \times 10^6 \times 86400}{\dfrac{9.534}{2} \times 10^{-13}} = 3.624 \times 10^{25}$

Number of deuterium nuclei $= \dfrac{g}{2} \times 6 \times 10^{23}$; $\quad 3.624 \times 10^{25} = \dfrac{g}{2} \times 6 \times 10^{23}$

$$g = \frac{2 \times 3.624 \times 10^{25}}{6 \times 10^{23}} = 120.83 \text{ gm/day}.$$

11.1 Thermonuclear Fusion

To generate useful amount of energy, nuclear fusion must occur in bulk matter. The best hope for bringing this about is to raise the temperature of the material until the particles have enough energy-due to their thermal motions alone-to penetrate the Coulomb barrier. We call this process thermonuclear fusion.

In thermonuclear studies, temperatures are reported in terms of the kinetic energy K of interacting particles via the relation K=3kT/2

In which K is the average kinetic energy of the interacting particles, k is the Boltzmann constant, and the temperature T is in kelvins. Thus, rather than saying,

Fusion in Sun: Among the celestial bodies in which energy is produced, the sun is relatively cooler. There are stars with temperature around $10^8 K$ inside. In sun and other stars, where the temperature is less than or around $10^7 K$, fusion takes place dominantly by proton-proton cycle as follows:

$$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu$$

$$^2H + {}^1H \rightarrow {}^3He + \gamma$$

$$^3He + {}^3He \rightarrow {}^4He + 2{}^1H$$

$$4{}^1H \rightarrow {}^4He + 2e^+ + 2\nu + 2\gamma$$

Note that the first two reactions should occur twice to produce two ^3He nuclei and initiate the third reaction. As a result of this cycle, effectively, four hydrogen nuclei combine to form a helium nucleus. About 26.7 MeV energy is released in the cycle. Thus, hydrogen is fuel which 'burns' into helium to release energy. The sun is estimated to have been radiating energy for the last 4.5×10^9 years and will continue to do so till all the hydrogen in it is used up. It is estimated that the present store of hydrogen in the sun is sufficient for the next 5×10^9 years.

11.2 Lawson Criterion

J. D. Lawson showed that in order to get an energy output greater than the energy input, a fusion reactor should achieve $n\tau > 10^{14} s\,cm^{-3}$

Where n is the density of the interacting particles and τ is the confinement time. The quantity $n\tau$ in $s\,cm^{-3}$ is called Lawson number.

The ratio of the energy output to the energy input is known as Q of the fusion machine. For a viable fusion machine, Q should be greater than 1.

11.3 Tokamak Design

In one of the method receiving serious attention, one uses the so-called Tokamak design. The deuterium plasma is contained in a toroidal region by specially designed magnetic field. The directions and magnitudes of the magnetic field are so managed in the toroidal space that whenever a charged plasma particle attempts to go out, the $q\vec{v} \times \vec{B}$ force tends to push it back into the toroidal volume. It is a difficult task to design a magnetic field which will push the particles moving in random directions with random speeds into a specified volume, but it is possible and has been done. The plasma is, therefore, confined by the magnetic field. Such confinement has been achieved for short durations (≈few microseconds) in which some fusion occur. Fusion thus proceeds in bursts or pulse. The heating is accomplished by passing high frequency oscillating current through the plasma gas. A schematic design is shown in Fig. 25.11.

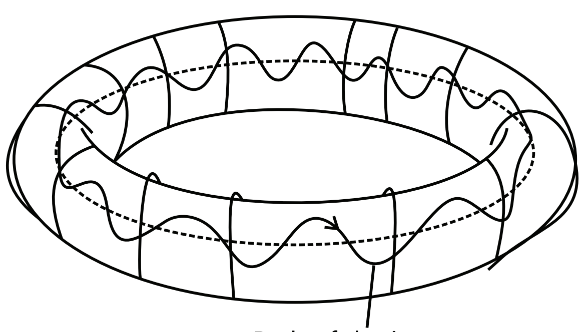


Figure 25.11

11.4 Inertial Confinement

In another method known as inertial confinement, laser beams are used to confine the plasma. A small solid pellet is made which contains deuterium and tritium. Intense laser beams are directed on the pellet from many directions distributed over all sides. The laser first vaporizes the pellet converting it into plasma and then compresses it from all directions because of the large pressure exerted. The density increases by 10^3 to 10^4 times the initial density and the temperature rises to high values. The fusion occurs in this period. The α-particles (He Nuclei) generated by the fusion are also forced to remain inside the plasma. Their kinetic energy is lost into the plasma itself contributing further rise in temperature. Again the lasers are operated in pulses of short duration.

The research in fusion energy is going on. Fusion is the definite and ultimate answer to our energy problems. The 'fuel' used for fusion on earth is deuterium which is available in natural water (0.03%). And with oceans as the almost unlimited source of water, we can be sure of fuel supply for thousands of years. Secondly, fusion reactions are neat and clean. Radioactive radiation accompanying fission reactors will not be there with fusion reactors.

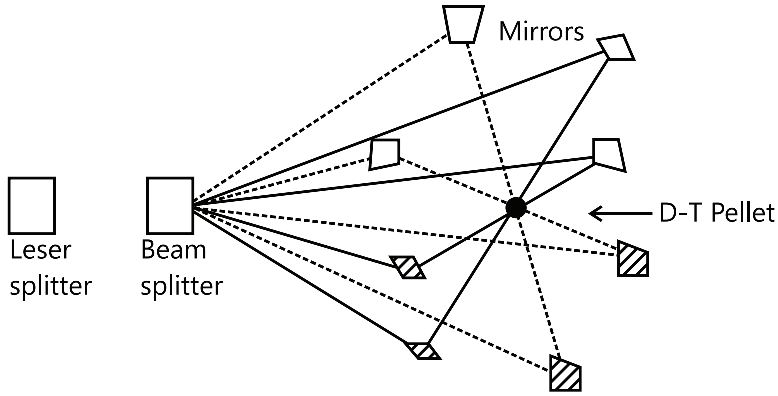


Figure 25.12

11.5 Nuclear Holocaust

Nuclear holocaust refers to a possible nearly complete annihilation of human civilization by nuclear warfare. Under such a scenario, all or most of the Earth is made uninhabitable by nuclear weapons in future word wars.

PROBLEM-SOLVING TACTICS

1. Problems from this section do not need any mathematically difficult involvement. One only needs to focus on exponential functions and its properties.
2. Questions related to energy can easily be solved by thinking.
3. For e.g. consider energy as money and think of it in terms of loss and gain, But overall total money is conserved i.e. total energy is conserved; only it is exchanged. One must not be worried with the relation $E = mc^2$ at this stage and just consider mass and energy as equivalent. So, if more clearly stated this equivalent quantity is conserved in every process.
4. Mostly, questions related to basic understanding of Nuclear force are asked rather than which involve complicated calculations.
5. Statistics must always be kept in mind while solving a problem of radioactive decay.

FORMULAE SHEET

1. After n half-lives

(a) Number of nuclei left $= N_0\left(\frac{1}{2}\right)^n$

(b) Fraction of nuclei left $= \left(\frac{1}{2}\right)^n$ and

(c) Percentage of nuclei left $= 100\left(\frac{1}{2}\right)^2$

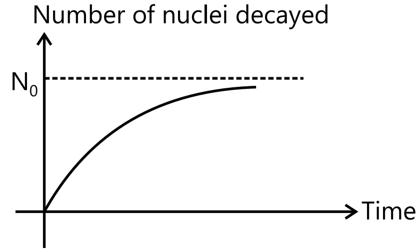


Figure 25.13

2. Number of nuclei decayed after time $t = N_0 - N$

$= N_0 - N_0e^{-\lambda t} = N_0(1 - e^{-\lambda t})$

The corresponding graph is as shown in Fig. 25.13.

3. Probability of a nucleus for survival of time t,

$$P(\text{survival}) = \frac{N}{N_0} = \frac{N_0e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

The corresponding graph is shown in Fig. 25.14.

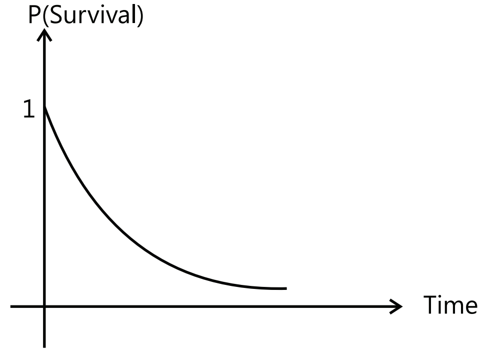


Figure 25.14

4. Probability of a nucleus to disintegrate in time t is,

$P(\text{disintergration}) = 1 - P(\text{survival}) = 1 - e^{-\lambda t}$

The corresponding graph is as shown.

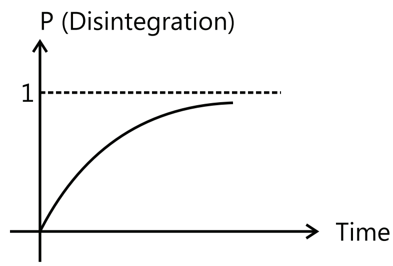


Figure 25.15

5. Half-life and mean life are related to each other by the relation, $t_{1/2} = 0.693\,t_{av}$ or $t_{av} = 1.44\,t_{1/2}$

6. As we said in point number (2), number of nuclei decayed in time t are $N_0(1 - e^{-\lambda t})$. This expression involves power of e. So to avoid it we can use, $\Delta N = \lambda N \Delta t$ where, ΔN are the number of nuclei decayed in time Δt, at the instant when total number of nuclei are N. But this can be applied only when $\Delta t << t_{1/2}$.

7. In same interval of time, equal percentage (or fraction) of nuclei are decayed (or left un decayed).

1. $R = R_0A^{1/3}$

2. $\Delta E_{be} = \sum(mc^2) - Mc^2$ (binding energy)

3. $\Delta E_{ben} = \frac{\Delta E_{be}}{A}$ (binding energy per nucleon.)

4. $\frac{dN}{N} = -\lambda dt$

5. $N = N_0e^{-\lambda t}$ (radioactive delay),

6. $\tau = \frac{1}{\lambda}$

7. $T_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2.$

Solved Examples

JEE Main/Boards

Example 1: Sun radiates energy in all direction. The average energy received at earth is 1.4 kW/m^2. The average distance between the earth and the sun is 1.5×10^{11} m. If this energy is released by conversion of mass into energy, then the mass lost per day by sun is approximately (use 1 day = 86400 sec)

Sol: The sun produces energy by fusion reaction of hydrogen atoms. The loss in mass of sun is calculated using $\Delta m = \frac{\Delta E}{c^2}$ where ΔE is the amount of energy released during the day.

The sun radiates energy in all directions in a sphere. At a distance R, the energy received per unit area per second is 1.4 KJ (given). Therefore the energy released in area $4\pi R^2$ per sec is $1400 \times 4\pi R^2$ J the energy released per day = $1400 \times 4\pi R^2 \times 86400$ J

Where $R = 1.5 \times 10^{11}$m, thus

$\Delta E = 1400 \times 4 \times 3.14 \times (1.5 \times 10^{11})^2 \times 86400$

The equivalent mass is $\Delta m = \Delta E / C^2$

$$\Delta m = \frac{1400 \times 4 \times 3.14 \times (1.5 \times 10^{11})^2 \times 86400}{9 \times 10^{16}}$$

$$\Delta m = 3.8 \times 10^{14} kg$$

Example 2: The energy released per fission of uranium (U^{235}) is about 200 MeV. A reactor using U^{235} as fuel is producing 1000 kilowatts power. The number of U^{235} nuclei undergoing fission per sec is, approximately

Sol: The number of Uranium nuclei undergoing fission is obtained by

$$N = \frac{\text{Energy produced}}{\text{Energy realesed during one fission}}.$$

The energy produced per second is

$$= 1000 \times 10^3 J = \frac{10^6}{1.6 \times 10^{-19}} eV = 6.25 \times 10^{24} eV$$

The number of fissions should be,

$$N = \frac{6.25 \times 10^{24}}{200 \times 10^6} = 3.125 \times 10^{16}$$

Example 3: A star initially has 10^{40} deuterons. It produces energy via the processes ${}_1^2H + {}_1^2H \rightarrow {}_1^3H + {}_1^1p$ and ${}_1^2H + {}_1^3H \rightarrow {}_2^4He + {}_0^1n$. If the average power radiated by the star is $10^{16}W$, in how much time the deuteron supply of the star get exhausted?

Sol: The star produces the energy by fusing deuterium and tritium into the helium, and releasing proton and neutron. Thus the mass defect is easily obtained per one such conversion. The time in which the deuterium supply is exhausted is found by

$$t = \frac{\text{Number of deutrons present in star}}{\text{number of deutrons used per sec}}$$

Here number of deuterons used per second

$$n = \frac{N \times \text{Power}}{\text{energy released per reaction}}$$

where N is the deuteron used per reaction.

Adding the two processes, we get

$$3\ {}_1^2H \rightarrow {}_2^4He + {}_1^1p + {}_0^1n$$

Mass defect $= 3 \times 2.014 - 4.001 - 1.007 - 1.008$

$= 0.026$ amu $= 0.026 \times 931 MeV$

Power of the star $= 10^{16} W = 10^{16} J/s$

Number of deuterons used in one second

$$= \frac{10^{16}}{0.026 \times 931 \times 10^6 \times 1.6 \times 10^{-19}} \times 3 = 7.75 \times 10^{27}$$

Now the time in which the deuterons supply exhausted

$$t = \frac{\text{Number of deutrons present in star}}{\text{number of deutrons used per sec}}$$

$$= \frac{10^{40}}{7.75 \times 10^{27}} = 1.3 \times 10^{12} \text{ sec}$$

Example 4: The mean lives of a radioactive material for α and β radiations are 1620 years and 520 years respectively. The material decays simultaneously for α and β radiation. The time after which one fourth of the material remains un-decayed is

Sol: The mean life of the radioactive material for simultaneous α and β decay is $\tau = \frac{\tau_\alpha \tau_\beta}{\tau_\alpha + \tau_\beta}$. The time in which the 3/4th of material decayed is $t = \frac{2.303}{\lambda}\log_{10} 4$.

We know that $\lambda \propto \frac{1}{\tau}$.

$$\tau = \frac{\tau_\alpha \tau_\beta}{\tau_\alpha + \tau_\beta} = \frac{1620 \times 520}{1620 + 520} = 394 \text{ years}$$

time of decay $t = \tau \times 2.303 \log_{10} \frac{N_0}{N}$

$$t = 394 \times 2.303\ \log_{10}(4) = 394 \times 2.303 \times 0.602$$

$$t = 546 \text{ years}$$

Example 5: A sample contains two substances P and Q, each of mass 10^{-2} kg. The ratio of their atomic weights is 1 : 2 and their half-lives are 4 s and 8 s respectively. The masses of P and Q that remain after 16s will respectively be-

Sol: The mass of radioactive element decaying after time t is given by $N = \frac{N_0}{2^n}$; $M = \frac{M_0}{2^n}$ where M is the mass (in kg) of the radioactive element. As half-lives are given, value of n is found as, number of half-life

$n = \frac{t}{t_{1/2}}$.

$\because N = \frac{N_0}{2^n}; \quad M = \frac{M_0}{2^n}; \qquad \text{for } P, n = \frac{16}{4} = 4$

$\therefore M_P = \frac{10^{-2}}{16} = 6.25 \times 10^{-4} Kg$

$\text{for } Q,\ n = \frac{16}{8} = 2 \therefore M_Q = \frac{10^{-2}}{2^2} = 2.5 \times 10^{-3}$

Example 6: There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half-life of neutrons is 700 sec, what fraction of neutrons will decay before they travel 10 m? Given mass of neutron $= 1.675 \times 10^{-27}$ kg.

Sol: The fraction of neutrons decayed in the distance of 10 m is calculated by $\frac{\Delta N}{N} = \frac{0.693}{T_{1/2}} \Delta t$. Here $T_{1/2}$ is the half-life of the neutron and Δt is the time taken to cover distance of 10 m.

From the given kinetic energy of the neutrons we first calculate their velocity, thus

$$\frac{1}{2} mu^2 = 0.0327 \times 1.6 \times 10^{-19}$$

$$\therefore u^2 = \frac{2 \times 0.0327 \times 1.6 \times 10^{-19}}{1.675 \times 10^{-27}}$$

$$= 625 \times 10^4 \text{ or } u = 2500 m/s$$

With this speed, the time taken by the neutrons to travel a distance of 10 m is,

$$t = \frac{10}{2500} = 4 \times 10^{-3} s$$

The fraction of neutrons decayed in time Δt second is,

$$\frac{\Delta N}{N} = \lambda \Delta t \text{ also,} \qquad \lambda = \frac{0.693}{T_{1/2}}$$

$$\frac{\Delta N}{N} = \frac{0.693}{T_{1/2}} \Delta t = \frac{0.693}{700} \times \left(4 \times 10^{-3}\right) = 3.96 \times 10^{-6}$$

Example 7: A radioactive sample has $6.0 x 10^{18}$ active nuclei at a certain instant. How many of these nuclei will still be in the same active state after two half-lives?

Sol: The number of radioactive nuclei remaining after n half-lives is calculated as $N = \frac{N_o}{2^n}$ where No is the number of nuclei present initially.

In one half-life the number of active nuclei reduces to half the original. Thus, in two half-lives the number is reduced to $\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)$ of the original number. The number of remaining active nuclei is, therefore,

$$6.0 x 10^{18}\ x \left(\frac{1}{4}\right) = 1.5 x 10^{18}$$

Example 8: The half-life of radium is about 1600 years. In how much time will 1 g of radium (a) reduce to 100 mg (b) lose 100 mg ?

Sol: The weight of radioactive nuclei remaining after n half-lives is calculated as $W = \frac{W_o}{2^n}$ where Wo is the mass present originally. Here $n = \frac{t}{T}$ where T is the half-life.

$$W = \frac{W_o}{2^{(t/T)}} \text{ where } T = 1600 \text{ yr.}$$

(a) W_o = 1gm, W = 0.1 gm. $2^{(t/T)} = 1/0.1 = 10$

Or $\frac{t}{T} \log 2 = 1$ or $t = \frac{T}{\log 2} = \frac{1600}{0.301} = 5{,}333 yr$

(b) $W_o = 1g, W = 1 - 0.1 = 0.9 gm$

$$2^{(t/T)} = \frac{1}{0.9} \text{ or } \frac{t}{T} \log 2 = 0.0458$$

$$t = \frac{0.0458 x 1600}{0.301} = 243.3 yr$$

Example 9: The activity of a radioactive sample falls from 600/s to 500/s in 40 minutes. Calculate its half-life.

Sol: The activity of any radioactive element is found by $A = A_0 e^{-\lambda t}$. The decay constant is found easily by above equation. The half-life is obtained by $T_{1/2} = \frac{\ln 2}{\lambda}$.

We have $A = A_0 e^{-\lambda t}$

or, $\quad 500 s^{-1} = (600 s^{-1}) e^{-\lambda t} \text{ or}, e^{-\lambda t} = \frac{5}{6}$

or, $\quad \lambda t = \ln(6/5) \text{or}, \lambda = \frac{\ln(6/5)}{t} = \frac{\ln(6/5)}{40 min}$

$\because T_{1/2} = \frac{\ln 2}{\lambda}, \quad \therefore$ The half-life is

$$T_{1/2} = \frac{\ln 2}{\ln(6/5)} x 40 min = 152 min.$$

JEE Advanced/Boards

Example 1: The disintegration rate of a certain radioactive sample initially is 4750 disintegrations per minute. Five minutes later the rate becomes 2700 disintegrations per minute. Calculate the half-life of the sample.

Sol: The decay constant is obtained using $\lambda = \frac{1}{t}\log_e\left(\frac{A_0}{A}\right)$ where A_o is the initial activity and A is the activity at the time t. As decay constant is obtained we can easily calculate the half-life of the sample using $T_{1/2} = \frac{\log_e 2}{\lambda}$.

Let N_0 is initial no of nuclei and N is no. of nuclei after five minutes

Initially $-\left(\frac{dN}{dt}\right)_0 = \lambda N_0$

Five minutes later, $-\left(\frac{dN}{dt}\right)_t = \lambda N$

$$\therefore \quad \frac{N_0}{N} = \left(\frac{dN}{dt}\right)_0 \Big/ \left(\frac{dN}{dt}\right)_t = \frac{4750}{2700} = 1.76$$

Also $N = N_0 e^{-\lambda t}$

$$\lambda = \frac{1}{t}\log_e\left(\frac{N_0}{N}\right) = \frac{2.3026}{5}\log_{10}(1.76)$$

= 0.11306 per min.

Further $T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.6931}{0.11306} = 6.13$ minutes.

Example 2: In the interior of the sun, a continuous process of 4 protons, fusing into a helium nucleus and pair of positron, is going on. Calculate

(a) The release of energy per process

(b) Rate of consumption of hydrogen to produce 1 MW power.

Given ${}_1H^1 = 1.007825$ a.m.u. (atom)

${}_2He^4 = 4.002603$ a.m.u. (atom)

$m_{e^+} = m_{e^-} = 5.5\text{x}10^{-4}$ a.m.u.

(Neglect the energy carried away by neutrons)

Sol: The energy produced in sun during one fusion reaction is $E = \Delta mc^2$ J $= \Delta m \times 931.5$ MeV. Take the ratio of the power required and the energy released from one reaction to get the number of reactions required per second.

(a) During fusion

(i) Initially $4\,{}_1^1H \rightarrow {}_2^4He + 2\,{}_{+1}^{\ 0}e$ and loses 2 bound electrons

$$\begin{bmatrix} 4\,{}_1H^1 \text{ has 4 bound electrons while} \\ {}_2He^4 \text{ has only 2 bound electrons} \end{bmatrix}$$

Energy released in fusion = $\Delta m \times 931.5$ MeV

$$= \left\{4\left[{}_1^1H\right] - 1\left[{}_2^4He\right] - 2\,{}_{+1}^{\ 0}e - 2\,{}_{-1}^{\ 0}e\right\} \times 931.5 \text{ MeV}$$

$$= 4(1.0078) - [4.0026 + 4 \times 0.0006] \times 931.5$$

MeV=24.685 MeV

(ii) Later the two positrons combine with 2 electrons to annihilate each other and release energy.

Energy release = $4(0.00055) \times 931\text{MeV} = 2.049$ MeV

$\therefore$ Total energy release per fusion = $24.685 + 2.049 = 26.734\,\text{MeV}$

(b) $26.735\text{MeV} = 4.277 \times 10^{-12}$ J

This energy corresponds to $4(1.007825)$ a.m.u.

i.e., 6.692×10^{-27} kg of ${}_1^1H$

1 MW power = 10^6 Js^{-1}

Mass of hydrogen required for producing energy of 10^6 J

$$= \frac{10^6 \times 6.692 \times 10^{-27}}{4.277 \times 10^{-12}} = 1.565 \times 10^{-9} \text{ kg}$$

$\therefore$ Rate of consumption of hydrogen required to produce 1 MW power $= 1.565 \times 10^{-9}\,\text{kgs}^{-1}$

Example 3: The element curium ${}_{96}^{248}Cm$ has a mean life of 10^{13} seconds. Its primary decay modes are spontaneous fission and α – decay, the former with a probability of 8% and the latter with probability of 92%. Each fission releases 200 MeV of energy. The masses involved in are as follows:

${}_{99}Cm^{248} = 248.07220$u

${}_{94}Pu^{244} = 244.064100$u

And ${}_2He^4 = 4.002603$u

Calculate the power output from a sample of 10^{20} Cm atoms. $\left(1\text{amu} = 931\frac{\text{MeV}}{c^2}\right)$

Sol: The energy released in each transformation is found by $E = \Delta m \times c^2$ J. As the probabilities of each fission is given the total energy Et released in respective transformation is Probability × E where E is the energy liberated during any one fission reaction. And the power liberated during the entire process is given by $P = \frac{E_T}{\tau}$ where ET is the total energy released during fission of all the molecules of the sample.

α - decay of Cm takes place as follows:

$${}_{96}Cm^{248} \rightarrow {}_{94}Pu^{244} + {}_{2}He^{4}$$

$\therefore$ Mass defect = Δm; $\Delta m = (M)_{cm} - \left[(M)_{pu} + M_\alpha\right]$

$$\Delta m = (248.07220) - [244.064100 + 4.002603]$$

$$\Delta m = 0.005517u$$

Energy released per $\alpha -$decay

$= (0.005517)(931)$ MeV $= 5.136$ MeV

Probability of spontaneous fission=8%

Probability of $\alpha -$decay = 92%

Energy released in each ${}^{248}_{96}Cm$ transformation

$= (0.08 \times 200 + 0.92 \times 5.136)$ MeV = 20.725 MeV

Energy released by 10^{20} atoms

= 20.725×10^{20} MeV

Mean life time = 10^{13} sec

$$\text{power} = \frac{20.725 \times 10^{20} \text{MeV}}{10^{13} \text{sec}}$$

$$= 20.725 \times 10^{7} \times \left(1.6 \times 10^{-13}\right) \frac{\text{joule}}{\text{sec}} = 3.316 \times 10^{-5} \text{watt.}$$

Example 4: In the chain analysis of a rock, the mass ratio of two radioactive isotopes is found to be 100:1. The mean lives of the two isotopes are 4×10^9 year and 2×10^9 year respectively. If it is assumed that at the time of formation of the rock, both isotopes were in equal proportion, calculate the age of the rock. Ratio of atomic weights of the two isotopes is 1.02:1.

$\left(\log_{10} 1.02 = 0.0086\right)$.

Sol: The number of the radioactive nuclei remaining at time t is given as $N_t = N_o e^{-\lambda t}$. Here the ratio of the masses are given. The ratio of number of atoms are given by $\frac{N_1}{N_2} = \frac{m_1}{m_2}\frac{M_2}{M_1}$. Find the value of t from the ratio $\frac{N_{t1}}{N_{t2}}$.

At the time of formation of the rock, both isotopes have the same number of nuclei N_0. Let λ_1 and λ_2 be the decay constants of the two isotopes. If N_1 and N_2 are the number of their nuclei after a time t, we have

$$N_1 = N_0 e^{\lambda_1 t} \text{ and } N_2 = N_0 e^{\lambda_2 t} \quad \frac{N_1}{N_2} = e^{(\lambda_1 - \lambda_2)t} \quad \text{... (i)}$$

Let the masses of the two isotopes at time t be m_1 and m_2 and let their respective atomic weights be M_1 and M_2. We have $m_1 = N_1 M_1$ and $m_2 = N_2 M_2$

$$\frac{N_1}{N_2} = \frac{m_1}{m_2}\frac{M_2}{M_1} \quad \text{... (ii)}$$

Substituting the value given in the problem, we get

$$\frac{N_1}{N_2} = \frac{100}{1} \times \frac{1}{1.02} = \frac{100}{1.02}$$

Let t_1 and t_2 be the mean lives of the two isotopes.

Then $t_1 = \frac{1}{\lambda_1}$ and $t_2 = \frac{1}{\lambda_2}$

Which gives $\lambda_1 - \lambda_2 = \frac{t_1 - t_2}{t_1 t_2} = \frac{2 \times 10^9 - 4 \times 10^9}{\left(2 \times 10^9\right) \times \left(4 \times 10^9\right)}$

$$= -0.25 \times 10^{-9}$$

Setting this value in Eqn. (i), we get

$$\frac{N_1}{N_2} = e^{\left(0.25 \times 10^{-9}\right)t} \Rightarrow t = \frac{1}{0.25 \times 10^{-9}} \log_e \frac{100}{1.02}$$

= 18.34×10^9 year

Example 5: A small quantity of solution containing ${}^{24}_{11}Na$ radioactive nuclei (half-life 15 hours) of activity 1.0 μ Ci is injected into the blood of a person. A sample of the blood of volume 1 cc taken after 5 hours showed an activity of 296 disintegrations per minute. Determine the total volume of blood in the body of the person. Assume that the radioactive solution mixed uniformly in the blood of the person.

(1 Curie =3.7× 10^{10} disintegration per second)

Sol: The activity of the radioactive nuclei is given by $A_o = \lambda N_0$ where λ is the decay constant of the radioactive nuclei. Find the number of radioactive nuclei N_0 present initially. Also find the number of nuclei in the sample of the blood initially. The ratio of these two gives the volume.

We know that $T_{1/2} = \frac{0.693}{\lambda}$ or

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{15 \times 3600} = 1.283 \times 10^{-5} \text{ / sec.} \quad \text{... (i)}$$

Now activity $A_o = \frac{dN}{dt} = \lambda N_o$ ... (ii)

Where A_o = 1 micro curie = $1 \times 3.7 \times 10^4$

= 3.7×10^4 disintegrations / sec

From equation (ii) we have

$3.7 \times 10^4 = 1.283 \times 10^{-5} \times N_0$

$N_0 = \frac{3.7 \times 10^4}{1.283 \times 10^{-5}} = 2.883 \times 10^9$

Let the number of radioactive nuclei present after 5 hours be N_1 in 1 cc sample of blood.

Then $\frac{dN}{dt} = \lambda N_1$ or $\frac{296}{60} = \frac{0.693}{15 \times 3600} N_1$

or $N_1 = \frac{296 \times 15 \times 3600}{60 \times 0.693} = 3.844 \times 10^5$

Let N_0' be the number of radioactive nuclei in per cc of sample, then

Then $N_0' = (2)^{t/T} \times N_1$

$N_0' = (2)^{5/15} \times N_1 = (2)^{1/3} \times 3.844 \times 10^5$

$= 1.269 \times 3.844 \times 10^5 \left[(2)^{1/3} = 1.269\right] = 4.878 \times 10^5$

Volume of blood $V = \frac{N_0}{N_0'} = \frac{2.883 \times 10^9}{4.878 \times 10^5}$

$= 0.5910 \times 10^4 \, cm^3 = 5.91$ litres.

$T = \frac{0.693}{\lambda} = \frac{0.693}{0.113} \min = 6.14 \min$

Example 6: The half-life of radium is 1620 years. How many radium atoms decay in 1s in a 1g sample of radium? The atomic weight of radium is 226 g/mol.

Sol. Number of radioactive nuclei disintegrated in 1 second is found by $\frac{\Delta N}{\Delta t} = \lambda N$ here λ is the decay constant and N is the number of nuclei present in 1 g sample of radium.

Number of atoms in 1g sample is

$N = \left(\frac{1}{226}\right)\left(6.02 \times 10^{23}\right) = 2.66 \times 10^{21}$ atoms.

The decay constant is

$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(1620)\left(3.16 \times 10^7\right)} = 1.35 \times 10^{-11} s^{-1}$

Taking 1 yr = 3.16×10^7 s;

Now, $\frac{\Delta N}{\Delta t} = \lambda N = \left(1.35 \times 10^{-11}\right)\left(2.66 \times 10^{21}\right)$

$= 3.6 \times 10^{10} \, s^{-1}$

Thus, 3.6×10^{10} nuclei decay in one second.

Example 7: Determine the age of an ancient wooden piece if it is known that the specific activity of C^{14} nuclide in it amounts to 3/5 of that in freshly felled trees. The half-life of C^{14} nuclide is 5570 years.

Sol: Find the age of wooden piece using equation $A = A_0 e^{-\lambda t}$. Here A is $\frac{3}{5} A_o$ and λ is the decay constant.

Specify activity is the activity per unit mass of the substance.

$A = A_0 e^{-\lambda t}$; Here $A = (3/5) A_0$

$\therefore \frac{3}{5} A_0 = A_0 e^{-\lambda t}$ or $t = \frac{\ln \frac{5}{3}}{\lambda}$

or $t = \ln\left(\frac{5}{3}\right) / (\ln 2 / T) = 5570 \left(\ln \frac{5}{3}\right)$

$= 4.1 \times 10^3$ years

JEE Main/Boards

Exercise 1

Nuclear Physics

Q.1 Some amount of radioactive substance (half-life= 10 days) is spread inside a room and consequently the level of radiation becomes 50 times permissible level for normal occupancy of the room. After how many days the room will be safe for occupation?

Q.2 The mean lives of a radioactive substance are 1620 and 405 years for α – emission and $\beta-$emission respectively. Find out the time during which three forth of a sample will Decay if it is decaying both by α – emission and $\beta-$emission simultaneously.

Q.3 A radioactive element decays by $\beta-$emission. A detector records n-beta particles in 2 Seconds and in next 2 seconds it records 0.75 n beta particles. Find mean life correct to nearest whole number. Given log2=0.6931, log3=1.0986.

Q.4 Nuclei of radioactive element A are being produced at a constant rate $\propto$. The element has a decay constant λ. At time t = 0 there are N_0 nuclei of the element.

(a) Calculate the number N of nuclei of A at time t.

(b) If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life of A and also the limiting value of N at $t\to\infty$

Q.5 Polonium $\left({}^{210}_{84}Po\right)$ emits ${}^{4}_{2}\alpha$ - particles and is converted into lead $\left({}^{206}_{82}Pb\right)$. The Reaction is used for producing electric power in a space mission. ${}^{210}_{84}Po$ has half of 138.6days. Assuming an efficiency of 10% of the thermoelectric machine, how much ${}^{210}_{84}Po$ is required to produce $1.2\times10^{7}\,J$ of electric energy per day at the end of 693 days? Also find the initial activity of the material. (Given masses of the nuclei ${}^{210}_{84}Po$ =209.98264 amu, ${}^{206}_{82}Pb$ =205.97440 amu, ${}^{4}_{2}\alpha$ =4.00260 amu, 1 amu = 931 MeV and Avogadro number= 6×10^{23} / mol).

Q.6 A nuclear explosion is designed to deliver 1MW of heat energy, how many fission events must be required in a second to attain this power level. If this explosion is designed with nuclear fuel consisting of uranium -235 to run a reactor at this power level for one year, then calculate the amount of fuel needed. You can assume that the amount of energy released per fission event is 200 MeV.

Q.7 Draw a diagram to show the variation of binding energy per nucleon with mass number for different nuclei. State with reason why light nuclei usually undergo nuclear fusion.

Q.8 Define decay constant of radioactive sample. Which of the following radiations, $\alpha-$rays, $\beta-$rays, $\gamma-$rays

(i) Are similar to X-rays/

(ii) Are easily absorbed by matter?

(iii) Travel with greatest speed?

(iv) Are similar in nature to cathode rays?

Q.9 Calculate the energy released in the following nuclear reaction:

${}^{6}_{3}Li+{}^{1}_{0}n\to{}^{4}_{2}He+{}^{3}_{1}H$ (Given; mass of ${}^{6}_{3}Li$ =6.015126 u, mass of ${}^{1}_{0}n$ = 1.008665 u, mass of ${}^{4}_{2}He$ = 4.002604 u, mass of ${}^{3}_{1}H$ = 3.016049 u and 1 atomic mass unit (1 u)) = 931 MeV)

Q.10 Explain with an example, whether the neutron-proton ration in a nucleus increases or decreases due to beta (β) decay.

Q.11 Define the herms; 'half-life period' and 'decay constant' of radioactive sample. Derive the relation between these terms.

Q.12 When a deuteron of mass 2.0141 u and negligible kinetic energy is absorbed by a lithium$\left({}^{6}Li_{3}\right)$ nucleus of mass 6.0155 u, the compound nucleus disintegrates spontaneously into two alpha particles, each of mass 4.0026 u. Calculate the energy in joules carried by each alpha particle. $\left(1u = 1.66\times10^{-27}kg\right)$

Q.13 A radioactive sample contains 2.2 mg of pure $^{11}_{6}C$ which has half-life period of 1224 seconds. Calculate

(i) The number of atoms present initially.

(ii) The activity when 5 μg of the sample will be left.

Q.14 Define the terms half-life period and decay constant of a radioactive substance. Write their S.I. units. Establish the relationship between the two.

Q.15 A neutron is absorbed by a $^{6}_{3}Li$ nucleus with the subsequent emission of an alpha particle.

(i) Write the corresponding nuclear reaction.

(ii) Calculate the energy released, in MeV, in this reaction.

Given mass $^{6}_{3}Li$ = 6.015126 u;

Mass (neutron) = 1.0086554 u;

Mass (alpha particle) = 4.0026044 u and

Mass (triton) = 3.0100000 u. Take 1 u =931 MeV/C^2.

Q.16 Define the term 'activity' of a radionuclide. Write its SI unit.

Q.17 Draw graph showing the variation of potential energy between a pair of nucleons as a function of their separation. Indicate the regions in which the nuclear force is

(i) Attractive (ii) Repulsive

Q.18 Draw the graph to show variation of binding energy per nucleon with mass number of different atomic nuclei. Calculate binding energy/nucleon of $^{40}_{20}Ca$ nucleus.

Q.19 State two characteristic properties of nuclear force.

Q.20 Calculate the energy, released in MeV, in the following nuclear reaction

$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He + Q$ $\left[\text{Mass of } ^{238}_{92}U = 238.05079 \text{ u}\right.$

Mass of $^{234}_{90}Th = 234.043630$ u Mass of $^{4}_{2}He = 4.002600$ u

$\left. 1u = 931.5 MeV / c^2 \right]$

Q.21 Two nuclei have mass numbers in the ration 1:8. What is the ration of their nuclear radii?

Q.22 The mass of a nucleus in its ground state is always less than the total mass of its Constituents neutrons and protons. Explain.

Q.23 Draw a plot showing the variation of binding energy per nucleon versus the mass number A. Explain with the help of this plot the release of energy in the processes of nuclear fission and fusion.

Q.24 Define the activity of a radionuclide. Write its S.I. unit. Give a plot of the activity of a radioactive species versus time.

Q.25 Draw a plot of the binding energy per nucleon as a function of mass number for a large Number of nuclei, $2 \le A \le 240$. How do you explain the constancy of binding energy per nucleon in the range $30 < A < 170$ using the property that nuclear force is short-range?

Radioactivity

Q.26 Classify each of the following nuclides as "beta $\left(^{0}_{-1}\beta\right)$ emitter", or "positron $\left(^{0}_{-1}\beta\right)$ emitter": $^{49}_{20}Ca$ $^{195}_{80}Hg$ $^{5}_{8}B$ $^{150}_{67}Ho$ $^{30}_{13}Al$ $^{94}_{36}Kr$. Note: $^{84}_{36}Kr$ $^{200}_{80}$,Hg and $^{165}_{67}Ho$ are stable.

Q.27 Of the three isobars $^{114}_{48}Cd$ $^{114}_{49}In$ and $^{114}_{50}Sn$, which is likely to be radioactive? Explain your choice.

Q.28 Complete the following nuclear equations;

(a) $^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ...$

(b) $^{9}_{4}Be + ^{4}_{2}He \rightarrow ^{12}_{6}C + ...$

(c) $^{9}_{4}Be(p, \alpha)........$

(d) $^{30}_{15}P \rightarrow ^{30}_{14}S + ...$

(e) $^{3}_{1}H \rightarrow ^{3}_{2}He + ...$

(f) $^{43}_{20}Ca(\propto, ...) \rightarrow ^{46}_{21}Sc$

Q.29 The activity of the radioactive sample drops to 1/64 of its original value in 2 hr find the decay constant (λ).

Q.30 The nucleidic ration of ^{3}H to $^{1}_{1}H$ in a sample of water is $8.0\times10^{-18}:1.$ Tritium undergoes decay tritium atoms would 10.0 g of such a sample contains 40 year after the original sample is collected?

Q.31 The half-life period of $^{125}_{53}I$ is 60 days. What % of radioactivity would be present after 240 days?

Q.32 At any given time a piece of radioactive material $\left(t_{1/2}=30\,\text{days}\right)$ contains 10^{12} atoms.

Calculate the activity of the sample in dps.

Q.33 Calculate the age of a vegetarian beverage whose tritium content is only 15% of the level in living plants. Given $t_{1/2}$ for $^{3}_{1}H$ = 12.3 years.

Q.34 An isotopes of potassium $^{40}_{19}K$ has a half-life of 1.4×10^{9} year and decays to Argon $^{40}_{18}Ar$ which is stable.

(i) Write down the nuclear reaction representing this decay.

(ii) A sample of rock taken from the moon contains both potassium and argon in the ratio 1/3. Find age of rock.

Q.35 At a given instant there are 25% undecayed radioactive nuclei in a sample. After 10 sec the number of undecayed nuclei remain 12.5%. Calculate :

(i) mean-life of the nuclei and

(ii)The time in which the number of undecayed nuclear will further reduce to 6.25% of the reduced number.

Q.36 Calculate the energy released in joules and MeV in the following nuclear reaction : $^{2}_{1}H+^{2}_{1}H\rightarrow^{3}_{2}He+^{1}_{0}n$ Assume that the masses of $^{2}_{1}H$, $^{3}_{2}He$ and neutron (n) respectively are 2.020, 3.0160 and 1.0087 in amu.

Q.37 (a) Calculate number of α – and β-particles emitted when $^{238}_{92}U$ changes into radioactive $^{206}_{82}Pb$.

(b) Th^{234} disintegrates and emits $6\beta-$ and $7\alpha-$particles to form a stable element. Find the atomic number and mass number of the stable product.

Q.38 One of the hazards of nuclear explosion is the generation of Sr^{90} and its subsequent incorporation in bones. This nuclide has a half-life of 28.1 year. Suppose one microgram was absorbed by a new-born child, how much Sr^{90} will remain in his bones after 20 years?

Q.39 (i) $^{210}_{84}Po$ decays with $\alpha-$particle to $^{206}_{82}Pb$ with a half-life of 138.4 day. If 1.0 g of $^{210}_{84}Po$ is placed in a sealed tube, how much helium will accumulate in 69.2 day? Express the answer in cm^{3} at 1atm and 273K. Also report the volume of He formed if 1 g of $^{210}_{84}Po$ is used.

(ii) A sample of U^{238} (half-life = 4.5×10^{9} yr) ore is found to contain 23.8 g of U^{238} and 20.6 g of Pb^{206}. Calculate the age of the ore.

Q.40 Ac^{227} has a half-life of 22 year w.r.t radioactive decay. The decay follows two parallel paths, one leading the Th^{227} and the other leading to Fr^{223}. The percentage yields of these two daughters nucleides are 2% and 98% respectively. What is the rate constant in yr^{-1}, for each of these separate paths?

Exercise 2

Nuclear Physics

Single Correct Choice Type

Q.1 Let u be denoted one atomic mass unit. One atom of an element of mass number A has mass exactly equal to Au

(A) For any value of A

(B) Only for A = 1

(C) Only for A = 12

(D) For any value of A provided the atom is stable

Q.2 The surface area of a nucleus varies with mass number A as

(A) $A^{2/3}$ (B) $A^{1/3}$ (C) A (D) None

Q.3 Consider the nuclear reaction $X^{200}\rightarrow A^{110}+B^{90}$ If the binding energy per nucleon for X,A and B is 7.4 MeV, 8.2. MeV and 8.2 MeV respectively, what is the energy released?

(A) 200 MeV (B) 160 MeV

(C) 110 MeV (D) 90 MeV

Q.4 The binding energy per nucleon for C^{12} is 7.68 MeV and that for C^{13} is 7.5 MeV. The energy required to remove a neutron from C^{13} is

(A) 5.34 MeV (B) 5.5 MeV

(C) 9.5 MeV (D) 9.34 MeV

Q.5 The binding energies of nuclei X and Y are E_1 and E_2 respectively. Two atoms of X fuse to give one atom of Y and an energy Q is released. Then:

(A) $Q = 2E_1 - E_2$ (B) $Q = E_2 - 2E_1$

(C) $Q = 2E_1 + E_2$ (D) $Q = 2E_2 + E_1$

Q.6 There are two radio-nuclei A and B. A is an alpha emitter and B is a beta emitter. Their disintegration constants are in the ratio of 1:2. What should be the ratio of number of atoms of two at time $t = 0$ so that probabilities of getting α – and β – particles are same at time t=0.

(A) 2:1 (B)1:2 (C)e (D) e^{-1}

Q.7 A certain radioactive substance has a half-life of 5 years. Thus for a particular nucleus in a sample of the element, the probability of decay in ten years is

(A) 50% (B) 75% (C) 100% (D) 60%

Q.8 Half-life of radium is 1620 years. How many radium nuclei decay in 5 hours in 5 gm radium? (Atomic weight of radium = 223)

(A) 9.1×10^{12} (B) 3.23×10^{15}

(C) 1.72×10^{20} (D) 3.3×10^{17}

Q.9 The decay constant of the end product of a radioactive series is

(A) Zero

(B) Infinite

(C) Finite (non zero)

(D) Depends on the end product.

Q.10 A radioactive nuclide can decay simultaneously by two different processes which have decay constants λ_1 and λ_2. The effective decay constant of the nuclide is λ, then :

(A) $\lambda = \lambda_1 + \lambda_2$ (B) $\lambda = 1/2(\lambda_1 + \lambda_1)$

(C) $\frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$ (D) $\lambda = \overline{\lambda_1\lambda_2}$

Radioactivity

Single Correct Choice Type

Q.11 $^{27}_{13}Al$ is a stable isotope. $^{29}_{13}Al$ is expected to be disintegrated by

(A) α emission (B) β emission

(C) Positron emission (D) Proton emission.

Q.12 Loss of a $\beta -$particle is equivalent to

(A) Increase of one proton only

(B) Decrease of one neutron only

(C) Both (A) and (B)

(D) None of these

Q.13 Two radioactive material A_1 and A_2 have decay constant of $10\lambda_0$ and λ_0. If initially they have same number of nuclei, then after time $\frac{1}{9\lambda_0}$ the ratio of number of their undecayed nuclei will be

(A) $\frac{1}{e}$ (B) $\frac{1}{e^2}$ (C) $\frac{1}{e^3}$ (D) $\frac{\sqrt{e}}{1}$

Q.14 The half-life of a radioactive isotopes is three hours. If the initial mass of the isotope were 256 g, the mass of it remaining undecayed after 18 hours would be

(A) 16.0 g (B) 4.0 g (C) 8.0 (D) 12.0 g

Q.15 A consecutive reaction $A \xrightarrow{k_1} B \xrightarrow{k_2} C$ is characterised by

(A) Maxima in the concentration of A

(B) Maxima in the concentration of B

(C) Maxima in the concentration of C

(D) High exothermicity

Q.16 Consider the following nuclear reactions: $^{238}_{92}M \rightarrow ^{X}_{Y}N + 2\,^{4}_{2}He;\ ^{X}_{Y}N \rightarrow ^{A}_{B}L + 2\beta^{+}$

The number of neutrons in the element L is

(A) 142 (B)144 (C)140 (D)146

Q.17 The half-life of a radioisotope is four hours. If the initial mass of the isotope was 200 g, the mass remaining after 24 hours undecayed is

(A) 1.042 g (B) 2.084 g (C) 3.125 g (D) 4.167 g

Q.18 Helium nuclei combines to form an oxygen nucleus. The binding energy per nucleon of oxygen nucleus is if $m_0 = 15.834$ amu and $m_{He} = 4.0026$ amu

(A) 10.24 MeV (B) 0 MeV

(C) 5.24 MeV (D) 4 MeV

Q.19 A radioactive element gets spilled over the floor of a room. Its half-life period is 30 days. If the initial activity is ten times the permissible value, after how many days will it be safe to enter the room?

(A) 1000 days (B) 300 days

(C) 10 days (D) 100 days

Q.20 Which of the following nuclear reactions will generate an isotope ?

(A) neutron particle emission

(B) position emission

(C) α-particle emission

(D) β-particle emission

Q.21 Read the following:

(i) The half-life period of a radioactive element X is same as the mean-life time of another radioactive element Y. Initially both of them have the same number of atoms. Then Y will decay at a faster rate than X.

(ii) The electron emitted in beta radiation originates from decay of a neutron in a nucleus

(iii) The half-life of^{215} at is 100 ms. The time taken for the radioactivity of a sample of of^{215} At to decay to 1/16th of its initial value is 400 us.

(iv) The volume (V) and mass (m) of a nucleus are related as $V \propto m$.

(v) Given a sample of Radium-226 having half-life of 4 days. Find the probability. A nucleus disintegrates within 2 half-lives is ¾

Select the correct code for above.

(A) TTTTT (B) TFTTF

(C) FTFTF (D) FTTTF

Q.22 The radioactive sources A and B of half-lives of t hours respectively, initially contain the same number of radioactive atoms. At the end of t hours, their rates of disintegration are in the ratio:

(A) $2\sqrt{2}:1$ (B) 1:8

(C) $\sqrt{2}:1$ (D) n:1

Q.23 The ratio of ^{14}C to ^{12}C in a living matter is measured to be $\frac{^{14}C}{^{14}C} = 1.3 \times 10^{-12}$ at the present time. Activity of 12.0 gm carbon sample is 180 dpm. The half-life of ^{14}C is nearly ________x 10^{-12} sec. [Given: $N_A = 6x10^{23}$]

(A) 0.18 (B) 1.8 (C) 0.384 (D) 648

Q.24 Which of the following processes represent a gamma – decay?

(A) ${}^{A}_{Z}X + y \rightarrow {}^{A}_{Z-1}X + a + b$

(B) ${}^{A}_{Z}X + {}^{1}_{0}n \rightarrow {}^{A-3}_{Z-2}X + c$

(C) ${}^{A}_{Z}X \rightarrow {}^{A}_{Z}X + f$

(D) ${}^{A}_{Z}X + {}_{-1}e \rightarrow {}^{A}_{Z-1}X + g$

Q.25 Let F_{pp}, F_{pn} and F_{nn} denote the magnitudes of net force by a proton on a proton, by a proton on a neutron and by a neutron on a neutron respectively. Neglect gravitational force. When the separation is 1 fm,

(A) $F_{pp} > F_{pn} = F_{nn}$ (B) $F_{pp} = F_{pn} = F_{nn}$

(C) $F_{pp} > F_{pn} > F_{nn}$ (D) $F_{pp} < F_{pn} = F_{nn}$

Q.26 The average (mean) life at a radio nuclide which decays by parallel path is

$A \xrightarrow{\lambda_1} B;\ \lambda_1 = 1.8x10^{-2}\ sec^{-1}$

$2A \xrightarrow{\lambda_2} C;\ \lambda_2 = 10^{-3}\ sec^{-1}$

(A) 52.63 sec (B) 500 sec

(C) 50 sec (D) None

Q.27 Two radioactive nuclides A and B have half lives of 50 min respectively. A fresh sample contains the nuclides of B to be eight time that of A. How much time should elapse so that the number of nuclides of A becomes double of B

(A) 30 (B) 40 (C) 50 (D) None

Q.28 A sample of $^{14}CO_2$ was to be mixed with ordinary CO_2 for a biological tracer experiment. In order that $10\ cm^3$ of diluted gas should have 10^4 dis/min, what activity (in μCi) of radioactive carbon is needed to prepare 60 L of diluted gas at STP. [1 Ci= $3.7x10^{10}$ dps]

(A) 270μCi (B)27μCi (C) 2.7μCi (D)2700μCi

Q.29 Wooden article and freshly cut tree show activity of 7.6 and 15.2 $min^{-1}\ gm^{-1}$ of carbon ($t_{1/2} = 5760$ years) respectively. The age of article in years. Is

(A) 5760 (B) $5760x\left(\frac{15.2}{7.6}\right)$

(C) $5760\times\left(\frac{7.6}{15.2}\right)$ (D) 5760 x (15.2-7.6)

Q.30 A radioactive sample had an initial activity of 56 dpm (disintegration per min it was found to have an activity of 28 dpm. Find the number of atoms in a sample having an activity of 10 dpm.

(A) 693 (B) 1000 (C) 100 (D) 10,000

Q.31 The radioactivity of a sample is R_1 at a time T_1 and R_2 at a time T_2. If the half-life of the specimen is T, the number of atoms that have disintegrated in the time $(T_2 - T_1)$ is proportional to

(A) $(R_1T_1 - R_2T_2)$ (B) $(R_1 - R_2)$

(C) $(R_1 - R_2)/T$ (D) $(R_1 - R_2)T/0.693$

Previous Years' Questions

Q.1 The half-life of the radioactive radon is 3.8 days. The time, at the end of which 1/20th of the radon sample will remain undecayed, is (given $\log_{10} 3$ =0.4343) ***(1981)***

(A) 3.8 days (B) 16.5 days

(C) 33 days (D) 76 days

Q.2 Beta rays emitted by a radioactive material are ***(1983)***

(A) Electromagnetic radiations

(B) The electrons orbiting around the nucleus

(C) Charged particles emitted by the nucleus

(D) Neutral particles

Q.3 The equation ; ***(1987)***

$4\,{}^1_1H \rightarrow {}^4_2He^{2+} + 2e^- + 26\ MeV$ represents

(A) β -Decay (B) γ -Decay

(C) Fusion (D) Fission

Q.4 During a negative beta decay ***(1987)***

(A) An atomic electron is ejected

(B) An electron which is already present within the nucleus is ejected

(C) A neutron in the nucleus decays emitting an electron

(D) A part of the binding energy of the nucleus is converted into an electron

Q.5 A star initially has 10^{40} deuterons. It produces energy via the processes ${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3\,p$ and ${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n$. If the average power radiated by the star is 10^{16} W, the deuteron supply of the star is exhausted in a time of the order of ***(1993)***

(A) 10^6 s (B) 10^8 s (C) 10^{12} s (D) 10^{16} s

Q.6 Fast neutrons can easily be slowed down by ***(1994)***

(A) The use of lead shielding

(B) Passing them through heavy water

(C) Elastic collisions with heavy nuclei

(D) Applying a strong electric field

Q.7 Consider α -particles, β -particles and λ -rays each having an energy of 0.5 MeV. In increasing order of penetrating powers, the radiations are ***(1994)***

(A) α,β,γ (B) α,γ,β (C) β,γ,α (D) γ,β,α

Q.8 A radioactive sample S_1 having an activity of 5 μCi has twice the number of nuclei as another sample S_2 which has an activity of 10 μCi. The half lives of S_1 and S_2 can be ***(2008)***

(A) 20 yr and 5 yr, respectively

(B) 20 yr and 10 yr, respectively

(C) 10 yr each

(D) 5 yr each

Q.9 The radioactive decay rate of a radioactive element is found to be 10^3 disintegration /second at a certain time. If the half-life of the element is one second, the decay rate after one second is .... And after three seconds is .... ***(1983)***

Q.10 In the uranium radioactive series the initial nucleus is ${}^{238}_{92}U$ and the final nucleus is ${}^{206}_{92}Pb$. When the uranium nucleus decays to lead, the number of α -particles emitted is.... And the number of β -particles emitted is............ ***(1985)***

Q.11 Consider the reaction: ${}^2_1H + {}^2_1H = {}^4_2He + Q$. Mass of the deuterium atom = 2.0141u. Mass of helium atom = 4.0024u. This is a nuclear .... reaction in which the energy Q released is .... MeV. ***(1996)***

Q.12 This question contains Statement-I and Statement-II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement–I: Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion.

and

Statement–II: For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decrease with increasing Z. ***(2008)***

(A) Statement-I is false, statement-II is true.

(B) Statement-I is true, statement-II is true; statement-II is correct explanation for statement-I.

(C) Statement-Iis true, statement– 2 is true; statement-II is nota correct explanation for statement-I.

(D) Statement-I is true, statement-II is False.

Q.13 Suppose an electron is attracted towards the origin by a force k/r where 'k' is a constant and 'r' is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the n^{th} orbital of the electron is found to be 'r_n' and the kinetic energy of the electron to be T_n. Then which of the following is true? ***(2008)***

(A) $T_n \propto 1/n^2$, $r_n \propto n^2$ (B) T_n independent of n, $r_n \propto n$

(C) $T_n \propto 1/n$, $r_n \propto n$ (D) $T_n \propto 1/n$, $r_n \propto n^2$

Q.14 The above is a plot of binding energy per nucleon E_b, against the nuclear mass M; A, B, C, D, E, F correspond to different nuclei. Consider four reactions: ***(2009)***

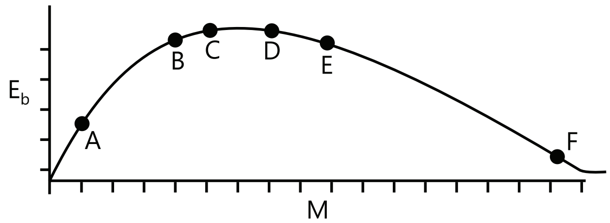
(i) A + B $\to$ C + ε

(ii) C $\to$ A + B + ε

(iii) D + E $\to$ F + ε and

(iv) F $\to$ D + E + ε

where ε is the energy released? In which reactions is ε positive?



(A) (i) and (iv) (B) (i) and (iii)

(C) (ii) and (iv) (D) (ii) and (iii)

Q.15 The transition from the state n = 4 to n = 3 in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from ***(2009)***

(A) 2 $\to$1 (B) 3 $\to$2

(C) 4 $\to$2 (D) 5$\to$3

Q.16 The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then ***(2010)***

(A) $E_2 = 2E_1$ (B) $E_1 > E_2$

(C) $E_2 > E_1$ (D) $E_1 = 2E_2$

Q.17 The speed of daughter nuclei is ***(2010)***

(A) $c\frac{\Delta m}{M+\Delta m}$ (B) $c\sqrt{\frac{2\Delta m}{M}}$

(C) $c\sqrt{\frac{\Delta m}{M}}$ (D) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$

Q.18 A radioactive nucleus (initial mass number A and atomic number Z) emits 3 a-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be **(2010)**

(A) $\frac{A-Z-8}{Z-4}$ (B) $\frac{A-Z-4}{Z-8}$

(C) $\frac{A-Z-12}{Z-4}$ (D) $\frac{A-Z-4}{Z-2}$

Q.19 Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is: ***(2011)***

(A) 36.3 eV (B) 108.8 eV

(C) 122.4 eV (D) 12.1 eV

Q.20 The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 and $\frac{1}{3}$ of it had decayed is : ***(2011)***

(A) 14 min (B) 20 min (C) 28 min (D) 7 min

Q.21 Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be ***(2012)***

(A) 2 (B) 3 (C) 5 (D) 6

Q.22 Assume that a neutron breaks into a proton and an electron. The energy released during this process is (Mass of neutron = 1.6725×10^{-27} kg; mass of proton = 1.6725×10^{-27} kg; mass of electron = 9×10^{-31} kg) ***(2012)***

(A) 0.73 MeV (B) 7.10 MeV

(C) 6.30 MeV (D) 5.4 MeV

Q.23 As an electron makes a transition from an excited state to the ground state of a hydrogen - like atom/ion: ***(2015)***

(A) its kinetic energy increases but potential energy and total energy decrease

(B) kinetic energy, potential energy and total energy decrease

(C) kinetic energy decreases, potential energy increases but total energy remains same

(D) kinetic energy and total energy decrease but potential energy increases

Q.24 Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively, Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be : ***(2016)***

(A) 4 : 1 (B) 1 : 4 (C) 5 : 4 (D) 1 : 16

JEE Advanced/Boards

Exercise 1

Nuclear Physics

Q.1 The binding energies per nucleon for deuteron $({}_1H^2)$ and helium $({}_2He^4)$ are 1.1 MeV and 7.0 MeV respectively. The energy released when two deuterons fuse to form a helium nucleus $({}_2He^4)$ is ______________

Q. 2 An isotopes of Potassium ${}^{40}_{19}K$ has a half-life of 1.4×10^9 year and decay to Argon ${}^{40}_{18}Ar$ which is stable.

(i) Write down the nuclear reaction representing this decay.

(ii) A sample of rock taken from the moon contains both potassium and argon in the ratio 1/7. Find age of rock.

Q.3 At t=0, a sample is placed in a reactor. An unstable nuclide is produced at a constant rate R in the sample by neutron absorption. This nuclide β-decays with half-life τ. Find the time required to produce 80% of the equilibrium quantity of this unstable nuclide.

Q.4 Suppose that the Sun consists entirely of hydrogen atom and releases the energy by the nuclear reaction, $4{}^1_1H \rightarrow {}^4_2He$ with 26 MeV of energy released. If the total output power of the Sun is assumed to remain constant at 3.9×10^{26} W, find the time it will take to burn all the hydrogen, Take the mass of the Sun as 1.7×10^{30} kg.

Q.5 U^{238} and U^{235} occur in nature in an atomic ratio 140:1. Assuming that at the time of earth's formation the two isotopes were present in equal amounts. Calculate the age of the earth.

(Halflife of u^{238} = 4.5 × 109 years and that of U^{235} = 7.13 × 10^8 years)

Q.6 The kinetic energy of an α-particle which flies out of the nucleus of a Ra^{226} atom in radioactive disintegration in 4.78 MeV. Find the total energy the escape of the α-particle.

Q.7 A small bottle contains powdered beryllium Be & gaseous radon which is used as a source of α-particles. Neutrons are produced when α-particles of the radon react with beryllium. The yield of this reaction is (1/4000) i.e. only one α-particle out of 4000 induces the reaction.

Find the amount of radon (Rn^{222}) originally introduced into the source. If it produces 1.2 x 10^6 neutrons per second after 7.6 days. $[T_{1/2}\ of\ R_a = 3.8\ days]$

Q.8 An experiment is done to determine the half-life of radioactive substance that emits one β-particle for each decay process. Measurement show that an average of 8.4β are emitted each second by 2.5 mg of the substance. The atomic weight of the substance is 230. Find the half-life of the substance.

Q.9 A wooden piece of great antiquity weighs 50 gm and shows C^{14} activity of 320 disintegrations per minute. Estimate the length of the time which has elapsed since this wood was part of living tree, assuming that living plant show a C^{14} activity of 12 disintegrations per minute per gm. The half-life of C^{14} is 5730 yrs.

Q.10 When two deuterons $({}_1^2H)$ fuse to from a helium nucleus ${}_2He^4$, 23.6 MeV energy is released. Find the binding energy of helium if it is 1.1 MeV for each nucleon of deutrim.

Q.11 A π^+ meson of negligible initial velocity decays to a μ^+ (muon) and a neutrino. With what kinetic energy (in eV) does the muon move? (The rest mass of neutrino can be considered zero. The rest mass of the π^+ meson is 150 MeV and the rest mass of the muon is 100 MeV.) Take neutrino to behave like a photon.

Take $\sqrt{3} = 1.41$.

Q.12 A body of mass m_0 is placed on a smooth horizontal surface. The mass of the body is decreasing exponentially with disintegration constant λ. Assuming that the mass is ejected backward with a relative velocity u. Initially the body was at rest. Find the velocity of body after time t.

Q.13 Show that in a nuclear reaction where the outgoing particle is scattered at an angle of 90^0 with the direction of the bombarding particle, the Q-value is expressed as

$$Q = K_p\left(1+\frac{m_p}{M_o}\right) - K_1\left(1+\frac{m_1}{M_o}\right)$$

Where, I=incoming particle, P=product nucleus, T=target nucleus, O=outgoing particle.

Radioactivity

Q.14 In a nature decay chain series starts with ${}_{90}Th^{232}$ and finally terminates at ${}_{82}Pb^{208}$. A thorium ore sample was found to contain 8 x 10^{-5} ml of helium at 1 atm & 273 K and 5 x 10^{-7} gm of Th^{232}. Find the age of ore sample assuming that source of He to be only due to decay of Th^{232}. Also assume complete retention of helium within the ore. (Half-life of Th^{232} =1.39 x 10^{10} Y)

Q.15 A radioactive decay counter is switched on at t=0. A β-active sample is present near the counter. The counter registers the number of β-particles emitted by the sample. The counter registers 1 x 10^5 β-particles at t=36 s and 1.11 x 10^5 β-particles at t = 108 s. Find $T_{1/2}$ of this sample.

Q.16 A small quantity of solution containing ${}^{24}Na$ radionuclide (half-life 15 hours) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 hours shows an activity of 296 disintegrations per minute. Determine the total volume of blood in the body of the person. Assume that the radioactive solution mixes uniformly in the blood of the person. (1 Curie=3.7 x 10^{10} disintegrations per second)

Q.17 A mixture of ${}^{239}Pu$ and ${}^{240}Pu$ has a specific activity of 6 x 10^9 dis/s/g. The half lives of the isotopes are 2.44 x 10^4 y and 6.08 x 10^3 y respectively. Calculate the isotopic composition of this sample.

Q.18 Nuclei of a radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time t=0, there are N_0 nuclei of the element.

(a) Calculate the number N of nuclei of A at time t

(b) If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life of A & also the limiting value of N as $t \to \infty$

Q.19 In hydrogenation reaction at 25^oC, it is observed that hydrogen gas pressure falls from 2 atm to 1.2 atm in 50 min. Calculate the rate of reaction in molarity per sec. R=0.0821 litre atm degree $^{-1}mol^{-1}$

Q.20 ${}_{92}^{238}U$ by successive radioactive decays changes to ${}_{82}^{206}Pb$. A sample of uranium ore was analyzed and found to contain 1.0g of U^{238} and 0.1g of PB^{206}. Assuming that all the PB^{206} had accumulated due to decay of U^{238}, find out the age of the ore.

(Halflife of $U^{238} = 4.5 x 10^9$ years)

Q.21 $^{218}_{84}Po$ ($t_{/12}$ =3.05 min) decay to $^{214}_{82}Pb$ ($t_{/12}$ = 3.05 min) by α-emission, while Pb^{214} is a $\beta-$emitter. In an experiment starting with 1 gm atom of Pure Po^{218}, how much time would be required for the number of nuclei of $^{214}_{82}Pb$ to reach maximum?

Q.22 (a) On analysis a sample of uranium ore was found to contain 0.277g of $^{206}_{82}Pb$ and 1.667 g of $^{237}_{92}U$. The half-life period of U^{238} is 4.51 x 10^9 year. If all the lead were assumed to have come from decay of $^{238}_{92}U$, what is the age of earth?

(b) An ore of $^{238}_{92}U$ is found to contain $^{238}_{92}U$ and $^{236}_{92}U$ in the weight ratio of 1:0.1. The half-life period of $^{238}_{92}U$ is 4.5 x 10^9 year. Calculate the age of ore.

Q.23 An experiment requires minimum β-activity produced at the rate of 346 β-particles per minute. The half-life period of $^{99}_{42}Mo$ which is a β-emitter is 66.6 hr. Find the minimum amount of $^{99}_{42}Mo$ required to carry out the experiment in 6.909 hour.

Exercise 2

Nuclear Physics

Single Correct Choice Type

Q.1 The rest mass of the deuteron, ^2_1H, is equivalent to an energy of 1876 MeV, the rest mass of a proton is equivalent to 939 MeV and that of a neutron to 940 MeV. A deuteron may disintegrate to a proton and a neutron if it :

(A) Emits a γ-ray photon of energy 2 MeV

(B) Captures a γ-ray photon of energy 2 MeV

(C) Emits a γ-ray photon of energy 3 MeV

(D) Captures a γ-ray photon of energy 3 MeV

Q.2 A certain radioactive nuclide of mass number m_x disintegrates, with the emission of an electron and γ radiation only, to give second nuclide of mass number m_y. Which one of the following equation correctly relates m_x and m_y?

(A) $m_y = m_x + 1$ (B) $m_y = m_x - 2$

(C) $m_y = m_x - 1$ (D) $m_y = m_x$

Q.3 The number of α and β-emitted during the radioactive decay chain starting from $^{226}_{88}Ra$ and ending at $^{206}_{82}Pb$ is

(A) 3α & $6\beta^-$ (B) 4α & $5\beta^-$

(C) 5α & $4\beta^-$ (D) 6α & $6\beta^-$

Q.4 In an α-decay the Kinetic energy of α particle is 48 MeV Q-value of the reaction is 50 MeV. The mass number of the mother nucleus is : (Assume that daughter nucleus is in ground state)

(A) 96 (B) 100

(C) 104 (D) None of these

Q.5 In the uranium radioactive series the initial nucleus is $^{238}_{92}U$, and the final nucleus is $^{206}_{82}Pb$. When the uranium nucleus decays to lead, the number of α-particles emitted is. And the number of β-particles emitted.

(A) 6, 8 (B) 8, 6

(C) 16, 6 (D) 32, 12

Q.6 Activity of a radioactive substance is R_1 at time t_1 and R_2 at time $t_2 (t_2 > t_1)$. Then the $\frac{R_2}{R_1}$ is :

(A) $\frac{t_2}{t_1}$ (B) $e^{-\lambda(t_1+t_2)}$

(C) $e\left(\frac{t_1-t_2}{\lambda}\right)$ (D) $e^{\lambda(t_1-t_2)}$

Q.7 A particular nucleus in a large population of identical radioactive nuclei did survive 5 half lives of that isotope. Then the probability that this surviving nucleus will survive the next half-life :

(A) $\frac{1}{32}$ (B) $\frac{1}{5}$ (C) $\frac{1}{2}$ (D) $\frac{5}{2}$

Q.8 The activity of a sample reduces from A_0 to $A_0\sqrt{3}$ in one hour. The activity after 3 hours more will be

(A) $\frac{A_0}{3\sqrt{3}}$ (B) $\frac{A_0}{9}$ (C) $\frac{A_0}{9\sqrt{3}}$ (D) $\frac{A_0}{27}$

Q.9 The activity of a sample of radioactive material is A_1 at time t_1 and A_2 at time $t_2 (t_2 > t_1)$. Its mean life is T.

(A) $A_1 t_1 = A_2 t_2$ (B) $\frac{A_1 - A_2}{t_2 - t_1}$ =constant

(C) $A_2 = A_1 e^{(t_1 - t_2)/T}$ (D) $A_2 = A_1 e^{(t_1 - T t_2)}$

Q.10 A fraction f_1 of a radioactive sample decays in one mean life, and a fraction f_2 decays in one half-life.

(A) $f_1 > f_2$

(B) $f_1 < f_2$

(C) $f_1 = f_2$

(D) May be (A), (B) or (C) depending on the values of the mean life and half-life.

Q.11 A radioactive substance is being produced at a constant rate of 10 nuclei/s. The decay constant of the substance is $1/2\,\text{sec}^{-1}$. After what time the number of radioactive nuclei will become 10? Initially there are no nuclei present. Assume decay law holds for the sample.

(A) 2.45 sec (B) log (2) sec

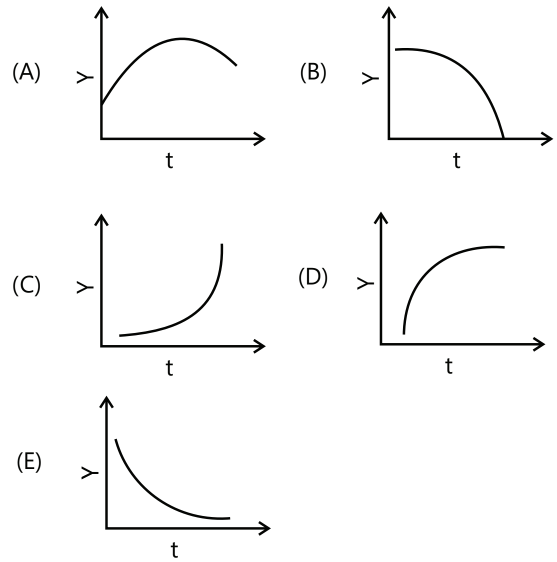
(C) 1.386 sec (D) $\frac{1}{\log(2)}$ sec

Q.12 The radioactivity of a sample is R_1 at time T_1 and T_2. If the half-life of the specimen is T. Number of atoms that have disintegrated in the $(T_2 - T_1)$ is proportional to

(A) $(R_1T_1 - R_2T_2)$ (B) $(R_1 - R_2)T$

(C) $(R_1 - R_2)/T$ (D) $(R_1 - R_2)(T_1 - T_2)$

Q.13 The radioactive nucleus of an element X decays to a stable nucleus of element Y. A graph of rate of formation of Y against time would look like



Q.14 A radioactive substance is dissolved in a liquid and the solution is heated. The activity of the solution

(A) Is smaller than that of element

(B) Is greater than that of element

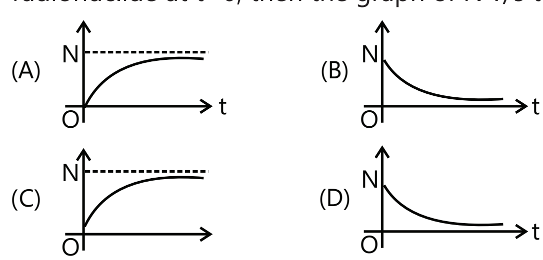
(C) Is equal to that of element

(D) Will be smaller or greater depending upon whether the solution is weak or concentrated.

Q.15 In a certain nuclear reactor, a radioactive nucleus is being produced at a constant rate =1000/s. The mean life of radionuclide is 40 minutes. At steady state, the number of radionuclide will be

(A) 4×10^4 (B) 24×10^4 (C) 24×10^5 (D) 24×10^6

Q.16 In the above question, if there were 20 x 10^5 radionuclide at t=0, then the graph of N v/s t is



Q.17 A free neutron is decayed into a proton but a free proton is not decayed into a neutron. This is because-

(A) Neutron is a composite particle made of a proton and an electron whereas proton is a fundamental particle

(B) Neutron is an uncharged particle whereas proton is a changed particle

(C) Neutron has larger rest mass than the proton

(D) Weak forces can be operated in a neutron but not in a proton

Multiple Correct Choice Type

Q.18 When a nucleus with atomic number Z and mass number A undergoes a radioactive decay process:

(A) Both Z and A will decrease, if the process is α decay

(B) Z will decrease but A will not change, if the process is β^+ decay

(C) Z will decrease but A will not change, if the process is β^- decay

(D) Z and A will remain unchanged, if the process is γ decay.

Q.19 When the atomic number A of the nucleus increases

(A) Initially the neutron-proton ratio is constant=1

(B) Initially neutron-proton ratio increases and later decreases

(C) Initially binding energy per nucleon increases when the neutron-proton ratio increases.

(D) The binding energy per nucleon increases when the neutron –proton ratio increases.

Q.20 Let m_p be the mass of a proton, m_a the mass of a neutron, M_1 the mass of a $^{20}_{10}Ne$ nucleus and M_2 the mass of a $^{40}_{20}Ca$ nucleus. Then

(A) $M_2 = 2M_1$ (B) $M_2 > 2M_1$

(C) $M_2 < 2M_1$ (D) $M_1 < 10(m_a + m_p)$

Q.21 The decay constant of a ratio active substance is 0.173 $(years)^{-1}$. Therefore :

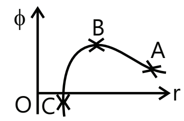
(A) Nearly 63% of the radioactive substance will decay in (1/0.173) year.

(B) Half-life of the radioactive substance is (1/0.173) year.

(C) One-fourth of the radioactive substance will be left after nearly 8 years.

(D) All the above statements are true.

Q.22 The graph shown by the side shows the variation of potential energy ϕ of a proton with its distance 'r' form a fixed sodium nucleus, as it approaches the nucleus, placed at origin O. Then the portion.



(A) AB indicates nuclear repulsion

(B) AB indicates electrostatic repulsion

(C) BC indicates nuclear attraction

(D) BC represents electrostatic interaction

Q.23 In β-decay, the Q-value of the process is E. Then

(A) K.E. of a β-particle cannot exceed E.

(B) K.E. of antineutrino emitted lies between Zero and E.

(C) N/X ratio of the nucleus is altered.

(D) Mass number (A) of the nucleus is altered.

Q.24 Consider the following nuclear reactions and select the correct statements from the option that follow.

Reaction I: $n \rightarrow p + e^- + \bar{v}$

Reaction II: $p \rightarrow n + e^+ + v$

(A) Free neutron is unstable, therefore reaction I is possible

(B) Free proton is stable, therefore reaction II is not possible

(C) Inside a nucleus, both decays (reaction I and II) are possible

(D) Inside a nucleus, reaction I is not possible but reaction II is possible

Q.25 When the nucleus of an electrically neutral atom undergoes a radioactive decay process, it will remain neutral after the decay if the process is:

(A) α decay (B) β-decay

(C) γ decay (D) K-capture

Q.26 The heavier nuclie tend to have larger N/Z ratio because-

(A) A neutron is heavier than a proton

(B) A neutron is an unstable particle

(C) A neutron does not exert electric repulsion

(D) Coulomb forces have longer range compared to the nuclear forces

Q.27 For nuclei with A>100

(A) The binding energy of the nucleus decreases on an average as A increases

(B) The binding energy per nucleon decreases on an average a A increases

(C) If the nucleus breaks into two roughly equal parts energy is released

(D) If two nuclei fuse to form a bigger nucleus energy is released

Q.28 A radioactive sample has initial concentration no. of nuclei-

(A) The number of undecayed nuclei present in the sample decays exponentially with time

(B) The activity (R) of the sample at any instant is directly proportional to the number of undecayed nuclei present in that sample at that time

(C) The no. of decayed nuclei grows exponentially with time

(D) The no. of decayed nuclei grow linearly with time

Q.29 A nuclide A undergoes α decay and another nuclide B undergoes β^- decay-

(A) All the α-particles emitted by A will have almost the same speed

(B) The α-particles emitted by A may have widely different speeds

(C) All the β-particles emitted by B will have almost the same speed

(D) The β-particles emitted by B may have widely different speeds.

Q.30 A nitrogen nucleus $^{14}_{7}N$ absorbs a neutron and can transform into lithium nucleus $_3Li^7$ under suitable conditions, after emitting :

(A) 4 protons and 3 neutrons

(B) 5 protons and 1 negative beta particle

(C) 1 alpha particle and 2 gamma particles

(D) 1 alpha particle, 4 protons and 2 negative beta particles

(E) 4 protons and 4 neutrons

Q.31 The instability of the nucleus can be due to various causes. An unstable nucleus emits radiations if possible to transform into less unstable state. Then the cause and the result can be

(A) A nucleus of excess nucleons is α − active

(B) An excited nucleus of excess protons is β^- active

(C) An excited nucleus of excess protons is β^+ active

(D) An nucleus of excess neutrons is β^- active

Assertion Reasoning Type

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is false.

Q.32 Half-life for certain radioactive element is 5 min. Four nuclei of that element are observed a certain instant of time. After five minutes

Statement-I: It can be definitely said that two nuclei will be left undecayed.

Statement-II: After half-life i.e. 5minutes, half of total nuclei will disintegrate. So only two nuclei will be left undecayed.

Q.33 Statement-I: Consider the following nuclear of an unstable $^{14}C_6$ nucleus initially at rest. The decay $^{14}_{6}C \rightarrow ^{14}_{7}N + ^{0}_{-1}e + \bar{\nu}$. In a nuclear reaction total energy and momentum is conserved experiments show that the electrons are emitted with a continuous range of kinetic energies upto some maximum value.

Statement-II: Remaining energy is released as thermal energy.

Q.34 Statement-I: It is easy to remove a proton from $^{40}_{20}Ca$ nucleus as compared to a neutron

Statement-II: Inside nucleus neutrons are acted on only by attractive forces but protons are also acted on by repulsive forces.

Q.35 Statement-I: It is possible for a thermal neutron to be absorbed by a nucleus whereas a proton or an α−particle would need a much larger amount of energy for being absorbed by the same nucleus.

Statement-II: Neutron is electrically neutral but proton and α− particle are positively charged.

Comprehension Type

Paragraph 1: (Q.36) A town has a population of 1 million. The average electric power needed per person is 300 W. A reactor is to be designed to supply power to this town. The efficiency with which thermal power is converted into electric power is aimed at 25%.

Q.36 Assuming 200 MeV of thermal energy to come from each fission event on an average the number of events that should take place every day.

(A) 2.24×10^{24} (B) 3.24×10^{24}

(C) 4.24×10^{24} (D) 5.24×10^{24}

Paragraph 2: A nucleus at rest undergoes a decay emitting an α particle of de-Broglie wavelength $\lambda = 5.76\times10^{-15}$ m. The mass of the daughter nucleus is 223.40 amu and that of α particle is 4.002 amu.

Q.37 The linear momentum of α particle and that of daughter nucleus is-

(A) 1.15×10^{-19} N – s & 2.25×10^{-19} N – s

(B) 2.25×10^{-19} N – s & 1.15×10^{-19} N – s

(C) Both 1.15×10^{-19} N – s

(D) Both 2.25×10^{-19} N – s

Q.38 The kinetic energy of α particle is-

(A) 0.01 Mev (B) 6.22 MeV

(C) 0.21 Mev (D) 0.31 MeV

Q.39 The kinetic energy of daughter nucleus is-

(A) 3.16 Mev (B) 4.16 MeV

(C) 5.16 MeV (D) 0.11 MeV

Match the Columns

Q.40

	Column I		Column II
(A)	In reaction $_1^2H + _1^2H \rightarrow _2^3He + X$ The X is	(p)	$_{82}^{206}Pb$
(B)	If $_{92}^{238}U$ decays by 8α & 6β the resulting nuclei is	(q)	$_0^1n$
(C)	Heavy water is	(r)	$_2^4He$
(D)	By emission of which particle the position in the periodic table is lowered by 2	(s)	$_7^{14}N$
(E)	When a deuterium is bombarded on $_8^{16}O$ nucleus, an α particle is emitted, the product nucleus is	(t)	D_2O

Q.41

	Column I		Column II
(A)	Nuclear Fusion	(p)	Some matter converted into energy
(B)	Nuclear Fission	(q)	Generally occurs in nuclei having low atomic number
(C)	β – decay	(r)	Generally occurs in nuclei having higher atomic number.
(D)	α – decay	(s)	Essentially occurs due to weak nuclear force.

Q.42

	Column I		Column II
(A)	1 Rutherford	(p)	1 dis/sec
(B)	1 Becquerel	(q)	3.7×10^{10} dis/sec
(C)	1 Curie	(r)	10^6 dis/sec
(D)	Activity of 1g Ra^{226}	(s)	10^{10} dis/sec

Radioactivity

Single Correct Choice Type

Q.43 The analysis of a mineral of Uranium reveals that ratio of mole of ^{206}Pb and ^{238}U in sample is 0.2. If effective decay constant of process $^{238}U \rightarrow {}^{206}Pb$ is λ then age of rock is

(A) $\frac{1}{\lambda}\ln\frac{5}{4}$ (B) $\frac{1}{\lambda}\ln\left(\frac{5}{1}\right)$ (C) $\frac{1}{\lambda}\ln\frac{4}{1}$ (D) $\frac{1}{\lambda}\ln\left(\frac{6}{5}\right)$

Q.44 The half-life of Tc^{99} is $6.0\,hr$. The delivery of a sample of Tc^{99} that must be shipped in order for the lab to receive 10.0 mg?

(A) 20.0 mg (B) 15.0 mg

(C) 14.1 mg (D) 12.5 mg

Q.45 A sample contains 0.1 gram-atom of radioactive isotope $^{A}_{Z}X\left(t_{1/2}=5\text{days}\right)$. How many number of atoms will decay during eleventh day? [N_A =Avogadro's number]

(A) $0.1\left(-e^{-\frac{0.693\times11}{5}}+e^{-\frac{0.693\times10}{5}}\right)$

(B) $0.1\left(-e^{\frac{0.693\times11}{5}}+e^{\frac{0.693\times10}{5}}\right)$

(C) $0.1\left(-e^{-\frac{0.693\times11}{5}}+e^{-\frac{0.693\times10}{5}}\right)N_A$

(D) $0.1\left(-e^{\frac{0.693\times11}{5}}+e^{\frac{0.693\times10}{5}}\right)N_A$

Multiple Correct Choice Type

Q.46 Which of the following statements are correct about half-life period?

(A) It is proportional to initial concentration for zero-th order.

(B) Average life=1.44 half-life for first order reaction

(C) time of 75% reaction is thrice of half –life period in second order reaction.

(D) 99.9% reaction takes place in 100 minutes for the case when rate constant is 0.0693 min^{-1} is 0.5

Q.47 C^{14} is a beta active nucleus. A sample of $C^{14}H_4$ gas kept in a closed vessel shows increase in pressure with time. This is due to

(A) The formation of $N^{14}\ H_3$ and H_2

(B) The formation of $B^{11}\ H_3$ and H_2

(C) The formation of $C^{14}\ H_4$ and H_2

(D) The formation of $C^{12}\ H_3, N^{14}\ H_2$ and H_2

Q.48 Select correct statement (s):

(A) The emission of gamma radiation involves transition between energy levels within the nucleus.

(B) $^{4}_{2}$He is formed due to emission of beta particle from tritium $^{3}_{1}H$.

(C) When positron ($^{o}_{+1}e$) is emitted, $\frac{n}{p}$ ratio increases.

(D) In general, adsorption is exothermic process.

Comprehension Type

Paragraph 1: Nuclei of a radioactive element 'A' are being produced at a constant rate, α. The element has a decay constant, λ. At time, t=0, there are N_0 nuclei of the element.

Q.49 The number of nuclei of A at time't' is

(A) $\frac{\alpha}{\lambda}(1-e^{-\lambda t})$ (B) $N_0 . e^{\lambda t}$

(C) $\frac{1}{\lambda}[\alpha-(\alpha-\lambda N_0)e^{-\lambda t}]$ (D) $\frac{N_0 .\alpha}{\lambda}\left[1-\left(1-\frac{\lambda}{\alpha}\right)e^{-\lambda t}\right]$

Q.50 If $\alpha = 2N_0\lambda$, the number of nuclei of A after one half-life of A becomes

(A) Zero (B) $2N_0$ (C) $1.5N_0$ (D) $0.5N_0$

Paragraph 2: Mass defect in the nuclear reactions may be expressed in terms of the atomic masses of the parent and daughter nuclides in place of their nuclides in place of their nuclear masses.

Q.51 The mass defect of nuclear reaction:

$_4Be^{10} \rightarrow {_5B^{10}} + \bar{e}$ is

(A) Δm=At. mass of $^{10}_{4}Be$ – At. mass of $^{10}_{5}B$

(B) Δm=At. mass of $^{10}_{4}Be$ – At. mass of $^{10}_{5}B$ – mass of one electron

(C) Δm=At. mass of $^{10}_{4}Be$ – At. mass of $^{10}_{5}B$ + mass of one electron

(D) Δm=At. mass of $^{10}_{4}Be$ – At. mass of $^{10}_{5}B$ – mass of two electron

Q.52 The mass defect of the nuclear reaction: $_5B^8 \rightarrow {_4Be^8} + e^+$ is

(A) Δm=At. mass of $^{8}_{5}B$ – At. mass of $^{8}_{4}Be$

(B) $\Delta m=$ At. mass of ${}_{5}^{8}B$ – At. mass of ${}_{4}^{8}Be$ -mass of one electron

(C) $\Delta m=$ At. mass of ${}_{5}^{8}B$ – At. mass of ${}_{4}^{8}Be$ + mass of one electron

(D) $\Delta m=$ At. mass of ${}_{5}^{8}B$ – At. mass of ${}_{4}^{8}Be$ - mass of two electron

Previous Years' Questions

Q.1 There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half-life of neutrons is 700 s, what fraction of neutrons will decay before they travel a distance of 10m? ***(1986)***

Q.2 It is proposed to use the nuclear fusion reaction;

${}_{1}^{2}H+{}_{1}^{2}H\rightarrow{}_{2}^{4}He$

In a nuclear reactor 200MW rating. If the energy from the above reaction is used with a 25 percent efficiency in the reactor, how many grams of deuterium fuel will be needed per day? (The masses of ${}_{1}^{2}H$ and ${}_{2}^{4}He$ are 2.0141 atomic mass units and 4.0026 atomic mass units respectively.) ***(1990)***

Q.3 A nucleus X, initially at rest, undergoes alpha-decay according to the equation ${}_{92}^{A}X\rightarrow{}_{z}^{228}Y+\alpha$.

(a) Find the values of A and Z in the above process.

(b) The alpha particle produced in the above process is found to move in a circular track of radius 0.11m in a uniform magnetic field of 3T. Find the energy (in Mev) released during the process and the binding energy of the parent nucleus X.

Given that $m(Y)=228.03u; m({}_{0}^{1}n)=1.009u$

$m({}_{2}^{4}He)=4.003u; m({}_{1}^{1}H)=1.008u.$ ***(1991)***

Q.4 A small quantity of solution containing Na^{24} radio nuclide (half-life=15h) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5h shows an activity of 296 disintegrations per minute. Determine the total volume of the blood in the body of the person. Assume that the radioactive solution mixes uniformly in the blood of the person.

(1 curie= 3.7×10^{10} disintegrations per second) ***(1994)***

Q.5 The element curium ${}_{96}^{248}Cm$ has a mean life of $10^{13}s$. Its primary decay modes are spontaneous fission and α-decay, the former with a probability of 8% and the latter with a probability of 92%, each fission releases 200 MeV of energy. The masses involved in decay are as follows: ***(1997)***

${}_{96}^{248}Cm=248.072220u,$

${}_{94}^{244}Pu=244.064100u$ and ${}_{2}^{4}He=4.002603u.$

Calculate the power output from a sample of 10^{20} Cm atoms. $(1u=931MeV/c^2)$

Q.6 Nuclei of a radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time $t=0$, there are N_0 nuclei of the element. ***(1998)***

(a) Calculate the number N of nuclei of A at time t.

(b) If $\alpha=2N_0\lambda$, calculate the number of nuclei of A after one hale-life of A and also the limiting value of N as $t\rightarrow\infty$.

Q.7 In a nuclear reactor ${}^{235}U$ undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10yr, find the total mass of uranium required. ***(2001)***

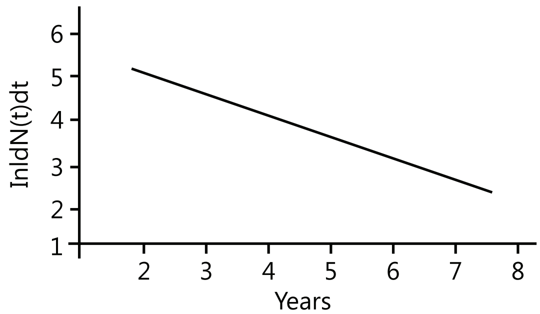
Q.8 A radioactive nucleus X decays to a nucleus Y with a decay constant $\lambda_x=0.1s^{-1}$, Y further decays to a stable nucleus Z with a decay constant $\lambda_y=1/30s^{-1}$. Initially, there are only X nuclei and their number is $N_0=10^{20}$. Set up the rate equations for the populations of X, Y and z. The population of Y nucleus as a function of time is given by

$$N_y(t)=\{N_0\lambda_x/(\lambda_x-\lambda_y)\}\left[\exp(-\lambda_y t)-\exp(-\lambda_x t)\right]$$

Find the time at which N_y is maximum and determine the populations X and Z at that instant. ***(2001)***

Q.9 A rock is 1.5×10^9 yr old. The rock contains ${}^{238}U$ which disintegrates to from ${}^{206}Pb$. Assume that there was no ${}^{206}Pb$ in the rock initially and it is the only stable product formed by the decay. Calculate the ratio of number of nuclei of ${}^{238}U$ to that of ${}^{206}Pb$ in the rock. Half-life of ${}^{238}U$ is 4.5×10^9 yr. $(2^{1/3}=1.259)$ ***(2004)***

Q.10 To determine the half-life of a radioactive element, a student plots a graph of $\ln\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16yr, the value of p is ***(2010)***



Q.11 The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^{-9} s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is ***(2011)***

Q.12 Some laws and processes are given in column I. Match these with the physical phenomena given in column II. ***(2006)***

	Column I		Column II
(A)	Nuclear Fusion	(p)	Converts some matter into energy
(B)	Nuclear Fusion	(q)	Generally possible for nuclei with low Atomic number
(C)	$\beta-$decay	(r)	Generally possible for nuclei with higher Atomic number.
(D)	Exothermic nuclear reaction	(s)	Essentially proceeds by weak nuclear forces

Q.13 In the core of nuclear fusion reactor, the gas becomes plasma because of ***(2009)***

(A) Strong nuclear force acting between the deuterons

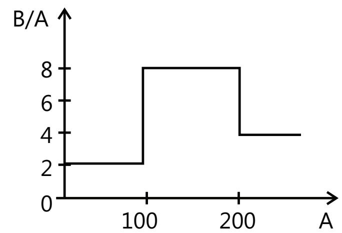
(B) Coulomb force acting between the deuterons

(C) Coulomb force acting between deuteron-electron pairs

(D) The high temperature maintained inside the reactor core

Q.14 Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other , each with kinetic energy 1.5kT, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature t required for them to reach a separation of 4×10^{-15}m is in the range ***(2009)***

Q.15 Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice(s) given below. ***(2008)***



(A) Fusion of two nuclei with mass numbers lying in the range of 1 < A < 50 will release energy

(B) Fusion of two nuclei with mass numbers lying in the range of 51 < A < 100 will release energy

(C) Fission of a nucleus lying in the mass range of 100 < A < 200 will release energy when broken into two equal fragments

(D) Fission of a nucleus lying in the mass range of 200 < A < 260 will release energy when broken into two equal fragments

Q.16 Results of calculations for four different designs of a fusion reactor using D-D reaction are given below.

Which of these is most promising based on Lawson criterion? ***(2009)***

(A) Deuteron density = 2.0 $\times10^{12}cm^{-3}$, confinement time = 5.0 $\times10^{-3}$s

(B) Deuteron density = 8.0 $\times10^{14}cm^{-3}$, confinement time = 9.0 $\times10^{-1}$s

(C) Deuteron density = 4.0 $\times10^{23}cm^{-3}$,confinement time = 1.0 $\times10^{-11}$s

(D) Deuteron density = 1.0 $\times10^{24}cm^{-3}$, confinement time = 4.0 $\times10^{-12}$s

Q.17 A freshly prepared sample of a radioisotope of half-life 1386s has activity 10^3 disintegrations per second.

Given that ln 2 = 0.693, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is ***(2013)***

(A) 4% (B) 5% (C) 5.5% (D) 3%

Q.18 A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5 % of the electrical power available form the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is ***(2014)***

(A) 2 (B) 5 (C) 3 (D) 4

Q.19 Match the nuclear processes given in column I with the appropriate option(s) in column II ***(2015)***

	Column I		Column II
(A)	Nuclear fusion	(p)	Absorption of thermal neutrons by $^{235}_{92}U$
(B)	Fission in a nuclear reactor	(q)	$^{60}_{27}$Co nucleus
(C)	β -decay	(r)	Energy production in stars via hydrogen conversion to helium
(D)	γ -ray emission	(s)	Heavy water
		(t)	Neutrino emission

Q.20 The isotope $^{12}_{5}$B having a mass 12.014 u undergoes β decay to $^{12}_{6}C$. $^{12}_{6}C$ has an excited state of the nucleus ($^{12}_{6}C^*$) at 4.041 MeV above its ground state. If $^{12}_{5}B$ decays to $^{12}_{6}C^*$, the (1 u = 931.5 MeV/c^2), where c is the speed of light in vacuum) ***(2016)***

Q.21 A radioactive sample S1 having an activity 5μCi has twice the number of nuclei as another sample S2 which has an activity of 10 μCi. The half lives of S1 and S2 can be ***(2008)***

(A) 20 years and 5 years, respectively

(B) 20 years and 10 years, respectively

(C) 10 years each

(D) 5 years each

Q.22 The electric field at r = R is ***(2008)***

(A) Independent of a

(B) Directly proportional to a

(C) Directly proportional to a^2

(D) Inversely proportional to a

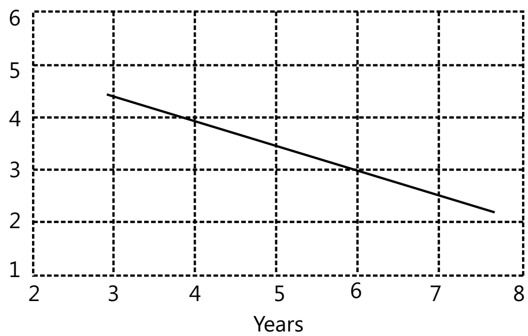
Q.23 For a = 0, the value of d (maximum value of ρ as shown in the figure) is ***(2008)***

(A) $\frac{3Ze}{4\pi R^3}$ (B) $\frac{3Ze}{\pi R^3}$ (C) $\frac{4Ze}{3\pi R^3}$ (D) $\frac{Ze}{3\pi R^3}$

Q.24 The electric field within the nucleus is generally observed to be linearly dependent on r. This implies. ***(2008)***

(A) a = 0 (B) $a = \frac{R}{2}$ (C) a = R (D) $a = \frac{2R}{3}$

Q.25 To determine the half-life of a radioactive element, a student plots a graph of $\log\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\frac{dN(t)}{dt}$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is ***(2009)***



Q.26 What is the maximum energy of the anti-neutrino? ***(2012)***

(A) Zero

(B) Much less than 0.8 × 10^6 eV

(C) Nearly 0.8 × 10^6 eV

(D) Much larger than 0.8 × 10^6 eV

Q.27 If the anti-neutrino had a mass of 3eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron? ***(2012)***

(A) $0 \le K \le 0.8 \times 10^6$ eV

(B) $3.0 \text{ eV} \le K \le 0.8 \times 10^6$ eV

(C) $3.0 \text{ eV} \le K < 0.8 \times 10^6$ eV

(D) $0 \le K < 0.8 \times 10^6$ eV

Q.28 The radius of the orbit of an electron in a Hydrogen-like atom is 4.5 a_0 where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck's constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is (are) ***(2013)***

(A) $\frac{9}{32R}$ (B) $\frac{9}{16R}$ (C) $\frac{9}{5R}$ (D) $\frac{4}{3R}$

Direction: The mass of nucleus $^{A}_{Z}X$ is less than the sum of the masses of (A-Z) number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of mass m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m3 and m4 can undergo complete fusion and form a heavy nucleus of mass M′ only if $(m_1 + m_4) > M'$. The masses of some neutral atoms are given in the table below:

$^{1}_{1}H$	1.007825 u	$^{2}_{1}H$	2.014102 u
$^{6}_{3}Li$	6.015123 u	$^{7}_{3}Li$	7.016004 u
$^{152}_{64}Gd$	151.919803 u	$^{206}_{82}Pb$	205.974455 u
$^{3}_{1}H$	3.016050 u	$^{4}_{1}He$	4.002603 u
$^{70}_{30}Zn$	69.925325 u	$^{82}_{34}Se$	81.916709 u
$^{209}_{83}Bi$	208.980388 u	$^{210}_{84}Po$	209.982876 u

Q.29 The correct statement is ***(2013)***

(A) The nucleus $^{6}_{3}$ Li can emit an alpha particle

(B) The nucleus $^{210}_{84}$ Po can emit a proton.

(C) Deuteron and alpha particle can undergo complete fusion.

(D) The nuclei $^{70}_{30}$ Zn and $^{82}_{34}$ Se can undergo complete fusion.

Q.30 The kinetic energy (in keV) of the alpha particle, when the nucleus $^{210}_{84}$ Po at rest undergoes alpha decay, is ***(2013)***

(A) 5319 (B) 5422 (C) 5707 (D) 5818

Q.31 Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists: ***(2013)***

	List I		List II
(i)	Alpha decay	(p)	$^{15}_{8}O \rightarrow ^{15}_{7}N + ...$
(ii)	β^{+} decay	(q)	$^{258}_{92}U \rightarrow ^{234}_{90}Th + ...$
(iii)	Fission	(r)	$^{185}_{83}Bi \rightarrow ^{184}_{82}Pb + ...$
(iv)	Proton emission	(s)	$^{239}_{94}Pu \rightarrow ^{140}_{57}La + ...$

Codes:

	p	q	r	s
(A)	(iv)	(ii)	(i)	(iii)
(B)	(i)	(iii)	(ii)	(iv)
(C)	(ii)	(i)	(iv)	(iii)
(D)	(iv)	(iii)	(ii)	(i)

Q.32 If λ_{cu} is the wavelength of K_{α} X-ray line of copper (atomic number 29) and λ_{Mo} is the wavelength of the K_{α} X-ray line of molybdenum (atomic number 42), then the ratio $\lambda_{cu}/\lambda_{Mo}$ is close to ***(2014)***

(A) 1.99 (B) 2.14 (C) 0.50 (D) 0.48

Q.33 An electron in an excited state of Li^{2+} ion has angular momentum $3h/2\pi$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is ***(2015)***

(A) πa_0 (B) $2\pi a_0$ (C) $4\pi a_0$ (D) $3\pi a_0$

Q.34 For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -\frac{dN}{dt}$ and R= dA $-\frac{dA}{dt}$, where N(t) is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q(mean life 2τ) have the same activity at t = 0. Their rates of change of activities at t = 2τ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is ***(2015)***

(A) $\frac{1}{2e}$ (B) $\frac{2}{e}$ (C) $\frac{3}{e}$ (D) $\frac{2}{3e}$

Q.35 A fission reaction is given by $^{236}_{92}U \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + x + y$, where x and y are two particles. Considering ${}^{236}_{92}U$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr}, K_x(2MeV) and K_y(2MeV), respectively. Let the binding energies per nucleon of ${}^{236}_{92}U$, ${}^{140}_{54}Xe$ and ${}^{94}_{38}Sr$ be 7.5 MeV, 8.5 MeV and 8.5 MeV respectively. Considering different conservation laws, the correct option(s) is(are) ***(2015)***

(A) x = n, y = n, K_{Sr} = 129MeV, K_{Xe} = 86 MeV

(B) x = p, y = e^-, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV

(C) x = p, y = n, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV

(D) x = n, y = n, K_{Sr} = 86 MeV, K_{Xe} = 129 MeV

Q.36 The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by $E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$

The measured masses of the neutron, ${}^{1}_{1}H$, ${}^{15}_{7}N$ and ${}^{15}_{8}O$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the ${}^{15}_{7}N$ and ${}^{15}_{8}O$ nuclei are same, 1 u = 931.5 MeV/c^2 (c is the speed of light) and $e^2/(4\pi\varepsilon_0)$ = 1.44 MeV fm. Assuming that the difference between the binding energies of ${}^{15}_{7}N$ and ${}^{15}_{8}O$ is purely due to the electrostatic energy, the radius of either of the nuclei is (1 fm = 10^{-15} m) ***(2016)***

(A) 2.85 fm (B) 3.03 fm

(C) 3.42 fm (D) 3.80 fm

Q.37 An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use? ***(2016)***

(A) 64 (B) 90 (C) 108 (D) 120

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q.5 Q.13 Q.28

Q.31 Q.34 Q.40

Exercise 2

Q.6 Q.11 Q.23

Q.24 Q.26 Q.27

Q.29

JEE Advanced/Boards

Exercise 1

Q. 7 Q.8 Q.13 Q.14

Q.15 Q.17 Q.19 Q.23

Exercise 2

Q.1 Q.12 Q.13 Q.22

Q.27 Q.37 Q.38 Q.39

Q.43 Q.44 Q.40 Q.49

Q.50

Previous Years' Question

Q.1 Q.7 Q.10 Q.12

Q.14

Answer Key

JEE Main/Boards

Exercise 1

Nuclear Physics

Q.1 56.45 days
Q.2 449.94 year
Q.3 7s
Q.4 $\frac{\alpha}{\lambda}$
Q.5 $4.57\times10^{21}\text{days}^{-1}$
Q.6 384.5g

Radioactivity

Q.26 beta emitter: ^{49}Ca, ^{30}Al, ^{94}Kr, positron emitter : ^{195}Hg, ^{8}B, ^{150}Ho

Q.27 $^{114}_{49}\text{In}$, odd number of neutrons
Q.28 $(a)\,^1_1H,\ (b)\,^1_0n,\ (c)\,^6_3Li,\ (d)\,^0_{+1}e,\ (e)\,^0_{-1}e,\ (f)\,^1_{+1}p$
Q.29 $\lambda = 2.078\,\text{hr}^{-1}$
Q.30 5.05×10^6 atoms
Q.31 6.25%
Q.32 $2.67\times10^5\ \text{sec}^{-1}$
Q.33 33.67 years
Q.34 (i) $^{40}_{19}K \to\, ^{40}_{18}Ar +\, ^0_1 e + v$ (ii) 2.8×10^9 years
Q.35 (i) $t_{means} = 14.43s$ (ii) 40 sec
Q.36 $\Delta E = 14.25$ Mev
Q.37 (a) No.of α -particles=8, No.of β -particles=6; (b) $^{206}_{82}Pb$
Q.38 6.13×10^{-7} g
Q.39 (i) 31.25 cm^3, 27.104 cm^3 (ii) 4.5×10^9 year
Q.40 $6.30\times10^{-4}\ yr^{-1}, 3.087\times10^{-2}\ yr^{-1}$

Exercise 2

Nuclear Physics

Single Correct Choice Type

Q.1 C	**Q.2** B	**Q.3** B	**Q.4** A	**Q.5** B	**Q.6** A
Q.7 B	**Q.8** B	**Q.9** A	**Q.10** A		

Radioactivity

Single Correct Choice Type

Q.11 B	**Q.12** C	**Q.13** A	**Q.14** B	**Q.15** B	**Q.16** B
Q.17 C	**Q.18** A	**Q.19** D	**Q.20** A	**Q.21** A	**Q.22** C
Q.23 A	**Q.24** C	**Q.25** A	**Q.26** C	**Q.27** C	**Q.28** B
Q.29 A	**Q.30** B	**Q.31** D			

Previous Years' Question

Q.1. B	**Q.2** C	**Q.3** C	**Q.5** C	**Q.6** B	**Q.7** A
Q.8 A	**Q.9** 125decays/sec	**Q.10** Alpha=8, beta=6		**Q.11** 24 Mev	**Q.12** D
Q.13 B	**Q.14** A	**Q.15** D	**Q.16** C	**Q.17** B	**Q.18** B
Q.19 B	**Q.20** B	**Q.21** D	**Q.22** A	**Q.23** A	**Q.24** C

JEE Advanced/Boards

Exercise 1

Nuclear Physics

Q.1 23.6 Mev

Q.2 (i) ${}^{40}_{19}K \rightarrow {}^{40}_{18}Ar + {}^{0}_{1}e + v$ (ii) 4.2×10^{9} years

Q.3 $t = \left(\frac{\ln 5}{\ln 2}\right)\tau$

Q.4 2.73×10^{18}sec

Q.8 1.7×10^{10} years

Q.9 5196 yrs

Q.10 28 Mev

Q.11 9.00×10^{6} eV

Q.12 $v=-u\lambda t$

Q.13 $\Delta T = \frac{0.2E_0\left[\alpha t - \frac{\alpha}{\lambda}\left(1-e^{-\lambda t}\right)\right]}{ms}$

Radioactivity

Q.14 $t = 4.89\times10^{9}$ years

Q.15 $\left(T_{1/2} = 10.8\,\text{sec}\right)$

Q.16 6 litre

Q.17 ^{239}Pu=44.7%, ^{240}Pu=55.3%

Q.18 (a) $N = \frac{1}{\lambda}\left[\alpha\left(1-e^{-\lambda t}\right) + \lambda N_0 e^{-\lambda t}\right]$ (b) $\frac{3N_0}{2}, 2N_0$

Q.19 0.833×10^{-5}mol/lit sec

Q.20 $t = 7.1\times10^{8}$ years

Q.21 4.125 min

Q.22 (a) 1.143×10^{9} year, (b) 7.097×10^{8} year

Q.23 3.43×10^{-18} mol

Exercise 2

Nuclear Physics

Single Correct Choice Type

Q.1 D	**Q.2** D	**Q.3** C	**Q.4** B	**Q.5** B	**Q.6** D
Q.7 C	**Q.8** B	**Q.9** C	**Q.10** A	**Q.11** C	**Q.12** B
Q.13 E	**Q.14** C	**Q.15** C	**Q.16** B	**Q.17** C	

Multiple Correct Choice Type

Q.18 A, B, D **Q.19** A, C **Q.20** C, D **Q.21** A, C **Q.22** B, C **Q.23** A, B, C
Q.24 A, B, C **Q.25** C, D **Q.26** C, D **Q.27** B, C **Q.28** A, B, C **Q.29** A, D
Q.30 D, E **Q.31** A, C, D

Assertion Reasoning Type

Q.32 D **Q.33** C **Q.34** A **Q.35** A

Comprehension Type

Q.36 B **Q.37** C **Q.38** B **Q.39** D

Matric Match Type

Q.40 A→q; B→p; C→t; D→r; E→s

Q.41 A→p, q; B→p, r; C→p, s; D→r, s

Q.42 A→r; B→p; C→q; D→q

Radioactivity

Single Correct Choice Type

Q.43 D **Q.44** C **Q.45** C

Multiple Correct Choice Type

Q.46 A, B, C, D **Q.47** B, C, D **Q.48** C, D

Comprehension Type

Q.49 C **Q.50** C **Q.51** A **Q.52** D

Previous Years' Questions

Q.1 3.96×10^{-6} **Q.2** 120.26 g **Q.3** 1823.2 MeV **Q.4** V=5.95 L **Q.5** 3.32×10^{-5} W

Q.7 3.847×10^{4}kg **Q.9** 3.861 **Q.10** 8 **Q.11** 1

Q.12 A → p, q; B → p, r; C → p, s; D → p, q, r **Q.13** D **Q.14** T = 1.4×10^{9} K

Q.15 B, D **Q.16** B **Q.17** A **Q.18** C

Q.19 A → r, t; B → p, s; C → p, q, r, t; D → p, q, r, t **Q.20** 9 MeV **Q.21** A

Q.22 A **Q.23** B **Q.24** C **Q.25** 8 **Q.26** C

Q.27 D **Q.28** A, C **Q.29** C **Q.30** A **Q.31** C

Q.32 B **Q.33** B **Q.34** B **Q.35** A **Q.36** C

Q.37 C

Solutions

JEE Main/Boards

Exercise 1

Sol 1: $t_{1/2} = \frac{\ln 2}{\lambda} = 10$

$\Rightarrow \lambda = \frac{\ln 2}{10}$ (days)$^{-1}$

Now, $\frac{N}{N_0} = \frac{1}{50}$ and $N = N_0 e^{-\lambda t}$

$\Rightarrow e^{-\lambda t} = \frac{1}{50} \Rightarrow \ln 50 = \lambda t$

$\Rightarrow t = \frac{10 \times \ln 50}{\ln 2} = 56.44$ days

Sol 2: $\lambda_1 = \frac{1}{1620}$ years^{-1} and $\lambda_2 = \frac{1}{405}$ years^{-1}

Now, $\frac{dN}{dt} = -(\lambda_1 t + \lambda_2 t) \Rightarrow \frac{dN}{dt} = -(\lambda_1 + \lambda_2)\, t$

$\Rightarrow N = N_0 \, . \, e^{-\lambda_{tot}.t}$

So, $2t_{1/2} = \frac{2.\ln 2}{\lambda_{tot}} = \frac{2.\ln 2}{\frac{1}{1620} + \frac{1}{405}}$

$= \frac{810.\ln 2}{1 + \frac{1}{4}} = \frac{4 \times 810.\ln 2}{5} = 449$ years

Sol 3: $N = N_0 \, . \, e^{-\lambda t}$

So in 1st 2 sec,

$\Delta N_1 = N_0 \, . - N_0 \, . e^{-\lambda 2} = N_0 \, . \, (1 - e^{-2\lambda})$

in other 2 sec,

$\Delta N_2 = N_0 \, . \, e^{-2\lambda} - N_0 \, . \, e^{-4\lambda} = N_0 \, . \, e^{-2\lambda}(1 - e^{-2\lambda})$

Now, $\frac{N_0.(1 - e^{-2\lambda})}{N_0.e^{-2\lambda}.(1 - e^{-2\lambda})} = \frac{n}{0.75n} = \frac{4}{3}$

$\Rightarrow \frac{3}{4} = e^{-2\lambda}$

$\Rightarrow e^{2\lambda} = \frac{4}{3}$

$\Rightarrow 2\lambda = 2\ln 2 - \ln 3$

$\Rightarrow \lambda = (\ln 2 - (\ln 3)/2)$ sec^{-1}

Now mean life

$= \frac{1}{\lambda} = \left[\frac{1}{\ln 2 - \frac{(\ln 3)}{2}}\right]$ sec

$= \left[\frac{1}{0.6931 - \frac{(1.0986)}{2}}\right] = 6.9 \approx 7$ sec

Sol 4: (a) $\frac{dN}{dt} = \alpha - \lambda N \Rightarrow \frac{dN}{dt} + lN = a$

$\Rightarrow \int d\left[N.e^{\lambda t}\right] = \int\left[\alpha.e^{\lambda t}\right] . \, dt$

$\Rightarrow \left[N.e^{\lambda t}\right]_0^t = \left[\alpha.e^{\lambda t}\right]_0^t / \lambda$

$\Rightarrow N \, . \, e^{\lambda t} - N_0 = (\alpha \, . \, e^{\lambda t} - \alpha)/\lambda$

$\Rightarrow N = N_0 \, . \, e^{-\lambda t} + \frac{\alpha}{\lambda}(1 - e^{-\lambda t})$

(B) $\alpha = 2N_0 l$

After one half life, $\boxed{t_{1/2} = \frac{\ln 2}{\lambda}}$

So, $t = \frac{\ln 2}{\lambda}$

$N = N_0 \, . \, e^{-\ln 2} + \frac{\alpha}{\lambda} . \, (1 - e^{-\ln 2})$

$= \frac{N_0}{2} + \frac{\alpha}{\lambda} . \, (1 - 1/2)$

$\boxed{N = \frac{N_0}{2} + \frac{\alpha}{2\lambda} = \left(N_0 + \frac{\alpha}{\lambda}\right) \times \frac{1}{2}}$

Now, as $t \to \infty$,

$N = N_0\,(0) + \frac{\alpha}{\lambda}\,(1 - 0) \Rightarrow \boxed{N = \frac{\alpha}{\lambda}}$

Sol 5: ${}_{84}Po^{210} \longrightarrow {}_{2}\alpha^{4} + {}_{82}Pb^{206}$

So, $t_{1/2} = \frac{\ln 2}{\lambda} = 138.6$ days

$\Rightarrow \lambda = \frac{\ln 2}{138.6}$ (days)$^{-1}$

Now, Mass defect

= 209.98264 – (205.97440 + 4.00260)

= 0.00564 amu

= 5.251 MeV.

1.6×10^{-19} J = 1 eV

So, Mass defect = $1.6 \times 10^{-19} \times 10^{6} \times 5.251$

= 8.4×10^{-13} J

So to produce 1.2×10^{7} J energy (at 0.1 efficiency)

Number of reactions

$8.4 \times 10^{-13}\pi\left(\frac{dN}{dt}\right) \times 0.1 = 1.2 \times 10^{7}$

$\Rightarrow \left(\frac{dN}{dt}\right) = \frac{1.2}{8.4} \times 10^{21} = \frac{1}{7} \times 10^{21}$

Now, $\frac{dN}{dt} = \lambda N = \frac{1}{7} \times 10^{21}$

$\Rightarrow \lambda.N_0.e^{-\lambda t} = \frac{1}{7} \times 10^{21}$

$\Rightarrow N_0 = \frac{1}{7} \times 10^{21} \times e^{\lambda t} . \frac{1}{\lambda}$

$= \frac{1}{7} \times 10^{21} \times \left(e^{\frac{\ln 2}{138.6} \times 693}\right) \times \frac{1}{\ln 2} \times 138.6$

= $28.56 \times 32 \times 10^{21}$

$N_0 = 9.13 \times 10^{23}$

Now, number of moles = $\frac{N_o}{6 \times 10^{23}} = 1.52$

So mass = 1.52 × 210 gm = 319.2 gm

Initial activity = λN_0

$= \frac{\ln 2}{138.6} \times 9.13 \times 10^{23} = 4.6 \times 10^{21}$ days^{-1}

Sol 6: Energy per fission = 200 MeV

= 200×10^{6} eV

= 200×10^{6} eV

= $200 \times 10^{6} \times 1.6 \times 10^{-19}$ J

= 3.2×10^{-11} J

Now, number of fissions required / time

$= \frac{1 \times 10^{6}}{3.2 \times 10^{-11}} = \frac{10}{3.2} \times 10^{18}$

= 3.125×10^{16} fissions

Number of fissions in 1 year

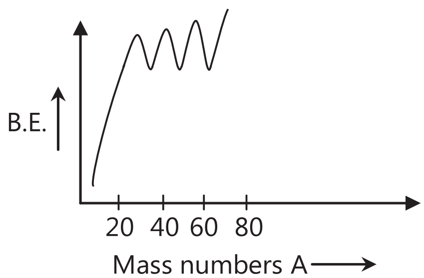
= $3.125 \times 10^{16} \times 365 \times 24 \times 60 \times 60$

= 9.855×10^{23}

Moles of uranium required = 1.637 moles

Mass of Uranium = 384.5 g

Sol 7:



Now higher the BE/nucleon higher the stability.

So light nuclei try to get high $\frac{BE}{Nucleon}$ ratio by going through nuclear fusion and hence increasing their atomic number.

Sol 8: Now number of particles decaying is directly proportional to the number of particles present in the reaction.

i.e. $\boxed{\frac{dN}{dt} \propto N}$

$\Rightarrow$ This is equated by a constant known as decaying constant.

$\boxed{\frac{dN}{dt} = \lambda N}$

(i) X-rays and gamma rays both electromagnetic.

(ii) γ-rays

(iii) γ-rays

(iv) β-rays (Both (–) ve)

Sol 9: Mass defect $m\left({}^{6}_{3}Li\right) + m\left({}^{1}_{0}n\right) - \left[m\left({}^{4}_{2}H\right) + m\left({}^{3}_{1}H\right)\right]$

$\Rightarrow$ (6.015126 + 1.008665)

– (4.002604 + 3.016049)

= 0.010697 amu = 9.96 MeV

Sol 10: n/p ratio decreases due to beta decay.

i.e. ${}_{6}C^{14} \longrightarrow {}_{7}N^{14} + \beta^{-1} + \bar{\nu}$

$\Rightarrow \frac{n}{p} = \frac{8}{6} = \frac{4}{3}, | \frac{n}{p} = \frac{7}{7} = 1$

Sol 11: Decay constant refer ans. 8

Half-life period: the time taken by a disintegration reaction to half the total number of particles in a sample.

Sol 12: Mass defect = 2.0141 + 6.0155 – 2x (4.0026)

= 0.0244 m

So energy transferred to KE

= (0.0244) × 931 MeV

= 22.7164 MeV

So energy for each particle

$= \frac{22.7164}{2} = 11.36$ MeV

$= 11.36 \times 1.6 \times 10^{-19} \times 10^{6}$

$= 18.176 \times 10^{-13} = 1.8176 \times 10^{-12}$ J

Sol 13: Number of moles $= \frac{2.2\times10^{-3}}{11}$

$= 0.2 \times 10^{-3} = 0.2 \times 10^{-4}$ moles

(i) Number of moles × A_0 = Number of particles

$= 6.0022 \times 10^{23} \times 2 \times 10^{-4}$

$= 12.044 \times 10^{19}$

(ii) Activity $= \lambda N = \frac{dN}{dt}$

$$\text{So} = \overbrace{\frac{\ln 2}{1224}}^{\lambda} \times \overbrace{\frac{5\times10^{-6}}{11}\times6\times10^{23}}^{N} = 1.54 \times 10^{14}$$

Sol 14: Half-life period: sec

Decay constant ⇒ $\sec^{-1} - \frac{dN}{dt} = 4\,N \times \lambda$

$\Rightarrow N = N_0 \cdot e^{-\lambda t}$

Now, $N = N_0 / 2$

$\Rightarrow e^{\lambda t_{1/2}} = 2$

$\Rightarrow \lambda t_{1/2} = \ln 2$

$\Rightarrow t_{1/2} = \frac{\ln 2}{\lambda}$

Sol 15: (i) ${}^{6}_{3}Li + {}^{1}_{0}n \rightarrow {}^{4}_{2}\alpha$ + triton

(ii) Mass defect

= mass before reaction – mass after reaction

=[(6.015126) + (1.0086554)] – [4.0026044 + 3.0100000]

= 0.011177 m

So energy released = 0.011177 × 931 MeV

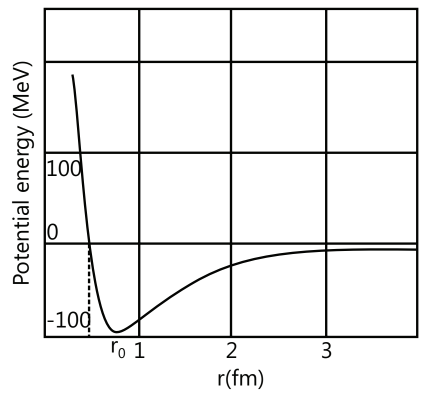
= 10.405 MeV

Sol 16: Activity = rate of change of number of particles in a disintegration reaction.

SI unit ⇒ $\frac{dN}{dt}$

⇒ SI Unit = $\sec^{-1}$

Sol 17: (i) Graph



(ii) For $r > r_0$ Attraction

(iii) For $r < r_0$ Repulsion

Sol 18: Refer Q.7

Mass defect = 20 × mass of proton + 20 × mass of neutron – [mass of ${}^{40}_{20}Ca$]

= 20×[1.0007825 + 1.008665] – 39.962589

= 0.226361 u

So, energy = (Δmc^2)

$= 0.226361 \times \frac{931}{c^2}$ MeV × c^2

= 210 MeV

Sol 19: (a) Nuclear forces are short-ranged. They are most effective only up to a distance of the order of a femtometre or less.

(b) Nuclear forces are much stronger than electromagnetic forces.

(c) Nuclear forces are independent of charge.

Sol 20: Mass defect = – [mass of $^{234}_{90}Th$

+ mass of $^{4}_{2}He$] + mass of $^{238}_{92}U$

= – [234.043630 + 4.002600] + 238.05079

= 0.00456 u

So energy released = 0.00456 × 931.5 MeV = 4.25 MeV

Sol 21: Radius = $R_0[A]^{1/3}$

So, $\frac{R_1}{R_2} = \frac{[A_1]^{1/3}}{[A_2]^{1/3}} \Rightarrow \frac{R_1}{R_2} = \left[\frac{1}{8}\right]^{1/3}$

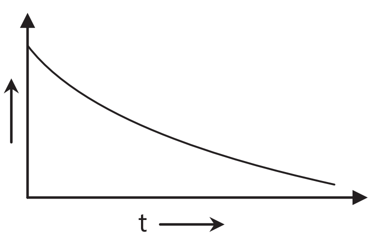
$\boxed{\frac{R_1}{R_2} = \frac{1}{2}}$

Sol 22: (a) It is because of the fact that the binding energy of the particle has to be (+) ve. [i.e. every system tries to minimise its energy] some of its mass is converted into energy.

Sol 23: For stability, binding energy/nucleon should be high. Since it is highest at some intermediate atomic number, the elements with large atomic number try to increase the binding energy/nucleon by fission. Similarly elements with small atomic number tries to increase B.E./nucleon using fusion.

Sol 24: Refer Q.16

Plot: $-\frac{dN}{dt} = \lambda N$ = activity



Sol 25: For A > 30, the stability of the nucleus increases as more and more nucleons are introduced because of minimization of potential energy. Because of this, initially the B.E./nucleon increases.

But at high mass number, the size of the nucleus starts to increase and because of this, as the nucleons are weaker for larger distance, the electrostatic repulsion between the protons starts to dominate over them and thus on further increase in the mass number (the nucleus starts to become unstable).

Radioactivity

Sol 26: For same atomic No.

If mass No. of an isotope > mass no. of most stable isotope

Then isotope is a beta emitter → n/p ratio increases otherwise positron emitter → n/p ratio decreases

$^{49}_{20}Ca$; $^{30}_{13}Al$; $^{94}_{36}Cr$ → beta emitter

Sol 27: Odd no. of neutrons

Sol 28: (a) $^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$

(b) $^{4}_{2}Be + ^{4}_{2}He \rightarrow ^{12}_{6}C + ^{1}_{0}n$

(c) $^{9}_{4}Be\ (p, \alpha) \rightarrow ^{6}_{3}L_i + ^{4}_{2}He + ^{0}_{1}P$

(d) $^{30}_{15}P \rightarrow ^{30}_{14}S + ^{0}_{+1}e$

(e) $^{3}_{1}H \rightarrow ^{3}_{2}He + ^{0}_{-1}e$

(f) $^{43}_{20}Ca + ^{4}_{2}He \rightarrow ^{46}_{21}Sc + ^{1}_{1}P$

Sol 29: $\frac{1}{64} = \frac{1}{2^6}$

$6t_{1/2} = 2 \times 3600$ s

$t_{1/2} = 1200$ sec

$t_{1/2} = \frac{\ln 2}{\lambda}$; $\lambda = \frac{\ln 2}{t_1 / 2}$

$\lambda = \frac{0.693}{1200}$

$\lambda = 5.775 \times 10^{-4}\ sec^{-1}$

$\lambda = 2.079\ hr^{-1}$

Sol 30: $n = \frac{40}{12.3} = 3.252$

No. of atoms = $\frac{\frac{10}{1} \times N_A \times 8 \times 10^{-18}}{2^{3.252}}$

= 5.05×10^6 atoms.

Sol 31: % of radiation = $100 \times \frac{1}{2^4}$ % =6.25%

Sol 32: $k = \frac{0.693}{t_{\frac{1}{2}}}$

$t_{\frac{1}{2}}$ = 30 days

$k = \frac{0.693}{30 \times 24 \times 60 \times 60}\ sec^{-1}$

As N = 10^{11} atoms

$-\frac{dN}{dt} = kN$

$-\frac{dN}{dt} = \frac{0.693 \times 10^{11}}{30 \times 24 \times 60 \times 60}\ sec^{-1}$

= $2.67 \times 10^5\ sec^{-1}$

Sol 33: $\frac{1}{2^{\frac{t}{t_{1/2}}}} = \frac{15}{100}$

$\Rightarrow 2^{\frac{t}{t_{1/2}}} = \frac{100}{15}$

$\frac{t}{t_{1/2}} = \frac{\ln\left(\frac{100}{15}\right)}{\ln 2}$

$t = \frac{t_{1/2} \times \ln\left(\frac{100}{15}\right)}{\ln 2}$

t = 33.66 yrs

Sol 34: $t_{1/2} = 1.4 \times 10^9$

Nuclear reaction: -

(i) ${}^{40}_{19}K \rightarrow {}^{40}_{18}Ar + {}^{0}_{1}e + v$

(ii) Age = $2t_{1/2} = 2.8 \times 10^{18}$ years

Sol 35: $t_{1/2}$ = 10 sec

(i) $t_{mean} = 1.443 \times t_{1/2}$ = 14.43 sec

(ii) $2^n = \frac{100}{6.25}$

$2^n = 16$

n = 4

t = 4 $t_{1/2}$ = 40 sec

Sol 36: ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n$

$\Delta m = 2 \times 2.020 - (3.0160 + 1.0087)$

Δm = 0.0153 amu

$\Delta m = \frac{0.0153 \times 10^{-3}}{6.022 \times 10^{23}} = 2.54 \times 10^{-29}$ kg

$E = \Delta m \times c^2 = 2.28 \times 10^{-12}$ J

$E = \frac{2.28 \times 10^{-12}}{1.6 \times 10^{-19}}$

E = 14.25 MeV

Sol 37: (a) ${}_{92}U^{238} \rightarrow {}_{82}Pb^{206} + 8\ {}^{4}_{2}He + 6\ {}^{0}_{-1}e$

α-Particles = 8

β-Particles = 6

(b) ${}_{90}Th^{234} \rightarrow {}^{206}_{82}Pb + 7\ {}^{4}_{2}He + 6\ {}^{0}_{-1}e$

Sol 38: Remains of $Sr^{40} = 1 \times 10^{-6} \times 2^{-\frac{20}{28.1}}$

= 0.613 mg

Sol 39: (i) ${}_{84}Po^{210} \rightarrow {}_{82}Pb^{206} + {}^{4}_{2}He$

Moles of helium produced = $\left(1-\frac{1}{2}\right) \times \frac{1}{210}$

$V = \frac{nRT}{P} = \frac{\left(1-\frac{1}{\sqrt{2}}\right) \times \frac{1}{210} \times 8.314 \times 273}{1.01325 \times 10^5} = 31.25\ cm^3$

(ii) $V' = \frac{V \times m_{PoO_2}}{m_{Po}} = 27.104\ cm^3$

(iii) U^{238} $t_{1/2} = 4.5 \times 10^9$ yrs

–0.1 mole U^{238} 0.1 mole Pb^{206}

Age of ore = $t_{1/2} = 4.5 \times 10^9$ yr.

Sol 40: $\lambda = \lambda_1 + \lambda_2 ; \lambda_1 + \lambda_2 = \frac{\ln 2}{t_{1/2}}$

$\lambda_1 + \lambda_2 = \frac{\ln 2}{22} ; \lambda_1 = \frac{1}{49}\lambda_2$

$\lambda_1 = \frac{\ln 2}{22 \times 50} = 6.301 \times 10^{-4}\ year^{-1}$

$\lambda_2 = \frac{49 \times \ln 2}{22 \times 50} = 3.087 \times 10^{-2}\ year^{-1}$

Exercise 2

Single Correct Choice Type

Sol 1: (C) Carbon-12 is taken as standard

Sol 2: (B) $R = R_0 . A^{1/3}$

Sol 3: (B) 0

Energy released

$= (8.2 \times 90 + 8.2 \times 110 - 7.4 \times 200)$ MeV

$= (0.8 \times 200)$ MeV = 160 MeV

Sol 4: (A) Energy = $(7.5 \times 13 - 12 \times 7.68)$ MeV

= 5.34 MeV

Sol 5: (B) $2X \rightarrow Y + Q$

Binding energy is the (–) ve energy

From energy conservation

$-2E_1 = -E_2 + Q \Rightarrow Q = E_2 - 2E_1$

Sol 6: (A) We know that half life is given as

$$T = \frac{0.693}{\lambda}$$

Given that $\lambda' = 1 : 2$

$$\therefore \frac{T}{T'} = \frac{\lambda'}{\lambda} = \frac{2}{1}$$

Thus, for probabilities of getting α and β particles at the same time t = 0, the ratio will be the same 2 : 1

Sol 7: (B) Half-life = 5 years, time given = 10 years = 2 half-lives

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

Or $N = \left(\frac{1}{2}\right)^2 N_0$

Or $N = \frac{1}{4} N_0 = 0.25\, N_0$

$\therefore$ 25% substance left hence probability of decay

= 100 – 25 = 75%

Sol 8: (B) $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{1620}\ (\text{years})^{-1}$

$= \frac{\ln 2}{1620 \times 365 \times 24}\ (\text{hours})^{-1}$

Now, $N = N_0 . e^{-\lambda t}$

where $N_0 = \frac{5}{223} \times 6.022 \times 10^{23}$

So $N = N_0 . e^{-\lambda \times 5}$

So decayed particles

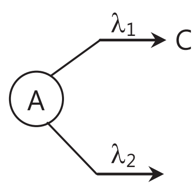
$= N_0 - N = N_0(1 - e^{-\lambda 5})$

$= \frac{5}{223} \times 6.022 \times 10^{23} \left[1 - e^{-\frac{5\ln 2}{1620 \times 365 \times 24}}\right]$

$= 3.29 \times 10^{15}$

Sol 9: (A) The end product of radioactive series is stable and hence the decay constant is zero.

Sol 10: (A)



Now, $\frac{dN}{dt} = -(\lambda_1 N + \lambda_2 N)$

$\Rightarrow \frac{dN}{dt} = -(\lambda_1 + \lambda_2)N.$

Radioactivity

Single Correct Choice Type

Sol 11: (B) ${}^{29}_{13}Al \rightarrow {}^{27}_{13}Al + 2\,{}^{1}_{1}n + 2\,{}^{0}_{-1}B$

Sol 12: (C) ${}^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}\beta$

Sol 13: (A) $\frac{N_1}{N_2} = \frac{A_1}{A_2} = \frac{e^{-10\lambda_0 t}}{e^{-\lambda_0 t}} = \frac{e^{-10/9}}{e^{-1/9}} = e^{-1}$

Sol 14: (B) $m = \frac{256}{2^6}$ g = 4g

$$\frac{k_1 e^{-k_1 t}}{(k_2 - k_1)} = \frac{k_2 e^{-k_2 t}}{(k_2 - k_1)};\ \frac{k_1}{k_2} = e^{(k_1 - k_2)t}$$

$$t_{max.} = \frac{\ln\left(\frac{k_1}{k_2}\right)}{(k_1 - k_2)} = \frac{\ln(k_2/k_1)}{(k_2 - k_1)}$$

Sol 15: (B) Reaction need not be exothermic.

Sol 16: (B) ${}^{238}_{92}M \rightarrow {}^{x}_{y}N + 2\,{}^{4}_{2}He$; ${}^{x}_{y}N \rightarrow {}^{A}_{B}L + 2\,{}^{0}_{1}\beta^{+}$

X = 230 A = 230

Y = 88 B = 86

Neutrons = 230 – 86 = 144

Sol 17: (C) $m = \frac{200}{2^6} = \frac{200}{64} = 3.125$ g

Sol 18: (A) $4\,{}^{4}_{2}He \rightarrow {}^{16}_{8}O$

$\Delta m = 4 \times 4.0026 - 15.834$

$= 16.0104 - 15.834 = 0.1764$ amu

B.E. per nucleon

$= \frac{1}{16} \times 0.1764 \times 931\,MeV = 10.24$ MeV

Sol 19: (D) $\frac{1}{2n} \le \frac{1}{10}$

$2^{t/30} \ge 10$

$\Rightarrow t/30 \ge \log_2 10$

$\Rightarrow t \ge \frac{30\ln 10}{\ln 2}$

$\Rightarrow t \ge 99.65 \approx 100$

Sol 20: (A) ${}^{a}_{b}X \rightarrow {}^{a-1}_{b}X + {}^{1}_{0}n$

Sol 21: (A) (i) $t_{1/2x} = \frac{t_{1/2y}}{\ln 2}$

$t_{1/2x} > t_{1/2y}$

$\therefore$ Y Decays faster.

(ii) True

(iii) $4t_{1/2} = 400\ \mu s$

(iv) $v \propto m$

(v) No. of disintegrated nucleus $= \frac{3}{4}N_0$

Probability $= \frac{3}{4}$

Sol 22: (C) $R = \frac{dN}{dt} = -\lambda N$

$I = \frac{\ln 2}{t_{1/2}}$; $\frac{\lambda_1}{\lambda_2} = \frac{t_{1/2_2}}{t_{1/2_1}} = 2$

$\frac{N_1}{N_2} = \frac{N/2}{N/\sqrt{2}}$

$\frac{R_1}{R_2} = \frac{\lambda_1 N_1}{\lambda_2 N_2} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$

Sol 23: (A) $\frac{\ln 2}{t_{1/2}} \times N = \frac{180}{60}$

$\frac{\ln 2}{t_{1/2}} \times 6.022 \times 10^{23} \times 1.3 \times 10^{-12} = 3$

$t_{1/2} = \frac{6.022 \times 1.3 \times \ln 2}{3} \times 10^{-11}$

$= 1.808 \times 10^{11} = 0.18 \times 10^{-12}$ sec

Sol 24: (C) In a γ decay energy of atom is reduced, atomic mass and atomic number remains the same.

Sol 25: (A) 1 fm << radius of atom

$\therefore$ Repulsive forces dominate.

$F_{pp} > F_{pn} = F_{nn}$

F_{pn} and F_{nn} would be negligible compared to repulsive forces of protons.

Sol 26: (C) $A \xrightarrow{\lambda_1} B$ $\lambda_1 = 1.8 \times 10^{-2}\ sec^{-1}$

$2A \xrightarrow{\lambda_2} C$ $\lambda_2 = 10^{-3}\ sec^{-1}$

$\lambda = \lambda_1 + 2\lambda_2 = 18 \times 10^{-3} + 2 \times 10^{-3}$

$= 2 \times 10^{-2}$

$t_{mean} = \frac{1}{\lambda} = \frac{1}{2 \times 10^{-2}} = 50$ sec

Sol 27: (C) Initially, $N_B = 8N_A$

Finally, $N'_A = 2N'_B$

$\frac{N_A}{N'_A} = 2^{t/50}$ $\frac{N_B}{N'_B} = 2^{t/10}$

$\frac{8N_A}{N'_A} = \frac{N_B}{2N'_B}$

$16 \times 2^{t/50} = 2^{t/10}$

$2^{\frac{t}{50}+4} = 2^{\frac{t}{10}} \Rightarrow \frac{t}{50} - \frac{t}{10} = -4$

$\frac{4t}{50} = 40 \Rightarrow t = 50$ min

Sol 28: (B) $A = 10^4 \times \frac{60\times 10^3}{10}$

$= 6 \times 10^7$ dis/min $= 10^6$ dps

Activity $= \frac{10^6}{3.7\times 10^{10}}$ Curie = 27 μCi

Sol 29: (A) Age = $t_{1/2}$

Sol 30: (B) $t_{1/2}$ = 69.3 min.

$\lambda = \frac{0.693}{69.3} = \text{min}^{-1} = \frac{1}{100}\ \text{min}^{-1}$

$\lambda N = 10$; $N = 10/\lambda = 1000$ atoms

Sol 31: (D) $R_1 = \lambda N_1$

$R_2 = \lambda N_2$

Atoms disintegrated = $(N_1 - N_2)$

$= \left(\frac{R_1 - R_2}{\lambda}\right) = \left(\frac{R_1 - R_2}{\ln 2}\right) T$

Previous Years' Questions

Sol 1: (B) Using $N = N_0 e^{-\lambda t}$

where $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln(2)}{3.8}$ ∴ $\frac{N_0}{20} = N_0 e^{-\frac{\ln(2)}{3.8}t}$

Solving this equation with the help of given data we find: t = 16.5 days

Sol 2: (C) Beta particles are fast moving electrons which are emitted by the nucleus.

Sol 3: (C) During fusion process two or more lighter nuclei combine to form a heavy nucleus.

Sol 4: Following nuclear reaction takes place

${}_0n^1 \rightarrow {}_1H^1 + {}_{-1}e^0 + \bar{\nu}$

$\bar{\nu}$ is antineutrino

Sol 5: (C) The given reaction are :

${}_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + p$

${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n$

$3\,{}_1H^2 \rightarrow {}_2He^4 + n + p$

Mass defect

$\Delta m = (3\times 2.014 - 4.001 - 1.007 - 1.008)$ amu

= 0.026 amu

Energy released = 0.026 × 931 MeV

$= 0.026 \times 931 \times 1.6 \times 10^{-13}$ J

$= 3.87 \times 10^{-12}$ J

This is the energy produced by the consumption of three deuteron atoms.

∴ Total energy released by 10^{40} deuterons

$= \frac{10^{40}}{3} \times 3.87 \times 10^{-12}$ J

$= 1.29 \times 10^{28}$ J

The average power radiated is $P = 10^{16}$ W or 10^{16} J/s

Therefore, total time to exhaust all deuterons of the star will be

$t = \frac{1.29\times 10^{28}}{10^{16}} = 1.29 \times 10^{12}\ s \approx 10^{12}$ s

Sol 6: (B) Heavy water is used as moderators in nuclear reactors to slow down the neutrons.

Sol 7: (A) Penetrating power is maximum for γ-rays, then of β-particles and then α-particles because basically it depends on the velocity. However, ionization power is in reverse order.

Sol 8: (A) Activity of $S_1 = \frac{1}{2}$(activity of S_2)

or $\lambda_1 N_1 = \frac{1}{2}(\lambda_2 N_2)$ or $\frac{\lambda_1}{\lambda_2} = \frac{N_2}{2N_1}$

or $\frac{T_1}{T_2} = \frac{2N_1}{N_2}$ (T = half-life$=\frac{\ln 2}{\lambda}$)

Given $N_1 = 2N_2$ ∴ $\frac{T_1}{T_2} = 4$

∴ Correct option is (A).

Sol 9: $R = R_0\left(\frac{1}{2}\right)^n$

Here R_0 = initial activity =1000 disintegration/s

and n = number of half-lives.

At t = 1s, n = 1

$\therefore$ R = $10^3\left(\frac{1}{2}\right)$ = 500 disintegration/s

At t = 3s, n = 3

R = $10^3\left(\frac{1}{2}\right)^3$ = 125 disintegration/s

Sol 10: Number of α-particles emitted

$n_1 = \frac{238-206}{4} = 8$

and number of β-particles emitted are say n_2, then 92 – 8 × 2 + n_2 = 82

$\therefore \quad n_2 = 6$

Sol 11: Q = (Δm in atomic mass unit) × 931.4 MeV

= (2 × mass of $_1H^2$ – mass of $_2He^4$) × 931.4 MeV

= (2 × 2.0141 – 4.0024) × 931.4 MeV

$Q \approx 24$ MeV

Sol 12: (D) Binding energy per nucleon increases for lighter nuclei and decreases for heavy nuclei.

Sol 13: (B) $\frac{k}{r} = \frac{mv^2}{r}$

mv^2 = k (independent or r)

$n\left(\frac{h}{2\pi}\right) = mvr \Rightarrow r \propto n$ and $T = \frac{1}{2}mv^2$ is independent of n.

Sol 14: (A) 1st reaction is fusion and 4th reaction is fission.

Sol 15: (D) IR corresponds to least value of $\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

i.e. from Paschen, Bracket and Pfund series. Thus the transition corresponds to 5 →3.

Sol 16: (C) After decay, the daughter nuclei will be more stable hence binding energy per nucleon will be more than that of their parent nucleus.

Sol 17: (B) Conserving the momentum $0 = \frac{M}{2}V_1 - \frac{M}{2}V_2$

$V_1 = V_2$ ... (i)

$\Delta mc^2 = \frac{1}{2}\cdot\frac{M}{2}V_1^2 + \frac{1}{2}\cdot\frac{M}{2}\cdot V_2^2$ ... (ii)

$\Delta mc^2 = \frac{M}{2}V_1^2$

$\frac{2\Delta mc^2}{M} = V_1^2$

$V_1 = c\sqrt{\frac{2\Delta m}{M}}$

Sol 18: (B) In positive beta decay a proton is transformed into a neutron and a positron is emitted.

$p^+ \to n^0 + e^+$

no. of neutrons initially was A – Z

no. of neutrons after decay (A – Z) – 3 x 2 (due to alpha particles) + 2 x 1 (due to positive beta decay)

The no. of proton will reduce by 8. [as 3 x 2 (due to alpha particles) + 2(due to positive beta decay)]

Hence atomic number reduces by 8.

Sol 19: (B) $E_n = -13.6\frac{Z^2}{n^2}$

$E_{Li^{++}} = -13.6 \times \frac{9}{1} = -122.4\text{eV}$

$E_{Li^{+++}} = -13.6 \times \frac{9}{9} = -13.6\text{eV}$

$\Delta E = -13.6 - (-122.4)$

= 108.8 eV

Sol 20: (B) $t_{\frac{1}{2}} = 20$ minutes

$N = N_0 e^{-\lambda t_2} \lambda t_1 = \ln 3$

$\frac{2}{3}N_0 = N_0 e^{-\lambda t_2} t_1 = \frac{1}{\lambda}\ln 3$

$\frac{2}{3}N_0 = N_0 e^{-\lambda t_2}$

$t_2 = \frac{1}{\lambda}\ln\frac{3}{2}$

$t_2 - t_1 = \frac{1}{\lambda}\left[\ln\frac{3}{2} - \ln 3\right] = \frac{1}{\lambda}\ln\left[\frac{1}{2}\right] = \frac{0.693}{\lambda} = 20$ min

Sol 21: (D) Number of spectral lines from a state n to ground state is = $\frac{n(n-1)}{2} = 6$

Sol 22: (A) $\Delta m(m_p + m_e) - m_n$ = 9 x 10^{-31} kg.

Energy released = (9 x 10^{-31} kg)c^2 joules

$$= \frac{9\times10^{-31}\times(3\times10^{8})^2}{1.6\times10^{-13}}\text{MeV} = 0.73 \text{ MeV.}$$

Sol 23: (A) KE $\propto \left(\frac{Z}{n}\right)^2$ as n decreases KE increases and TE, PE decreases

Sol 24: (C)

A B

T_A = 20 min T_B = 40 min

$$\frac{\left(1-\frac{N}{N_0}\right)_A}{\left(1-\frac{N}{N_0}\right)_B} = \frac{1-\frac{1}{2^{t/t_{1/2}}}}{1-\frac{1}{2^{t/t_{1/2}}}} = \frac{1-\frac{1}{2^{\frac{80}{20}}}}{1-\frac{1}{2^{\frac{80}{40}}}} = \frac{1-\frac{1}{16}}{1-\frac{1}{4}} = \frac{\frac{15}{16}}{\frac{3}{4}} = \frac{5}{4}$$

JEE Advanced/Boards

Exercise 1

Sol 1: ${}_1H^2 + {}_1H^2 \longrightarrow {}_2He^4$

Binding energy of deuteron

= (1. 1) × 2 MeV

= 2.2 MeV

Binding energy of helium

= ${}_2He^4$ = 7 × 4

= 28 MeV

So total energy released

= 28 – 2.2 × 2 = 28 – 4.4

= 23.6 MeV

Sol 2: (i) ${}^{40}_{19}K \rightarrow {}^{40}_{18}Ar + {}^{0}_{+1}e + \nu$

(ii) 4.2 × 10^9 years

Sol 3: $\frac{dN}{dt} = R - \lambda N$

$\frac{dN}{dt} + \lambda N = R$

$\Rightarrow \int_{0,0}^{N,t} d\left(N.e^{\lambda t}\right) = \int_0^t R.e^{\lambda t}.dt$

N. $e^{\lambda t} = \frac{R}{\lambda}.[e^{\lambda t} - 1]$

$\Rightarrow N = \frac{R}{\lambda}[1 - e^{-\lambda t}]$

So for eq. as $t \to \infty$, $N \to R/\lambda$,

So for N = 0.8 R/l

$0.8\frac{R}{\lambda} = \frac{R}{\lambda}[1 - e^{-\lambda t}]$

$\Rightarrow \frac{4}{5} = 1 - e^{-\lambda t} \Rightarrow \frac{1}{5} = e^{-\lambda t}$

$\Rightarrow \lambda t = \ln 5$

$\Rightarrow \frac{\ln 5}{\lambda}$

and give, $\lambda = \frac{\ln 2}{\tau} \Rightarrow \boxed{t = \frac{\ln 5}{\ln 2}\times\tau}$

Sol 4: 4 hydrogen atom produces 26 MeV energy.

⇒ 4g (4 moles) hydrogen atom produces

⇒ [26 × 6.022×10^{23}] MeV(energy)

= 26 × 6.022 × 10^{23} × 1.6 × 10^{-19} × 10^6 Joule

= 26 × 6.022 × 1.6 × 10^{10} Joule

= 250.51 × 10^{10} Joules

⇒ 1.7 × 10^{30} kg = 1.7 × 10^{33} g H produces

$= \frac{250.51\times10^{10}}{4} \times 1.7 \times 10^{33}$

$= \frac{250.51\times1.7}{4} \times 10^{43}$ Joules

= 1.065 × 10^{45} Joules

Now power × time = total energy

$\Rightarrow$ time $= \frac{1.065\times10^{45}}{3.9\times10^{26}} = 2.73 \times 10^{18}$ sec

Sol 5: We have, N= $N_0.e^{-\lambda t}$

So, $N_{U_{235}} = N_0.e^{-\lambda_1 t}$

$N_{U_{238}} = N_0.e^{-\lambda_2 t}$

$\frac{N_{U_{238}}}{N_{U_{235}}} = e^{(\lambda_1-\lambda_2)t}$

Given $\frac{140}{1} = e^{(\lambda_1-\lambda_2)t}$

$\Rightarrow \ln 140 = (\lambda_1 - \lambda_2) \times t$

$$\Rightarrow t = \frac{\ln 140}{\lambda_1 - \lambda_2}$$

Now, $\lambda_1 = \dfrac{\ln 2}{7.13\times10^8}$, $\lambda_2 = \dfrac{\ln 2}{4.5\times10^9}$

$$\text{So } t = \frac{\ln 140}{\ln 2\left[\dfrac{1}{7.3\times10^8} - \dfrac{1}{4.5\times10^9}\right]} \text{ years}$$

$$= \frac{\ln 140}{\ln 2\left[\dfrac{10}{7.3} - \dfrac{1}{4.5}\right]} \times 10^9 \text{ years} = 6.21 \times 10^9 \text{ years}$$

Sol 6: Now, as the momentum of the nucleus and α-particle are same (momentum conservation)

Energy = $\dfrac{P^2}{2m}$, is divided in the inverse ratio of their respective masses.

So, let the energy of nucleus after disintegration be K, then

$$\frac{4.78}{K} = \frac{222}{4} = \frac{111}{2}$$

$$\Rightarrow K = \frac{2\times4.78}{111} = 0.086 \text{ MeV}$$

Total Energy = (4.78+0.086) MeV = 4.87 MeV

Sol 7: We have, $\dfrac{dN}{dt} = \lambda N = \lambda.N_0.e^{-\lambda t}$

Sol 8: We have Number of particles

$$= \frac{2.5\times10^{-3}}{230} \times 6.022 \times 10^{23}$$

$$\text{So } \left|\frac{dN}{dt}\right| = (-\lambda N) \Rightarrow \lambda N = 8.4 \text{ sec}^{-1}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{N}{8.4} \text{ sec}$$

$$\Rightarrow \frac{\ln 2}{\lambda} = t_{1/2} = \frac{\ln 2\times N}{8.4} \text{ sec}$$

$$= \frac{\ln 2\times2.5\times10^{-3}\times6.022\times10^{23}}{230\times8.4\times365\times24\times60\times60} \text{ years} = 1.7 \times 10^{10} \text{ years}$$

Sol 9: Activity / gm = 320/50 = 68.4 min^{-1}

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{5730} \text{ (years)}^{-1}$$

Now, initial activity = $\lambda.N_0$

Activity at some t = $\lambda.N_0.e^{-\lambda t}$

So, $6.4 = \lambda.N_0.e^{-\lambda t}$ and $12 = \lambda N_0$

$\Rightarrow e^{-\lambda t} = 6.4/12$

$\Rightarrow \lambda t = \ln (12/6.4)$

$$\Rightarrow \quad t = \frac{5730\times\ln(12/6.4)}{\ln 2} = 5196 \text{ years}$$

Sol 10: ${}^2_1H + {}^2_1H \rightarrow {}^4_2H + 23.6 \text{ MeV}$

Now we have, B.E. of He > B.E. of deuterium for the reaction to happen.

$\Rightarrow$ B.E. of helium = B.E. of Deuterium + 23.6 MeV

= 4 × 1.1 + 23.6 = 4.4 + 23.6 = 28 MeV

Sol 11: $\underset{\substack{\text{(meson)}\\ 150\text{MeV}}}{\pi^+} \rightarrow \underset{\substack{\text{(muon)}\\ 100\text{MeV}}}{\mu^+} + \underset{\text{(neutrino)}}{\bar{\nu}} \rightarrow P$

Now assume momentum of $\bar{\nu}$ = P

Now $150 = 100 + KE_{\mu+} + KE_{\bar{\nu}}$ (energy conservation)

$\Rightarrow 50 = KE_{\mu+} + KE_{\bar{\nu}}$

Also using momentum conservation

$\sqrt{2m_{\mu^+}KE_{\mu^+}} = P$ and $KE_{\bar{\nu}} = Pc$

$$\Rightarrow KE_{\bar{\nu}} = c.\sqrt{2m_{\mu^+}KE_{\mu^+}}$$

$$\Rightarrow 50 = KE_{\mu+} + \sqrt{2m_{\mu^+}c^2KE_{\mu^+}}$$

$$\Rightarrow 50 = KE_{\mu+} + \sqrt{200KE_{\mu+}}$$

$$\Rightarrow 50 = KE_{\mu+} + 10\sqrt{2KE_{\mu+}}$$

$\Rightarrow (50 - x)^2 = 200x \qquad (x = KE)$

$\Rightarrow x^2 - 100x - 200x + 2500 = 0$

$\Rightarrow x^2 - 300x - 2500 = 0$

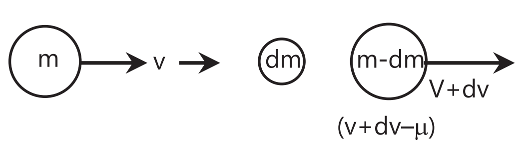
$$x = \frac{300 \pm \sqrt{90000 - 10000}}{2}$$

$$= \frac{300 \pm 200\sqrt{2}}{2} = \left(150 \pm 100\sqrt{2}\right) \text{MeV}$$

$$= \left(150 - 100\sqrt{2}\right)\text{MeV}$$

= (150 – 141) MeV= 9 MeV

Sol 12:



Momentum conservation

$mv = (m – dm)(v + dv) + dm(v + dv – u)$

$mv = mv + mdv – dmv + dmv – u.dm$

$\Rightarrow mdv = u.dm$

$\Rightarrow \int_0^v dv = u\int_{m_0}^{m} \frac{dm}{m}$

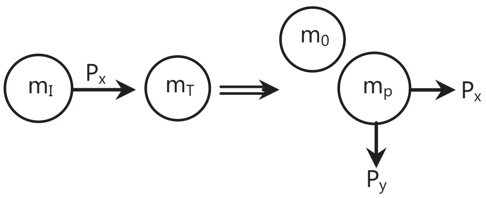
$\Rightarrow v = u \,.\, \ln m/m_0$

Now, $m = m_0.e^{-\lambda t} \Rightarrow m/m_0 = e^{-\lambda t}$

$\Rightarrow v = u.(-\lambda t) = -u\lambda t$

So v is opposite to u.

Sol 13:



From the conservation of momentum the above diagram can be deduced

Now, Q = K.E. after collision – K.E. before collision

$= \frac{P_y^2}{2m_0} + \frac{(P_x^2 + P_y^2)}{2m_p} - \frac{P_x^2}{2m_I}$

$= \frac{P_y^2}{2m_0} + \frac{P_x^2}{2m_0} - \frac{P_x^2}{2m_0} + \frac{(P_x^2 + P_y^2)}{2m_p} - \frac{P_x^2}{2m_I}$

$= \left[\frac{P_x^2 + P_y^2}{2m_p}\right].\left[1 + \frac{m_p}{m_0}\right] - \frac{P_x^2}{2m_0} - \frac{P_x^2}{2m_I}$

$= K_p.\left[1 + \frac{m_p}{m_0}\right] - \frac{P_x^2}{2m_I}\left(1 + \frac{m_I}{m_0}\right)$

$Q = K_p.\left[1 + \frac{m_p}{m_0}\right] - K_I.\left(1 + \frac{m_I}{m_0}\right)$

Radioactivity

Sol 14: ${}_{90}Th^{232} \rightarrow {}_{82}Pb^{208} + 6\,{}_2He^4 + 4\,{}^{0}_{+}\beta$

$n_{He} = \frac{1.01325 \times 10^5 \times 8 \times 10^{-5} \times 10^{-6}}{8.314 \times 273}$

$n_{He} = 3.571 \times 10^{-9}$

$n_{Th} = 2.155 \times 10^{-9}$

$n_{Th_0} = 2.75 \times 10^{-9}$

$\frac{n_{Th}}{n_{Th_0}} = 0.783 = 2^{\frac{-t}{T_{1/2}}}$

$T_{1/2} \times \frac{\ln 0.783}{\ln 2} = -t$

$t = \frac{1.39 \times 10^{10} \times 0.244}{0.693}$

$t = 4.906 \times 10^9$ yrs

Sol 15: $N_0(1 – e^{-36\lambda}) = 10^5$

$N_0(1 – e^{-108\lambda}) = 1.11 \times 5^5$

$\frac{1 - e^{-36\lambda}}{1 - e^{-108\lambda}} = \frac{100}{111}$

Let $e^{-36\lambda} = t$ $e^{-108\lambda} = t^3$

$\frac{1 - t}{1 - t^3} = \frac{100}{111}$

$111 – 111t = 100 – 100t^3$

$100t^3 – 111t + 11 = 0$

$(t – 1)(100t^2 + 100t – 11) = 0$

$t \neq 1$ $t = -1/2 + 3/5$

$t = 1/10$

$e^{-36\lambda} = \frac{1}{10} \Rightarrow -36\lambda = -\ln 10$

$\lambda = \frac{\ln 10}{36}$

$T_{1/2} = \frac{\ln 2}{\ln 10} \times 36$

$T_{1/2} = 10.8$ sec

Sol 16: $\frac{A}{A_0} = \frac{1}{2^{1/3}}$

$\frac{\frac{296}{60}}{3.7 \times 10^4 \times \frac{1}{V}} = \frac{1}{2^{1/3}}$

$V = \frac{3.7 \times 10^4 \times 60}{296 \times 2^{1/3}} \Rightarrow V = 5952.753\ cm^3$

$V = 5.592\ m^3$

Sol 17: $\lambda_1 = \frac{0.693}{2.44 \times 10^4} = 2.84 \times 10^{-5}\ yr^{-1}$

$\lambda_2 = \frac{0.693}{6.08 \times 10^3} = 1.139 \times 10^{-4}\ yr^{-1}$

$\lambda_1 = 9 \times 10^{-13}$; $\lambda_2 = 3.61 \times 10^{-12}$

$A = \lambda_1 N_1 + \lambda_2 N_2$

$6 \times 10^9 = 6.02\times10^{23}$

$$\left(\frac{9\times10^{-13}\times x}{239}+\frac{3.61\times10^{-12}(1-x)}{240}\right)$$

$$1 = \frac{90x}{239} + \frac{361(1-x)}{240}$$

$$1 = \frac{-64679x+86279}{239\times240}$$

$x = 0.447$

$\%^{239}Pu = 44.7\%$

$\%^{240}Pu = 55.3\%$

Sol 18: (a) $\frac{dN}{dt} = \alpha - \lambda N$

$$\frac{dn}{\alpha-\lambda N} = dt$$

$$\ln\left(\frac{\alpha-\lambda N}{\alpha-\lambda N_0}\right) = -\lambda t$$

$$\frac{\alpha-\lambda N}{\alpha-\lambda N_0} = e^{-\lambda t}$$

$$N = \frac{1}{\lambda}[\alpha-(\alpha-\lambda N_0)\,e^{-\lambda t}]$$

(b) $t = t_{1/2}$

$$N = \frac{1}{\lambda}\left(\alpha-\frac{(\alpha-\lambda N_0)}{2}\right)$$

$$= \frac{1}{\lambda}\left(\frac{\alpha}{2}+\frac{\lambda N_0}{2}\right) = \frac{1}{\lambda}\,(1.5\,\lambda N_0)$$

$N = 1.5\,N_0$

$$\lim_{t\to\infty} N = \frac{\alpha}{\lambda} = 2N_o$$

Sol 19: $\frac{dP_H}{dt} = -\,kP_H$

$$\frac{1.2}{2} = 2^{\frac{-50}{t_{1/2}}}$$

$$\ln\left(\frac{2}{1.2}\right) = \frac{50}{t_{1/2}}\ \ln 2$$

$$t_{1/2} = \frac{50\ln2}{\ln(2/1.2)} = 67.84\text{ min}$$

$$k = \frac{\ln2}{t_{1/2}} = 0.0102$$

$$\frac{dm}{dt} = \frac{d(n/v)}{dt} = \frac{1}{RT}\frac{dP_H}{dt}$$

$$= \frac{1}{RT}\times -0.0102\times1.2$$

$$= -\frac{0.0102\times1.2}{0.0821\times298}\ \text{molarity/min}$$

$= 0.833\times10^{-5}$ molarity/sec.

Sol 20: $n_U = \frac{1}{238}$, $n_{Pb} = \frac{0.1}{206}$

$$2^{\frac{t}{t_{1/2}}} = \frac{n_0}{n}$$

$$\frac{t}{t_{1/2}} = \frac{\ln(n_0/n)}{\ln2}$$

$$t_1 = \frac{t_{1/2}\times\ln(n_0/n)}{l\ \ln2}$$

$$\ell n\left(\frac{n_0}{n}\right) = \ln\left(\frac{1/238}{1/238-0.1/206}\right) = 0.119$$

$$t = \frac{4.5\times10^9\times0.1227}{\ln2}$$

$t = 7.75\times10^8$ years

Sol 21: ${}^{218}_{84}Po \xrightarrow{\lambda_1} {}^{214}_{82}Pb \xrightarrow{\lambda_2} {}^{214}_{83}Pb + {}^{o}_{-1}\beta$

$$\frac{dN}{dt} = \lambda_1 N_1 - \lambda_2 N$$

$$\frac{dN}{dt} = \lambda_1 N_o e^{-\lambda_1 t} - \lambda_2 N$$

$$N = \frac{\lambda_1 N_0\left(e^{-\lambda_1 t}-e^{-\lambda_2 t}\right)}{(\lambda_2-\lambda_1)}$$

$$\frac{dN}{dt} = 0$$

$\Rightarrow \lambda_1 N_0\ e^{-\lambda_1 t} = \lambda_2 N$

$$\lambda_1 N_0\ e^{-\lambda_1 t} = \frac{\lambda_1\lambda_2 N_0\left(e^{-\lambda_1 t}-e^{-\lambda_2 t}\right)}{(\lambda_2-\lambda_1)}$$

$$(\lambda_2-\lambda_1)\ e^{-\lambda_1 t} = \lambda_2\ e^{-\lambda_1 t} - \lambda_2\ e^{-\lambda_2 t}$$

$$(\lambda_1)\ e^{-\lambda_1 t} = \lambda_2\ e^{-\lambda_2 t}$$

$$e^{(\lambda_1-\lambda_2)t} = \frac{\lambda_1}{\lambda_2}$$

$$t = \frac{\ln(\lambda_1/\lambda_2)}{(\lambda_1-\lambda_2)}$$

$$t = \frac{\ln(2.68/3.05)}{\ln 2\left(\frac{1}{3.05} - \frac{1}{2.68}\right)} = 4.12 \text{ min}$$

Sol 22: (a) Given at time t; $^{238}_{92}U = 1.667g = \left(\frac{1.667}{238}\right)$ mole

$^{206}_{83}Pb = 0.277g = \left(\frac{0.277}{206}\right)$mole

Since all lead has been formed from U^{238} and therefore

moles of U decayed = Moles of Pb formed $= \left(\frac{0.277}{206}\right)$

∴ Total moles of U before decay (N_0) = moles of U at time t (N)

$$= \frac{1.667}{238} \times \frac{0.277}{206} \quad \because \ t = \frac{2.303}{\lambda}\log\frac{N_0}{N}$$

$$= \frac{2.303 \times 4.51 \times 10^9}{0.693}\log\frac{\left(\frac{1.667}{238}\right) + \left(\frac{0.277}{206}\right)}{\left(\frac{1.667}{238}\right)}$$

(a) t = 1.143 × 10^9 year

(b) 7.097 × 10^8 year

Sol 23: Minimum β-activity required = 346 min^{-1}

Number of β-activity required to carry out the experiment for 6.909 h = (346 min^{-1}) (6.909 × 60 min) = 143431

Amount of β-activity required

$$= \frac{143431}{6.022 \times 10^{23} \text{ mol}^{-1}} = 2.3818 \times 10^{-19} \text{ mol}$$

Now, the rate constant of radioactive decay is

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{66.6 \text{ h}} = 0.010404 \text{ h}^{-1}$$

Now using the integrated rate expression

$$\log\frac{n_0 - n_{consumed}}{n_0} = -\frac{\lambda t}{2.303},$$

We get $\log\frac{n_0 - 2.3818 \times 10^{-19} \text{ mol}}{n_0}$

$$= -\frac{(0.010404 \text{ h}^{-1})(6.909 \text{ h})}{2.303} = -0.03121 \text{ or}$$

$$\frac{n_0 - 2.3818 \times 10^{-19} \text{ mol}}{n_0} = 0.9306$$

Solving for n_0, we get

$$n_0 = \frac{2.3818 \times 10^{-19} \text{ mol}}{1 - 0.9306} = 3.43 \times 10^{-18} \text{ mol}$$

Exercise 2

Single Correct Choice Type

Sol 1: (D) $\underset{1876\,MeV}{{}_1H^2} \rightarrow \underset{939\,MeV}{{}_1P^1} + \underset{940\,MeV}{{}_0n^1}$ + Energy

So energy conservation gives

⇒ 1876 = 939 + 940 + E

⇒ E = – 3MeV

So a γ ray has to be absorbed

Sol 2: (C) Mass number is constant as no nucleoid is emitted.

Sol 3: (D) Mass of 20 is released and charge of 6 is released from nucleus 20 mass ⇒ 5α.

Sol 4: (B) Given that $k.E_\alpha$ = 48 Mev, Q = 50 Mev

We know that $k.E_\alpha = Q\left(\frac{A-4}{A}\right)$

Here, A is the mass number of mother nucleus

Putting the values, we get

⇒ 48 = 50 $\left(\frac{A-4}{A}\right)$ ⇒ 48 A = 50 A – 200

⇒ A = 100

Sol 5: (B) In the uranium radioactive series the initial nucleus is 8 alpha and 6 beta particles are released as it is a 4n + 2 series.

Sol 6: (D) Activity = $\frac{dN}{dt}$ = N × l

So λ . N_0 . $e^{-\lambda t}$ = acitivity (R)

$$\frac{R_2}{R_1} = \frac{\lambda.N_0.e^{-\lambda t_2}}{\lambda.N_0.e^{-\lambda t_1}} = e^{\lambda(t_1 - t_2)}$$

Sol 7: (C) Just like tossing of a coin, S heads won't change probability of next outcome, after any half-life, there is $\frac{1}{2}$ probability of any atom surviving.

Sol 8: (B) $\frac{A_0}{\sqrt{3}} = A_0.e^{-\lambda t} = A_0.e^{-l}$

$\Rightarrow \lambda = \frac{1}{2}\lambda n3$

Activity = $\lambda N = \lambda N_0.e^{-4\lambda} = A_0.e^{-4\lambda} = A_0/9$

Sol 9: (C) So act. $(t_1) = \lambda N_1 = \lambda.N_0.e^{-\lambda t_1} = A_1$

So act. $(t_2) = \lambda N_2 = \lambda.N_0.e^{-\lambda t_2} = A_2$

So $\frac{A_1}{A_2} = e^{\lambda(t_2-t_1)} \Rightarrow A_2 = A_1 \cdot e^{(t_2-t_1)/T}$

Sol 10: (A) $f_1 > f_2 \Rightarrow$ 63° decays in mean life

Sol 11: (C) $\frac{dN}{dt} = R - \lambda N$

$\Rightarrow \lambda N + \frac{dN}{dt} = R$

$\Rightarrow \int_{0,0}^{N,t} d\left[N.e^{\lambda t}\right] = \int_0^t R.e^{\lambda t}.dt$

$N.e^{\lambda t} = \frac{R}{\lambda}.\left[e^{\lambda t} - 1\right]$

$\Rightarrow N = \frac{R}{\lambda}.\left[1 - e^{-\lambda t}\right]$

So, N = 10, R = 10

$\Rightarrow 10 = \frac{10}{1/2}\left[1 - e^{-t/2}\right]$

$\Rightarrow e^{-t/2} = 1/2$

$\Rightarrow \ln 2 = t/2$

$\Rightarrow t = 2\ln 2 = 0.693$

Sol 12: (B) $R_1 = \lambda.N_0.e^{-\lambda T_1}$ and $R_2 = \lambda.N_0.e^{-\lambda T_2}$

$t_{1/2} = T = \frac{\ln 2}{\lambda} \Rightarrow \boxed{\lambda = \frac{\ln 2}{T}}$

Number of atoms disintegrated = $N_1 - N_2$

$= N_0.\, e^{-\lambda T_1} - N_0.\, e^{-\lambda T_2}$

$= \frac{R_1 - R_2}{\lambda} = \frac{T(R_1 - R_2)}{\ln 2}$

Sol 13: (E) Rate = $\frac{dN}{dt} = \lambda N(t) = \lambda N_0.e^{-\lambda t} \Rightarrow$(E)

Sol 14: (C) depends on the number of elements and activities inside nucleus.

Sol 15: (C) Mean life = $1/\lambda$

$\Rightarrow \lambda = 1/40\ (\text{min}^{-1}) = \frac{1}{2400}\ \text{sec}^{-1}$

So $\frac{dN}{dt} = -\lambda N + R$

$\Rightarrow N = \frac{R}{\lambda}.\left[1 - e^{-\lambda t}\right]$

At steady state, $t \to \infty$,

$\Rightarrow N = \frac{R}{\lambda} = \frac{10^3}{1/2400} = 24 \times 10^5$

Sol 16: (B) at t = 0, N and at $t \to \infty$, N = const.

Sol 17: (C) Because neutron has larger rest mass than proton.

Multiple Correct Choice Type

Sol 18: (A, B, D) × nuclear attraction is there, (no rep.)

(B) ✓as r ↑ the energy ↓ ⇒ it is electrostatic

(C) ✓ Nuclear attraction

(D) ×

Sol 19: (A, C) Refer theory.

Sol 20: (C, D) Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less than the sum of masses of its constituent particles.

$^{20}_{10}Ne$ is made up of 10 protons plus 10 neutrons. Therefore, mass of $^{20}_{10}Ne$ nucleus, $M_1 < 10\,(m_p + m_n)$

Also, heavier the nucleus, more is the mass defect.

Thus, $20(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$

Or $10(m_p + m_n) > M_2 - M_1$

Or $M_2 < M_1 + 10(m_p + m_n)$

Now since $M_1 < 10(m_p + m_n)$

$\therefore \quad M_2 < 2M_1$

Sol 21: (A, C) $T = \frac{0.693}{\lambda} = 2$

$\therefore$ Decay time = n × Half life.

$\therefore \quad n = \frac{8}{4} = 2$

$\therefore \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \frac{1}{4}$

Sol 22: (B, C) A, B, negative slope, +F;

B, C, positive slope, -F

Sol 23: (A, B, C) (A) ✓true (cons. of energy)

(B) ✓True (cons. of energy). [energy can't be generated from anywhere alse]

(C) ✓ as either N ↓ (β^-) and P ↑ or N ↑ or P ↓ (β^+)

(D) × mass number is const. so (ABC)

Sol 24: (A, B, C) (A) ✓ free neutron is unstable

(B) ✓free proton is stable

(C) ✓ $B^-$and B^+ decay

(D) × both are possible ABC

Sol 25: (C, D) (A) × as $^{4}_{2}He^{2+}$ has charge in it

(B) × as (+) 1 charge is there in neutron

(C) ✓ γ decay (no charge transfer)

(D) ✓ inside the atom, no change in charge.

(C, D)

Sol 26: (C, D)

(C) ✓

(D) ✓

Because there is comparatively more distance between protons inside the nucleus, electric repulsion is more because nuclear forces are small as compared to electrostatic when distance is high.

Sol 27: (B, C)

(B) ✓

(C) ✓ At high A, BE /nucleon is more

Sol 28: (A, B, C) (A) ✓ $N = N_0.e^{-\lambda t}$

(B) ✓ $\frac{dN}{dt} = -\lambda N$

(C) ✓ $N = N_0(1 - e^{-\lambda t})$

(D) ×

Sol 29: (A, D) $^{A}_{Z}A \longrightarrow {}^{A-4}_{z-2}A + {}^{4}_{2}He^{2+}$

$^{A}_{Z}B \longrightarrow {}^{A}_{z+1}B + \beta + \bar{\nu}$

(A) ✓ Now as the α-particle is alone, the energy could transfer to the α-particle only and from momentum cons., the particles will have same v.

(C) ×

(D) ✓ Now during β-decay, the anti-neutrino is also emitted with the β-particle and thus energy can be distributed between them.

Sol 30: (D, E) $^{14}_{7}N + {}^{1}_{0}n \longrightarrow {}_{3}Li^{7} + \text{'}\overline{\text{some more}}\text{'}$ elements

Now mass number should be same

$14 + 1 = 7 + x \Rightarrow x = 8$

(So the products should have mass number = 8) (D) ✓ and (E) ✓

Now charge also has to balance in D and E.

$1\alpha \Rightarrow 2 + 4 - 2 = 4$

Similarly, (E) is also correct and $4P^o + 2B$

So (D), (E)

Sol 31: (A, C, D) (A) more nucleons ⇒ release of nucleons as α particles

(B) Protons in excess ⇒ B^+ release ⇒ (C) is ✓

(D) β^- is reduced then protons are increased and neutrons are decreased in a nucleus.

⇒ (A), (C), (D)

Assertion Reasoning Type

Sol 32: (D) Because the statement is valid for large number of nuclei

Sol 33: (C) Remaining energy is given to the anti-neutrino particles

Sol 34: (A) True exp.

Sol 35: (A) True exp.

Comprehension Type

Paragraph 1

Sol 36: (B) 1 million = 10,00,000 = 10^6 person

Electric power = $300 \times 10^6 = 3 \times 10^8$ watts

Thermal power = $\frac{3\times10^8}{0.25}$ = 12×10^8 watts

$t = 24 \times 60 \times 60$

So $N \times 200 \times 10^6 \times 1.6 \times 10^{-19}\,J = 12 \times 10^8 \times t$

$\Rightarrow N = \frac{12\times10^8\times24\times60\times60}{200\times1.6\times10^{-13}}$

$= \frac{12\times24\times60\times60}{2\times1.6}\times10^{21} = 3.24 \times 10^{24}$

Paragraph 2

Sol 37: (C) Now, $\frac{h}{p} = 5.76 \times 10^{-15}$ m

$\Rightarrow P = \frac{6.6\times10^{-34}}{5.76\times10^{-15}} = 1.15 \times 10^{-19}$ N-s

As from cons. of momentum, their mom should be same

Sol 38: (B) $P = mv = \frac{P^2}{2m} = KE$

$\Rightarrow KE = \frac{(1.15\times10^{-19})^2}{2\times4\times1.6\times10^{-27}}\,J$

$= \frac{(1.15)^2\times10^{-38}}{8\times1.6\times10^{-27}}J = \frac{(1.15)^2\times10^{-11}}{8\times1.6\times1.6\times10^{-19}}$ eV

$= \frac{(1.15)^2}{8\times1.6\times1.6}\times10^8$ eV $= 6.22\times10^6$

Sol 39: (D) $KE = \frac{P^2}{2m}$

$= \frac{(1.15\times10^{-19})^2}{2\times223.4\times1.6\times10^{-27}}J$

$= \frac{(1.15)^2\times10^{-11}}{2\times223.4\times1.6\times1.6\times10^{-19}}$ eV

$= \frac{(1.15)^2}{2\times223.4\times(1.6)^2}\times10^8$ eV

= 0.11 MeV

Match the Columns

Sol 40: (A) → q charge balance

(B) → p (238 – 32) = 206 Þ (B)

(C) → t (Theory)

(D) → r ($Z_{new} = Z - 2$)

(E) → s 16 + 2 – 4 = 14 (mass no.)

Sol 41: (A) → p and q

Matter into energy (mass defect is observed)

Materials combine (low atomic no.)

(B) → (p) mass defect

(r) big nucleus disintegrates into smaller ones

(C) → (p) mass defect

(r) weak nuclear forces are Responsible

(D) → (r)

(s)

Sol 42: (A) → r (from def.)

(B) → p (from def.)

(C) → q (from def.)

(D) → q (from def.)

Radioactivity

Single Correct Choice Type

Sol 43: (D) $\frac{1}{2^{\frac{t}{t_{1/2}}}} = \frac{5}{6} \Rightarrow 2^{\frac{t}{t_{1/2}}} = \frac{6}{5}$

$\frac{t}{t_{1/2}} = \frac{\ln 6/5}{\ln 2}$

$t = \frac{t_{1/2}\ln 6/5}{\ln 2} = \frac{1}{\lambda}\ln 6/5$

Sol 44: (C) For $Tc^{99} = t_{1/2}$

$t_{1/2} = 6.0$ hr

Let the minimum amount be x

Concentration after 3 hr = $\frac{x}{\sqrt{2}}$

$\frac{x}{\sqrt{2}} \geq 10.0$ mg $\Rightarrow x \geq 10.0 \times \sqrt{2}$ mg

$x \geq 14.1$ mg

Sol 45: (C) $N_{11} = 0.1\ e^{-\lambda\times 11}$

$N_{10} = 0.1\ e^{-\lambda\times 10}$

Atoms decaying during 11th day

$= N_{10} - N_{11} = 0.1\ (e^{-10\lambda} - e^{-11\lambda})$

$$= 0.1\left(-e^{-\frac{\ln 2\times 11}{5}} + e^{-\frac{\ln 2\times 10}{5}}\right)$$

Multiple Correct Choice Type

Sol 46: (A, B, C, D) (A) $t_{1/2} \propto C^{1-n}$

where n is the order of the reaction.

(B) $t_{avg} = \frac{1}{\lambda} = \frac{t_{1/2}}{\ln 2}$

(C) $\frac{dN}{dt} = -\lambda N^2$

$\frac{dN}{N^2} = -\lambda dt; \quad \frac{1}{N} = \lambda t; \quad N = \frac{1}{\lambda t}$

(D) $t_{1/2} = \frac{0.693}{0.0693} = 10$ min

% Reactant $= \frac{100}{2^{10}} = \frac{100}{1024} = 0.098$

Reaction completions = 99.92%

Sol 47: (B, C, D) ${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}\beta$

${}_2C^{14}H_4 \rightarrow 2N^{14}H_3 + H_2$

(A)

Sol 48: (C, D) (A) Within the atom not nucleus

(B) ${}^{3}_{1}H \rightarrow {}^{3}_{2}He + {}^{0}_{-1}\beta$

(C) ${}^{1}_{1}P \rightarrow {}^{0}_{+1}\beta + {}^{1}_{0}n$

$\therefore$ n/p ratio increases

(D) True

Comprehension Type

Paragraph 1

Sol 49: (C) $\frac{dN}{dt} = \alpha - \lambda N;$

$\frac{dN}{\alpha - \lambda N} = dt$

$$\frac{-1}{\lambda}\ln\left(\frac{\alpha-\lambda N}{\alpha-\lambda N_0}\right) = t \Rightarrow \frac{\alpha-\lambda N}{\alpha-\lambda N_0} = e^{-\lambda t}$$

$\alpha - \lambda N = (\alpha - \lambda N_0)\ e^{-\lambda t} \Rightarrow \lambda N = \alpha - (\alpha - \lambda N_0)\ e^{-\lambda t}$

$\Rightarrow N = \frac{1}{\lambda}\ [\alpha - (\alpha - \lambda N_0)\ e^{-\lambda t}]$

Sol 50: (C) $t_{1/2} = \frac{\ln 2}{\lambda}$

$$N = \frac{1}{\lambda}\ (\alpha - (\alpha - \lambda N_0)\ e^{-\lambda\frac{\ln 2}{\lambda}}) = \frac{1}{\lambda}\left(\alpha - \left(\frac{\alpha-\lambda N_0}{2}\right)\right)$$

$$= \frac{1}{\lambda}\left(\frac{\alpha}{2}+\frac{\lambda N_0}{2}\right) = \frac{1}{\lambda}\left(\frac{2N_0\lambda}{2}+\frac{\lambda N_0}{2}\right)$$

$N = 1.5\ N_0$

Paragraph 2

Sol 51: (A) ${}^{10}_{4}Be \rightarrow {}^{10}_{5}Be + e^-$

$\Delta m = (4m_p + 6m_n) - (5m_p + 5m_n) - m_e$

$+4m_e - 5m_e + m_e$

= At. mass of ${}_4Be^{10}$ – At mass of ${}_5B^{10}$

Sol 52: (D) ${}^{8}_{5}Be \rightarrow {}^{8}_{4}Be + e^+$

$\Delta m = (5m_p + 3m_n) - (4m_p + 4m_n) - m_e$

$+5m_e - 4m_e - m_e$

Δm = At. mass of ${}_5B^8$ – At mass of ${}_4Be^8$

– mass of two electrons

Previous Years' Questions

Sol 1: Speed of neutrons

$$= \sqrt{\frac{2K}{m}}\ \left(\text{from } K = \frac{1}{2}mv^2\right)$$

$$\text{or } v = \sqrt{\frac{2\times 0.0327\times 1.6\times 10^{-19}}{1.675\times 10^{-27}}} \approx 2.5\times 10^3 \text{m/s}$$

Time taken by the neutrons to travel a distance of 10 m:

$t = \frac{d}{v} = \frac{10}{2.5\times 10^3} = 4.0\times 10^{-3}$

Number of neutrons decayed after time

t : $N = N_0(1 - e^{-\lambda t})$

$\therefore$ Fraction of neutrons that will decay in this time interval

$$= \frac{N}{N_0} = (1 - e^{-\lambda t}) = 1 - e^{-\frac{\ln(2)}{700}\times 4.0\times 10^{-3}} = 3.96\times 10^{-6}$$

Sol 2: Mass defect in the given nuclear reaction:

Δm = 2(mass of deuterium) – (mass of helium)

= 2(2.0141) – (4.0026) = 0.0256

Therefore, energy released

$\Delta E = (\Delta m)(931.48)$MeV = 23.85 MeV

$= 23.85 \times 1.6 \times 10^{-13}$ J $= 3.82\times 10^{-12}$J

Efficiency is only 25%, therefore,

25% of $\Delta E = \left(\frac{25}{100}\right)(3.82 \times 10^{-12})$ J

$= 9.55 \times 10^{-13}$ J

i.e., by the fusion of two deuterium nuclei, 9.55×10^{-13} J energy is available to the nuclear reactor.

Total energy required in one day to run the reactor with a given power of 200 MW:

$E_{total} = 200\times10^6\times24\times3600=1.728\times10^{13}$J

$\therefore$ Total number of deuterium nuclei required for this purpose

$$n = \frac{E_{total}}{\Delta E/2} = \frac{2\times1.728\times10^{13}}{9.55\times10^{-13}} = 0.362\times10^{26}$$

$\therefore$ Mass of deuterium required

= (Number of g-moles of deuterium required) × 2 g

$$= \left(\frac{0.362\times10^{26}}{6.02\times10^{23}}\right)\times 2 = 120.26 \text{ g.}$$

Sol 3: (a) A – 4 = 228

$\therefore$ A = 232

92 – 2 = Z or Z = 90

(b) From the relation,

$$r = \frac{\sqrt{2Km}}{Bq}$$

$$K_\alpha = \frac{r^2B^2q^2}{2m} = \frac{(0.11)^2(3)^2(2\times1.6\times10^{-19})^2}{2\times4.003\times1.67\times10^{-27}\times1.6\times10^{-13}}$$

= 5.21 MeV

From the conservation of momentum,

or $p_\gamma = p_\alpha$ or $\sqrt{2K_\gamma m_\gamma} = \sqrt{2K_\alpha m_\alpha}$

$$\therefore K_\gamma = \left(\frac{m_\alpha}{m_\gamma}\right)K_\alpha = \frac{4.003}{228.03}\times5.21 = 0.09 \text{ MeV}$$

$\therefore$ Total energy released $= K_\alpha + K_\gamma = 5.3$ MeV

Total binding energy of daughter products

= [92× (mass of proton) + (232 – 92) (mass of neutron) – (m_γ) – (m_α)] ×931.48 MeV

= [(92×1.008) + (140)(1.009) – 228.03 – 4.003] ×931.48 MeV

= 1828.5 MeV

$\therefore$ Binding energy of parent nucleus

= binding energy of daughter products – energy released

= (1828.5 – 5.3) MeV = 1823.2 MeV

Sol 4: λ = Disintegration constant

$$\frac{0.693}{t_{1/2}} = \frac{0.693}{15}\ h^{-1} = 0.0462\ h^{-1}$$

Let R_0 = initial activity = 1 microcurie

$= 3.7 \times 10^4$ disintegration per second.

r = Activity in 1 cm^3 of blood at t = 5 h

$= \frac{296}{60}$ disintegration per second

= 4.93 disintegration per second, and

R = Activity of whole blood at time t = 5 h

Total volume of blood should be

$$V = \frac{R}{r} = \frac{R_0e^{-\lambda t}}{r}$$

Substituting the values, we have

$$V = \left(\frac{3.7\times10^4}{4.93}\right)e^{-(0.0462)(5)}\ cm^3$$

$V = 5.95 \times 10^3\ cm^3$

or V = 5.95 L

Sol 5: The reaction involved in α-decay is

$${}^{248}_{96}Cm \rightarrow {}^{244}_{94}Pu + {}^{4}_{2}He$$

Mass defect

Δm = mass of ${}^{248}_{96}Cm$ – mass of ${}^{244}_{94}Pu$ – mass of ${}^{4}_{2}He$

(248.072220 – 244.064100 – 4.002603) u

= 0.005517 u

Therefore, energy released in α-decay will be

E_α = (0.005517 × 931) MeV = 5.136 MeV

Similarly, $E_{fission}$ = 200 MeV (given)

Means life is given as $t_{mean} = 10^{13}$ s $= \frac{1}{\lambda}$

$\therefore$ Disintegration constant $\lambda = 10^{-13}\ s^{-1}$

Rate of decay at the moment when number of nuclei are $10^{20} = \lambda N = (10^{-13})(10^{20})$

$= 10^7$ disintegration per second

Of these disintegrations. 8% are in fission and 92% are in α-decay

Therefore, energy released per second

$= (0.08 \times 10^7 \times 200 + 0.92 \times 10^7 \times 5.136)$ MeV

$= 2.074 \times 10^8$ MeV

$\therefore$ Power output (in watt)

= energy released per second (J/s)

$= (2.074 \times 10^8)(1.6 \times 10^{-13})$

$\therefore$ Power output $= 3.32 \times 10^{-5}$ W

Sol 6: (a) Let at time t, number of radioactive nuclei are N. Net rate of formation of nuclei of A

$$\frac{dN}{dt} = \alpha - \lambda N$$

$$\text{or } \frac{dN}{\alpha - \lambda N} = dt \qquad \text{or } \int_{N_0}^{N} \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

solving this equation, we get

$$N = \frac{1}{\lambda}[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}] \quad \text{... (i)}$$

(b) (i) Substituting $\alpha = 2\lambda N_0$ and $t = t_{1/2} \frac{\ln(2)}{\lambda}$ in equation (i) we get,

$$N = \frac{3}{2}N_0$$

(ii) Substituting $\alpha = 2\lambda N_0$ and $t \to \infty$ in Equation (i), we get

$$N = \frac{\alpha}{\lambda} = 2N_0 \quad \text{or } N = 2N_0$$

Sol 7: The reactor produces 1000 MW power or 10^9 J/s. The reactor is to function for 10 yr. Therefore, total energy which the reactor will supply in 10 yr is

E = (power)(time)

$= (10^9\ J/s)(10 \times 365 \times 24 \times 3600\ s)$

$= 3.1536 \times 10^{17}$ J

But since the efficiency of the reactor is only 10%, therefore actual energy needed is 10 times of it or 3.1536×10^{18} J. One uranium atom liberates 200 MeV of energy or $200 \times 1.6 \times 10^{-13}$ J or 3.2×10^{-11} J of energy. So, number of uranium atoms needed are

$$\frac{3.1536\times10^{18}}{3.2\times10^{-11}} = 0.9855 \times 10^{29}$$

Or number of kg-moles of uranium needed are

$$n = \frac{0.9855\times10^{29}}{6.02\times10^{26}} = 163.7$$

Hence, total mass of uranium required is

$m = (n)M = (163.7)(235)$ kg

or $m \approx 38470$ kg or $m = 3.847 \times 10^4$ kg

Sol 8: (a) Let at time t = t, number of nuclei of Y and Z are N_Y and N_Z. Then

Rate equations of the populations of X, Y and Z are

$$\left(\frac{dN_X}{dt}\right) = -\lambda_X N_X \quad \text{... (i)}$$

$$\left(\frac{dN_Y}{dt}\right) = \lambda_X N_X - \lambda_Y N_Y \quad \text{... (ii)}$$

$$\text{and } \left(\frac{dN_Z}{dt}\right) = \lambda_Y N_Y \quad \text{... (iii)}$$

(b) Given $N_Y(t) = \frac{N_0\lambda_X}{\lambda_X - \lambda_Y}[e^{-\lambda_Y t} - e^{-\lambda_X t}]$

For N_y to be maximum

$$\frac{dN_Y(t)}{dt} = 0$$

i.e., $\lambda_X N_X = \lambda_Y N_Y$ ... (iv)

[from Equation (ii)]

$$\text{or } \lambda_X(N_0 e^{-\lambda_X t}) = \lambda_Y \frac{N_0\lambda_X}{\lambda_X - \lambda_Y}[e^{-\lambda_Y t} - e^{-\lambda_X t}]$$

$$\text{or } \frac{\lambda_X - \lambda_Y}{\lambda_Y} = \frac{e^{-\lambda_Y t}}{e^{-\lambda_X t}} - 1; \quad \frac{\lambda_X}{\lambda_Y} = e^{(\lambda_X - \lambda_Y)t}$$

$$\text{or } (\lambda_X - \lambda_Y)t\ \lambda n(e) = \ln\left(\frac{\lambda_X}{\lambda_Y}\right)$$

$$\text{or } t = \frac{1}{\lambda_X - \lambda_Y}\ln\left(\frac{\lambda_X}{\lambda_Y}\right)$$

Substituting the values of λ_X and λ_Y, we have

$$t = \frac{1}{(0.1 - 1/30)}\ln\left(\frac{0.1}{1/30}\right) = 15\ln(3)$$

or t = 16.48 s.

(c) The population of X at this moment,

$N_X = N_0 e^{-\lambda_X t} = (10^{20})e^{-(0.1)(16.48)}$

$N_X = 1.92 \times 10^{19}$

$N_Y = \frac{N_X \lambda_X}{\lambda_Y}$ [From Equation (iv)]

$= (1.92 \times 10^{19})\frac{(0.1)}{(1/30)} = 5.76 \times 10^{19}$

$N_Z = N_0 - N_X - N_Y = 10^{20} - 1.92 \times 10^{19} - 5.76 \times 10^{19}$

or $N_z = 2.32 \times 10^{19}$

Sol 9: Let N_0 be the initial number of nuclei of ^{238}U.

After time t, $N_U = N_0\left(\frac{1}{2}\right)^n$

Here n = number of half-lives

$= \frac{t}{t_{1/2}} = \frac{1.5\times10^9}{4.5\times10^9} = \frac{1}{3}$

$N_U = N_0\left(\frac{1}{2}\right)^{1/3}$

and $N_{Pb} = N_0 - N_U = N_0\left[1-\left(\frac{1}{2}\right)^{1/3}\right]$

$\therefore \frac{N_U}{N_{Pb}} = \frac{\left(\frac{1}{2}\right)^{1/3}}{1-\left(\frac{1}{2}\right)^{3}} = 3.861$

Sol 10: $\left|\frac{dN}{dt}\right|$ = |Activity of radioactive substance|

$= \lambda N = \lambda N_0 e^{-\lambda t}$

Taking log both sides

$\ln\left|\frac{dN}{dt}\right| = \ln(\lambda N_0) - \lambda t$

Hence, $\ln\left|\frac{dN}{dt}\right|$ versus t graph is a straight line with slope $-\lambda$. From the graph we can see that,

$\lambda = \frac{1}{2} = 0.5\ yr^{-1}$

Now applying the equation,

$N = N_0 e^{-\lambda t} = N_0 e^{-0.5 \times 4.16}$

$= N_0 e^{-2.08} = 0.125\ N_0 = \frac{N_0}{8}$

i.e., nuclei decreases by a factor of 8.

Hence, the answer is 8.

Sol 11: Activity $\left(-\frac{dN}{dt}\right) = \lambda N = \left(\frac{1}{t_{mean}}\right) \times N$

$\therefore N = \left(-\frac{dN}{dt}\right) \times t_{mean}$ = Total number of atoms

Mass of one atom is 10^{-25} kg = m(say)

$\therefore$ Total mass of radioactive substance

= (number of atoms) × (mass of one atom)

$= \left(-\frac{dN}{dt}\right)(t_{mean})(m)$

Substituting the values, we get

Total mass of radioactive substance = 1 mg

$\therefore$ Answer is 1.

Sol 12: A $\rightarrow$ p, q; B $\rightarrow$ p, r; C $\rightarrow$ p, s; D $\rightarrow$ p, q, r

Sol 13: (D) It is only due to collision between high energy thermal deuterons which get fully ionized and release energy which increases the temperature inside the reactor

Sol 14: From conservation of mechanical energy, we have

$U_i + K_i = U_f + K_f$

$0 + 2(1.5\ KT) = \frac{1}{4\pi\varepsilon_0}.\frac{(e)(e)}{d} + 0$

Substituting the values, we get

$T = 1.4 \times 10^9$ K

Sol 15: (B, D) If $(BE)_{final} - (BE)_{initial} > 0$

Energy will be released.

Sol 16: (B) $nt_0 > 5 \times 10_{14}$ (as given)

Sol 17: (D) $f = (1-e^{-\lambda t}) = 1-e^{-\lambda t} \approx (1-\lambda t) = \lambda t$

f = 0.04

Hence % decay ≈ 4%

Sol 18: (C) $\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{T}}$

Where, A_0 is the initial activity of the radioactive material and A is the activity at t.

So, $\frac{12.5}{100} = \left(\frac{1}{2}\right)^{\frac{t}{T}}$ $\therefore t = 3T$

Sol 19: (C)

(A) → (r, t) ; (B) → (p, s) ; (C) → (p, q, r, t); (D) → (p, q, r, t)

Sol 20: (9) ${}^{12}_{5}B \to {}^{12}_{6}C^* + e^- + v$

We take the mass of ${}^{12}_{6}C$ as 12 amu

Rest energy of ${}^{12}_{6}C^*$ = 12 × 931.5 MeV + 4.041 MeV

Energy of ${}^{12}_{5}B$ = 12 × 931.5 MeV + 0.014 × 931.5

∴ Value of the reaction = 13.041 MeV – 4.041 MeV = 9 MeV

Maximum e^- energy = 9 MeV

Sol 21: (A) $5\mu Ci = \frac{\ln 2}{T_1}(2N_0)$

$10\mu Ci = \frac{\ln 2}{T_2}(N_0)$

Dividing we get $T_1 = 4T_2$

Sol 22: (A) The electric field at r = R

$E = \frac{KQ}{R^2}$

Q = Total charge within then nucleus = Ze

So, $E = \frac{KZe}{R^2}$

So electric field is independent of a.

Sol 23: (B) $q = \int_0^R \frac{d}{R}(R - x)\, 4\pi x^2 dx = Ze$

$d = \frac{3Ze}{\pi R^3}$

Sol 24: (C) If within a sphere ρ is constant $E \propto r$

Sol 25: (8) $N = N_0 e^{-\lambda t}$

$\ln |dN/dt| = \ln(N_0 \lambda) - \lambda t$

From graph, $\lambda = \frac{1}{2}$ per year

$t_{1/2} = \frac{0.693}{1/2} = 1.386$ year

4.16 yrs = $3t_{1/2}$

∴ p = 8

Sol 26: (C) KE_{max} of β^-

Q = 0.8 × 10^6 eV

$KE_p + KE_{\beta^-} + KE_{\bar{\nu}} = Q$

KE_p is almost zero

When $KE_{\beta^-} = 0$

Then $KE_{\bar{\nu}} = Q - KE_p \cong Q$

Sol 27: (D) $0 \le KE_{\beta^-} \le Q - KE_P - KE_{\bar{\nu}}$

$0 \le KE_{\beta^-} < Q$

Sol 28: (A, C) Given data

$4.5a_0 = a_0 \frac{n^2}{Z}$ ... (i)

$\frac{nh}{2\pi} = \frac{3h}{2\pi}$ ... (ii)

So n = 3 and z = 2

So possible wavelength are

$\frac{1}{\lambda_1} = RZ^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right] \Rightarrow \lambda_1 = \frac{9}{32R}$

$\frac{1}{\lambda_2} = RZ^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right] \Rightarrow \lambda_2 = \frac{1}{3R}$

$\frac{1}{\lambda_3} = RZ^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] \Rightarrow \lambda_3 = \frac{9}{5R}$

Sol 29: (C) ${}^{6}_{3}Li \to {}^{4}_{2}He + {}^{2}_{1}H$

$\frac{Q}{C^2} = 6.015123 - 4.002603 - 2.014102$

0 = – 0.001582 < 0

So no α-decay is possible

${}^{2}_{1}H + {}^{4}_{2}He \to {}^{6}_{3}Li$

$\frac{Q}{C^2} = 2.014102 + 4.002603 - 6.015123 = 0.001582 > 0$

So, this reaction is possible

${}^{70}_{30}Zn + {}^{82}_{34}Se \to {}^{152}_{64}Gd$

$\frac{Q}{C^2} = 69.925325 + 81.916709 - 151.919803 = -0.077769 < 0$

So this reaction is not possible

Sol 30: (A) ${}^{210}_{84}Po \to {}^{4}_{2}He + {}^{206}_{82}Pb$

Q = (209.982876 – 4.002603 – 205.97455)C^2

= 5.422 MeV

from conservation of momentum

$$\sqrt{2K_1(4)} = \sqrt{2K_2(206)}$$

$4K_1 = 206K_2$

$\therefore K_1 = \frac{103}{2}K_2$

$K_1 + K_2 = 5.422$

$$K_1 + \frac{2}{103}K_1 = 5.422$$

$$\Rightarrow \frac{105}{103}K_1 = 5.422$$

$\therefore K_1 = 5.319MeV = 5319\ KeV$

Sol 31: (C) $P \to (ii); Q \to (i); R \to (iv); S \to (iii)$

$^{15}_{8}O \to ^{15}_{7}N + ^{0}_{1}\beta$ (Beta decay)

$^{238}_{92}U \to ^{234}_{90}Th + ^{4}_{2}He$ (Alpha decay)

$^{185}_{83}Bi \to ^{184}_{82}Pb + ^{1}_{1}H$ (Proton emission)

$^{239}_{94}Ph \to ^{140}_{57}La + ^{99}_{37}Rb$ (fission)

Sol 32: (B) $\frac{\lambda_{Cu}}{\lambda_{Mo}} = \left(\frac{Z_{Mo}-1}{Z_{Cu}-1}\right)^2$

Sol 33: (B) $mvr = \frac{nh}{2\pi} = \frac{3h}{2\pi}$

de-Broglie Wavelength

$$\lambda = \frac{h}{mv} = \frac{2\pi r}{3} = \frac{2\pi}{3}\frac{a_0(3)^2}{z_{Li}} = 2\pi a_0$$

Sol 34: (B) $\lambda_P = \frac{1}{\tau}; \lambda_Q = \frac{1}{2\tau}$

$$\frac{R_P}{R_Q} = \frac{(A_0\lambda_P)e^{-\lambda_P t}}{A_0\lambda_Q e^{-\lambda_Q t}}$$

At $t = 2\tau$; $\frac{R_P}{R_Q} = \frac{2}{e}$

Sol 35: (A) Q value of reaction

= (140 + 94) × 8.5 –236 × 7.5 = 219 Mev

So, total kinetic energy of Xe and Sr

= 219 –2 –2 = 215Mev

So, by conservation of momentum, energy, mass and charge, only option (A) is correct

Sol 36: (C) $(BE)_{^{15}_{7}N} = 7m_p + 8m_n - m_{^{15}_{7}N}$

$$(BE)_{^{15}_{8}O} = 8m_p + 7m_n - m_{^{15}_{8}N}$$

$$\Rightarrow \Delta(BE) = (m_n + m_p) + \left(m_{^{15}_{8}O} - m_{^{15}_{7}N}\right)$$

= 0.00084 + 0.002956 = 0.003796 u

$$\Rightarrow \frac{3}{5} \times \frac{14 \times 1.44\ MeV\ f_m}{0.003796 \times 931.5\ MeV} = R$$

$\Rightarrow R = 3.42\ f_m$

Sol 37: (C) Activity A $\propto$ N (Number of atoms)

$$N = N_0\left(\frac{1}{2}\right)^n$$

where n $\to$ Number of half lives

If $N = \frac{N_0}{64}$

$$N_0\left(\frac{1}{2}\right)^n = \frac{N_0}{64}$$

$$\left(\frac{1}{2}\right)^n = \frac{1}{64} = \left(\frac{1}{2}\right)^6$$

n = 6

time = n × $T_{1/2}$

time = 6 × 18 days = 108 days