21. MOVING CHARGES AND MAGNETISM

1. INTRODUCTION

In the previous chapters on electrostatics and current electricity, we have studied about the electric force and electric field. Another important property associated with moving charges is the magnetic force and the magnetic field. The current flowing in a conductor produces a magnetic field and any charge moving in this field will experience a magnetic force which will depend on the velocity (both magnitude and direction) as well as on some property of the field. We will study the properties and laws governing the magnetic field and magnetic force in detail in this chapter.

There are a wide variety of industrial and medical applications of magnetic fields and forces. Common example, is the use of electromagnet to lift heavy pieces of metal. Magnets are used in CD and DVD players, computer hard drives,loud speakers, headphones, TVs, and telephones. We are surrounded by magnets. Right from our doorbells to cars to security alarm systems and in our hospitals, magnets are being used everywhere.

2. LORENTZ FORCE: DEFINITION OF MAGNETIC FIELD B

If electric field and magnetic field occur simultaneously in a region then the force acting on a point charge q in the region will depend both on the position of the charge as well as on its velocity. The force $\vec{F}$ will have two components, viz. the electric force $\vec{F}_e$ and magnetic force $\vec{F}_m$. The force $\vec{F}_e$ does not depend on the motion of the charge, but only on its position, while $\vec{F}_m$ depends both on charge's velocity and position (see Fig. 21.1). The magnitude of $\vec{F}_e$ is qE and direction is along $\vec{E}$ (q is positive).

To know the direction and magnitude of $\vec{F}_m$ we introduced a vector $\vec{B}$ called magnetic flux density ormagnetic induction, which characterizes the magnetic field at a particular point. Experiments show that the force $\vec{F}_m$ isproportional to the magnitude of charge q, to the velocity $\vec{v}$ of the charge and the magnitude of density $\vec{B}$, this force being always perpendicular to vector $\vec{v}$ as well as vector $\vec{B}$. Also, if the charge moves along the direction of $\vec{B}$ at a point then the magnetic force on it is zero. We can summarize all these experimental results with the following vector equation:

$$\vec{F}_m = q\vec{v} \times \vec{B}$$

That is, the force $\vec{F}_m$ on the point charge is equal to the

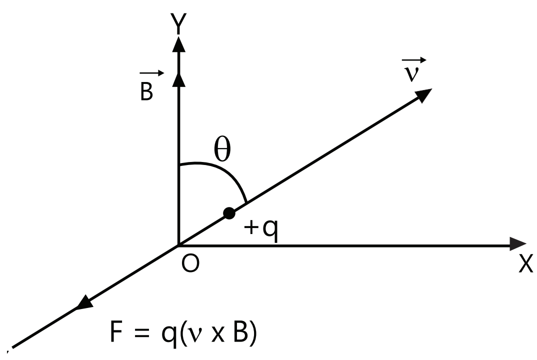

Figure 21.1: Magnetic Force on a Moving Charge

charge q times the cross product of its velocity $\vec{v}$ and the field $\vec{B}$ (all measured in the same reference frame). Using formula for the magnitude of cross product, we can write the magnitude of $\vec{F}_m$ as $F_m = |q|vB\sin\theta$ where θ is the angle between the velocity $\vec{v}$ and magnetic field $\vec{B}$.

If angle θ is 90°, then the above relation for magnetic force can be used to define the magnitude of magnetic flux density B as,

$$B = \frac{F_m}{|q|v_\perp}$$

where $v_\perp$ is the velocity component perpendicular to vector $\vec{B}$.

Thus, the total electromagnetic force acting on charge q is given as, $\vec{F} = \vec{F}_e + \vec{F}_m$

or $\vec{F} = q\vec{E} + q[\vec{v}\times\vec{B}]$

This is called Lorentz force.

The unit of B is Tesla abbreviated as T. If q=1 C, v=1 ms^{-1}, sin θ=1 for θ = 90°, and F_m = 1 N,then B=1 T = 1 Weber-m^{-2}. Thus 1 Tesla is defined as the unit of magnetic field strength in S.I units which when acting on 1 C of charge moving with a velocity of 1 ms^{-1} at right angles to the magnetic field exerts a force of 1 N in a direction perpendicular to that of field and velocity vectors. C.G.S. units of magnetic field strength or magnetic induction is 1 gauss or 1 oersted. 1 gauss = 1 oersted= 10^{-4}T.

Illustration 1: A 2 MeV proton is moving perpendicular to uniform magnetic field of 2.5 T. What is the magnetic force on the proton? (Mass of proton = 1.6 x 10^{-27} kg) **(JEE MAIN)**

Sol: Kinetic energy of proton is $K.E. = \frac{m_p v^2}{2}$. 1 MeV=1.6 x 10^{-13}J.

K.E = 2 MeV = 2 x 1.6 x 10^{-13}J or $\frac{1}{2}mv^2$= 3.2 x 10^{-13}J

$$\therefore V = \sqrt{\frac{2x3.2x10^{-13}}{m}} = \sqrt{\frac{2x3.2x10^{-13}}{1.6x10^{-27}}} = 2 \times 10^7 \text{ m s}^{-1}$$

Now, magnetic force on proton, F= ev B = 1.6 x 10^{-19} x 2 x 10^7 x 2.5 = 8.0 x 10^{-12} N

Illustration 2: A charged particle is projected in a magnetic field $\vec{B} = (3\hat{i} + 4\hat{j})x10^{-2}\,T$

The acceleration of the particle is found to be, $\vec{a} = (x\hat{i} + 2\hat{j})\ ms^{-2}$ Find the value of x. **(JEE MAIN)**

Sol: Magnetic force on a moving charge is perpendicular to the magnetic field. Therefore the dot product of force and magnetic field vector is zero.

As we have read $\vec{F}_m \perp \vec{B}$ i.e., the acceleration $\vec{a} \perp \vec{B}$ or $\vec{a}\cdot\vec{B} = 0$

or $(x\hat{i} + 2j)\cdot(3\hat{i} + 4j)x10^{-2} = 0$; (3x+8)x$10^2$=0 $\therefore\ x = -\frac{8}{3}ms^{-2}$

3. RELATION BETWEEN ELECTRIC AND MAGNETIC FIELD

Suppose in a particular inertial reference frame K, the electric field is zero and the magnetic field has a non-zero finite value. A point charge is moving with some velocity $\vec{v}$ in the frame K and thus experiences a magnetic force, and its velocity changes. Now suppose we have a frame K' translating with respect to frame K withconstantvelocity $\vec{v}$. In the frame K',the point charge is initially at rest, and so the magnetic force on it will be zero. Butas its velocity changes in the K frame, its velocity changes in the K' frame as well, i.e. it experiences a force in K' frame as well.

This initial force on it is the force $\vec{F}_e$ due to electric field in the K′ frame.Thus the magnetic field in K frame appears as a combination of electric field and magnetic field in K′ frame. The electric and magnetic fields are thus inter-dependent. We introduce a single physical entity called electromagnetic field. Whether the electromagnetic field will appear as electric field or magnetic field depends on the frame of reference. If we confine to a particular reference frame, we can treat electric fieldand magnetic field as separate entities. A field which is constant in one reference frame in the general case is found to vary in another reference frame.

4. MAGNETIC FIELD LINES

Magnetic field lines are used to represent the magnetic field in a region. The rules to construct the magnetic field lines are:-

(a) The direction of tangent to a magnetic field line at a point gives the direction of magnetic flux density vector $\vec{B}$ at that point.

(b) The density of the magnetic field lines at a point isproportional to the magnitude of vector $\vec{B}$ at that point. At points where the field lines are closer together, the magnetic field is stronger.

PLANCESS CONCEPTS

- In case of a bar magnet, the density of magnetic field lines is high at points near the poles, and the density at pointsnear the center of the magnet is low.
- If we place a magnetic compass at any point in the earth’s magnetic field, it will align itself in the direction of the magnetic field lines.

Vaibhav Krishnan (JEE 2009 AIR 22)

PLANCESS CONCEPTS

- Common misconception about magnetic field lines is that it is the path followed by a magnetic north pole in a magnetic field.
- This is not correct. It is the instantaneous direction of the magnetic force acting on the magnetic north pole in the magnetic field.

Vaibhav Gupta (JEE 2009 AIR 54)

5. EARTH’S MAGNETIC FIELD

Magnetic field is present everywhere near the earth’s surface. The line of earth’s magnetic field lies in a vertical plane coinciding with the magnetic north-south direction at that place i.e. the plane passing through the geomagnetic poles. This plane is called the Magnetic Meridian. This plane is slightly inclined to the plane passing through the geographic poles called the geographic meridian.The angle between the magnetic meridian and the geographic meridian at a point is called the declination at that point. The earth’s magnetic poles are opposite to the geographic poles i.e. at earth’s North Pole, its magnetic south pole is situated and vice versa.

In the magnetic meridian plane, the magnetic field vector of the earth at any point, is generally inclined to the horizontalat that pointby an angle called the magneticdip at that point. If magnetic field of the earth at that point is B and the dip is θ,

B_v = the vertical component of B in the magnetic meridian plane = B sinθ

B_H = the horizontal component of B in the magnetic meridian plane = B cosθ.

$$\frac{B_V}{B_H} = \tan\theta$$

6. MOTION OF CHARGED PARTICLE IN ELECTRIC AND MAGNETIC FIELD

6.1 Trajectory of a Charged Particle Moving in Uniform Electric Field

Let a positively charged particle having charge +q and mass m enter at origin O with velocity v along X-direction in the region where electric field Eis along the Y-direction (see Fig. 21.2).

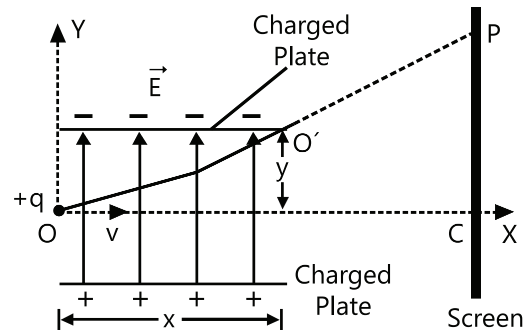

Figure 21.2: Charged particle moving in electric field

Force acting on the charge +q due to electric field E is given by

$$\vec{F} = q\vec{E}$$

Acceleration of the charged particle is $\vec{a} = \frac{\vec{F}}{m}$ or $\vec{a} = \frac{q\vec{E}}{m}$ ...(i)

The charged particle will accelerate in the direction of $\vec{E}$ and get deflected from its straight line path.

During its motion in the region of electric field, along x-axis we have u_x=v and a_x= 0 and x = vt

or $\quad t = \frac{x}{v}$ ...(ii)

Along y axis we have, $u_y = 0,\ a_y = \frac{qE}{m}$ (∴ Initially the particle was moving along x-direction)

$$y = \frac{1}{2}a_y t^2$$

$$\therefore \qquad y = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$$

Using Eq, (ii), we get $y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x}{v}\right)^2$ or $y = \frac{qEx^2}{2mv^2} = Kx^2$ ...(iii)

where $\quad K = \frac{qE}{2mv^2}$ is a constant.Thus the charged particle moves along a parabolic trajectory.

6.2 Trajectory of a Charged Particle Movingin Uniform Magnetic Field

(a) Magnetic force acting on a charged particle moving with velocity $\vec{v}$ parallel (θ=0) or antiparallel (θ=180°) to $\vec{B}$, will be zero. Thus the trajectory of the particle is a straight line.

(b) If velocity $\vec{v}$ of the particle is perpendicular to $\vec{B}$ i.e. θ =90°, then magnetic force is F= qvB and the direction of this force is always perpendicular to v. The charged particle moves in a circular trajectory (see Fig. 21.3).

(c) If velocity $\vec{v}$ of the charged particle makes an angle θ with $\vec{B}$, the particle moves in a helical path. The component v sin θ which is perpendicular to $\vec{B}$ drives the charged particle along a circular path whereas the component v cos θ , which is parallel or antiparallel to $\vec{B}$, remains unchanged as there is no magnetic force along the direction of $\vec{B}$. Thus the charged particle moves along a helical path (see Fig. 21.4).

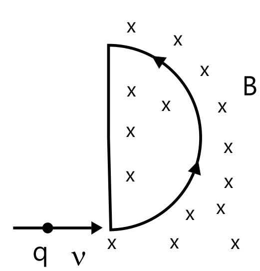

Figure 21.3: Charged particle moving in uniform magnetic field in electric field

(d) The magnetic force on the component of velocity perpendicular to the magnetic field provides the centripetal force to the charged particle to follow a circular trajectory of radius r.

$$qv_{\perp}B = \frac{mv_{\perp}^2}{r}$$

$$\text{or} \quad r = \frac{mv_{\perp}}{qB}$$

Angular velocity, $\omega = \frac{v_{\perp}}{r} = \frac{qB}{m}$

Frequency $f = \frac{qB}{2\pi m}$

Time period $T = \frac{2\pi m}{qB}$

Time period T is independent of v.

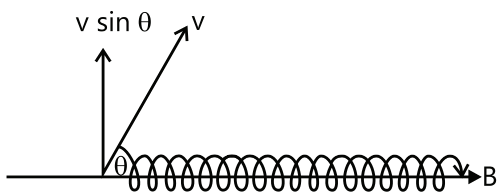

Figure 21.4: Charged particle moving in helical path in uniform magnetic field

7.DISCOVERY OF ELECTRON

The Fig. 21.5 shows the simplified version of Thomson's' experiment. An electric field $\vec{E}$ is established in the region between the deflecting plates by connecting a battery across their terminals. The magnetic field $\vec{B}$ in the region between the deflecting plates is directed into the plane of the figure.

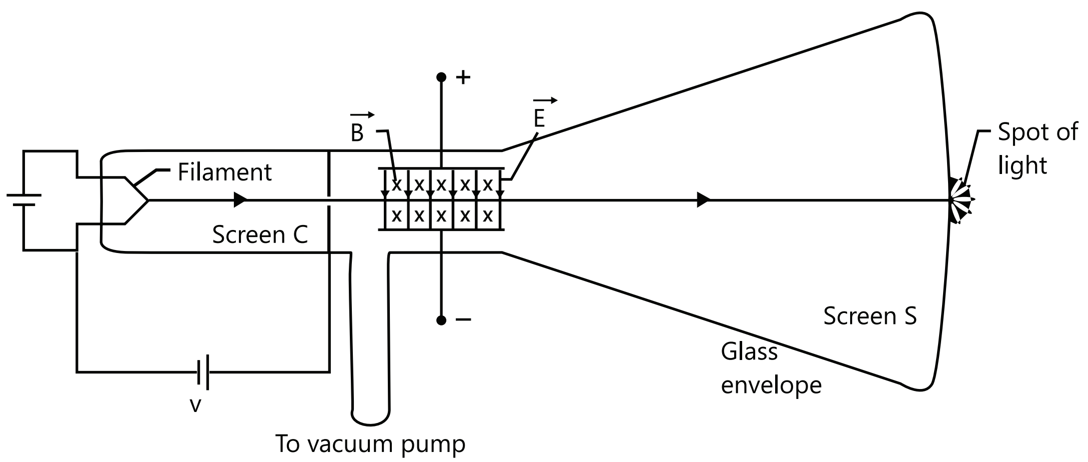

Figure 21.5: Thomson's experimental set up

Charged particles (electrons) are emitted by a hot filament at the rear of the evacuated cathoderay tube and are accelerated by an applied potential difference V. After they pass through a slit in screen C, they form a narrow beam. They then pass through the region between the deflecting plates, headed towards the center of fluorescent screen S, where they produce a spot of light. The crossed-fields $\vec{E}$ and $\vec{B}$ in the region between the deflecting plates can deflect them from the center of the screen. By controlling the magnitude and directions of the fields, $\vec{E}$ and $\vec{B}$ the deflection of the charged particles can be controlled.

When both the fields $\vec{E}$ and $\vec{B}$ are turned-off the beam of charged particles reaches the screen un-deflected.

When field $\vec{E}$ is turned-on the beam of charged particles is deflected.

Keeping the field $\vec{E}$ unchanged, field $\vec{B}$ is also turned-on.The magnitude of $\vec{B}$ is adjusted such that the deflection of the charged particles becomes zero. In this situation the electric force on the charged particles is balanced by the magnetic force.

$$q\,\vec{E} = -q\,\vec{v} \times \vec{B}$$

or $$\vec{E} = -\vec{v} \times \vec{B}$$

The ratio of magnitudes of $\vec{E}$ and $\vec{B}$ in this situation gives the speed of the charged particles.

$$v = \frac{E}{B}$$

When only field $\vec{E}$ is turned-on, the displacementof the charged particlesin the y-direction, when they reach the end of the plates, as derived in article 6.1 is

$$y = \frac{|q|EL^2}{2mv^2}$$

where v is the particle's speed along x-direction, mits mass, qits charge, and L is the length of the plates. The direction of deflection of charged particles show that the particles are negatively charged.

Substituting the value of v in terms of E and B we get,

$$y = \frac{|q|B^2L^2}{2mE}$$

or $$\frac{m}{|q|} = \frac{B^2L^2}{2yE}$$

Thus in this way the mass to charge ratio of electrons was discovered.

PLANCESS CONCEPTS

Charged particle motion as a points on wheel

- 1. Suppose electric and magnetic field are perpendicular to each other and a charged particle is projected perpendicular to magnetic field, its motion can be assumed as that of the motion of a particle on a wheel
- 2. The point could be inside, on or outside the wheel depending on the problem
- 3. Suppose in this field it is projected in any other way (expect along the magnetic field) its horizontal motion is still like that of a point on a wheel, while vertical motion will be uniform velocity motion
- 4. To such problem, just resolve the particle velocity in to along the magnetic field and perpendicular to it
- 5. If electric field is not perpendicular, resolve it also into along and perpendicular to magnetic field and solve accordingly.

Nitin Chandrol (JEE 2012 AIR 134)

8. HALL EFFECT

The Hall Effect is the production of a voltage difference (the Hall voltage) across acurrent carrying conductor, lying in a magnetic field perpendicular to the current. The hall voltage is produced in the direction transverse to the electric current in the conductor. It was discovered by Edwin Hall in 1879.Hall Effect allows us to find out whether the charge carries in a conductor are positively or negatively charged and the number of charge carries per unit volume of the conductor.

External magnetic field $\vec{B}$, points into the plane of a copper strip of width d, carrying a current I as shown in Fig. 21.6.The magnetic force $\vec{F}_m$ will act on each drifting electron, towards the right edge of the strip. As the electrons accumulate on the right edge, positive charges are induced on the left edge and an electric field $\vec{E}$ is produced within the strip, directed from left to right.This field exerts an electric force $\vec{F}_e$ on each electron, towards the left edge of the strip.The hall potential difference V across the width of the strip,due to the electric field $\vec{E}$ isV=Ed.

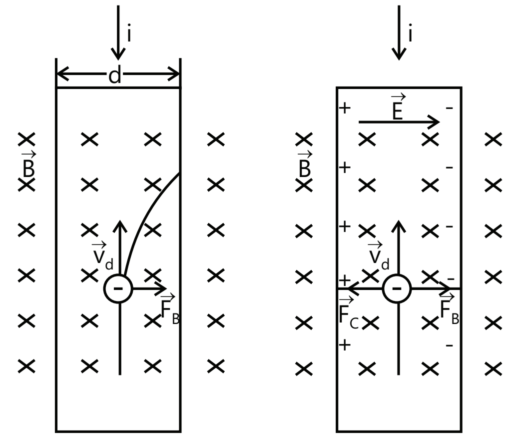

Figure 21.6: Hall Effect in conductor

When the electric and magnetic forces balance each other, $eE=ev_dB$ or $E = v_d B$

Thedrift speed v_d is given as $v_d = \frac{J}{ne} = \frac{I}{neA}$

So we obtain $n = \frac{BI}{V\ell e}$ where $\ell\left(= \frac{\text{Cross-section Area}}{\text{Width}}\right)$ is the thickness of the strip.

Illustration 3:Copper has 8.0×10^{28} conduction electrons per metre^3. A copper wire of length 1 m and cross-sectional area 8.0×10^{-6} m^2 carrying a current and lying at right angle to magnetic field of strength 5×10^{-3} T experiences a force of 8.0×10^{-2}N. Calculate the drift velocity of free electrons in the wire. **(JEE ADVANCED)**

Sol: If v is the drift speed of electrons then the magnetic force on the wire is

$$F = qvB\sin\theta = qvB\sin 90^o = qvB$$

where q is the total charge of electrons in the wire.

$n=8.0 \times 10^{28}$ m^{-3}

l= 1 m; $A=8.0 \times 10^{-6}$m^2

Charge on each electron, $e=1.6 \times 10^{-19}$ C

Number of electrons in the copper wire = n x volume of wire = n(A l)

Total charge in the wire, $q=n((Al)e$ or $q=8.0 \times 10^{28} \times 8.0 \times 10^{-6} \times 1 \times 1.6 \times 10^{-19}=1.024 \times 10^5$C

Using $F=qvB\sin\theta$, we have, $v = \frac{F}{qB\sin\theta} = \frac{8.0\times10^{-2}}{1.024\times10^5 \times 5\times10^{-3} \times \sin 90^o}$ = 1.563×10^{-4}m s^{-1}

9. MAGNETIC FORCE ON A CURRENT CARRYING WIRE

Suppose in a conductor number of free electrons per unit volume is n, then in an infinitesimal volume dV in the conductor, the total charge of free electrons will be

$$dq = ne\, dV$$

If the magnetic field at the location of the elementary volume is $\vec{B}$, and the drift velocity of free electrons is $\vec{v}_d$ then the magnetic force on the elementary volume will be

$$d\vec{F} = ne[\vec{v}_d \times \vec{B}]dV$$

Now we know that the current density is given as

$$\vec{j} = ne\vec{v}_d$$

So $\quad d\vec{F} = [\vec{j} \times \vec{B}]dV$

Introducing the vector $d\vec{\ell}$ in the direction of current we can write, $\vec{j}dV = \vec{j}\Delta S d\ell = I\,d\vec{\ell}$. Here ΔS is the area of cross-section and $d\ell$ the length of the elementary volume dV.

So $\quad d\vec{F} = I\,[d\vec{\ell} \times \vec{B}]$

The total magnetic force on the conductor is $\vec{F} = I\int [d\vec{\ell} \times \vec{B}]$

For a thin straight wire of length L, if the field $\vec{B}$ is constant throughout the length of the wire and perpendicular to it, we can write

$$F = I\,L\,B$$

In vector form we can write, $\vec{F} = I\,\vec{L} \times \vec{B}$, where $\vec{L}$ is a length vector that has magnitude L and is directed along the wire segment in the direction of the (conventional) current.

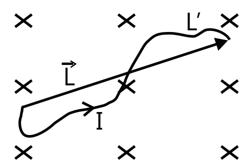

Figure 21.7: Current carrying conductor in uniform magnetic fieldr

Few important points regarding the force on current carrying conductor in magnetic field are given below:

(a) In a uniform magnetic field the force, dF= $IBd\ell\,\sin\theta$, does not depend on the position vector $\vec{r}$ of the current element.Thus this force is non-central. (Acentral force is a function of position vector $\vec{r}$, $\vec{F} = f(\vec{r})$)

(b) Theforce $d\vec{F}$ is always perpendicular to the plane containing $\vec{B}$ and $d\vec{\ell}$.Vectors $\vec{B}$ and $d\vec{\ell}$ may or may not be perpendicular to each other.

(c) As explained above, the total magnetic force on the conductor is

$\vec{F} = I\int [d\vec{\ell} \times \vec{B}]$

For uniform magnetic field, $\vec{B}$ can be taken out from the integral.

$\vec{F} = I\left[\int d\vec{\ell}\right] \times \vec{B}$

According to the law of vector addition $\int d\vec{\ell}$ is equal to the length vector $\vec{L}$ from initial to final point of the conductor as shown in Fig. 21.7. For a conductor of any arbitrary shape the magnitude of vector $\vec{L}$ is different from the actual length L' of the conductor.

$\therefore \quad \vec{F} = I\vec{L} \times \vec{B}$

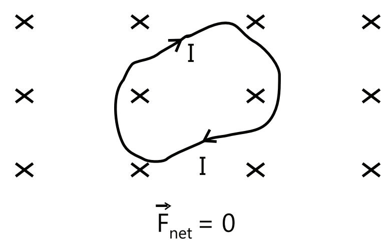

Figure 21.8: Current carrying loop in uniform magnetic field

(d) For a current carrying closed loop of any arbitrary shape placed in a uniform magnetic field (see Fig. 21.8),

$\vec{F} = I\left[\oint d\vec{\ell}\right] \times \vec{B} = 0$

Here as we add all the elementary vectors $d\vec{\ell}$ around the closed loop, the vector sum is zero because the final point is same as the initial point.

$\therefore \quad \oint d\vec{\ell} = 0$

Thus the net magnetic force on a current loop in a uniform magnetic field is always zero.

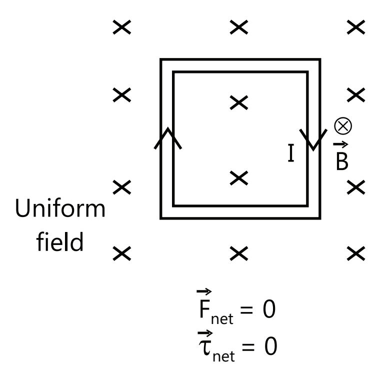

Figure 21.9: Area vector of closed loop is in direction of uniform magnetic field

However, different parts of the loop do experience different net forces, although the vector some of all these forces comes out to be zero.

So the loop may experience some infinitesimal contraction or expansion, thus may be under tension.

Although the resultant of magnetic forces acting on the loop is zero, the resultant torque due the magnetic forces may not be zero.

Thus the torque on a loop in a uniform magnetic field is not always zero.

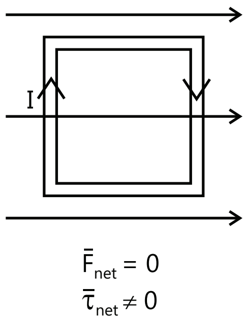

Figure 21.10: Area vector of closed loop is perpendicular to uniform magnetic field magnetic field

(e) When a current carrying closed loop is placed in a non-uniform magnetic field, in the general case it will experience non-zero net force as well as net torque.

Even a conductor of arbitrary shape not forming a loop, will experience a torque in a non-uniform field.

If the conductor is free to move, it will execute combined translational and rotational motion.

(f) When a current carrying conductor or closed loop translates or rotates in a magnetic field, the kinetic energy gained by it is, not due to the work done by magnetic forcesbut, at the expense of the energy supplied by the electric source which is maintaining current in the conductor/loop.

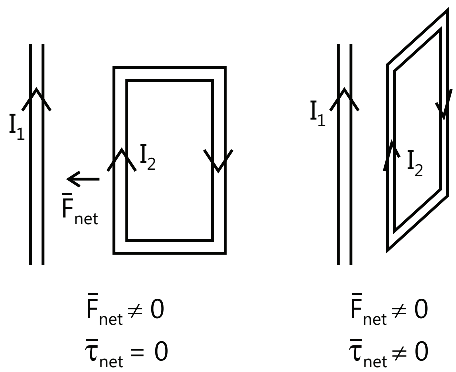

Figure 21.11: Closed loop in non-uniform magnetic field

The net work done by magnetic forces acting on a current carrying conductor is zero.

Though it may appear that,

$$W = \int \vec{F}.d\vec{r} = \int [I\int (d\vec{\ell} \times \vec{B})].d\vec{r} = \Delta K$$

but actually the kinetic energy is supplied by the electric source.

Illustration 4: A wire 12 cm long and carrying a current of 2 A is placed perpendicular to a uniform magnetic field. If a force of 0.8 N acts on it, calculate the value of the magnetic induction. **(JEE MAIN)**

Sol: This problem can be solved using formula $F = BI\ell\sin\theta$ for force on current carrying wire in uniform magnetic field.

ℓ = 12 cm = 12 x 10^{-2} m ; I = 2 A ; F = 0.8 N; $\theta = 90^o$

Using, F= BIlsin θ, we get $B = \dfrac{F}{I\ell\sin\theta} = \dfrac{0.8}{2x12x10^{-12}x\sin 90^o} = 3.3\,T$

9.1 Fleming's Left Hand Rule

If the thumb and the first two fingers of the left hand are stretched mutually perpendicular to each other and if the first finger points in the direction of the magnetic field and the second middle finger points in the direction of the current in the conductor, then the direction of thumb gives the direction of force on the conductor.

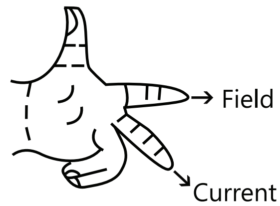

Figure 21.12: Fleming's Left hand Rule

10. TORQUE ON A CURRENT LOOP

Let us consider a square loop PQRS having side ℓ and area A=l^2 (See Figure). Let us introduce a unit vector $\hat{n}$ normal to the plane of the loop whose direction is related to the direction of current in the loop by the right-hand screw rule. Area of the loop can be written in vector form as $\vec{A} = \ell^2\,\hat{n}$.

If current I in the loop is anti-clockwise then the vector $\hat{n}$ will be directed along the perpendicular to the plane of the paper towards the reader as shown in the Fig. 21.13. Suppose the loop is placed in a uniform magnetic field $\vec{B}$ directed along the perpendicular to the plane of the paper towards the reader, i.e. along the vector $\hat{n}$.In this situation, the magnitude of magnetic force on each of the branches of the loop will be IℓB, i.e. $\left|\vec{F}_1\right| = \left|\vec{F}_2\right| = \left|\vec{F}_3\right| = \left|\vec{F}_4\right| = I\ell B$. The direction of force on each branch can be found by Fleming's left hand rule. We can easily see that $\vec{F}_1 = -\vec{F}_3$ and $\vec{F}_1$ and $\vec{F}_3$ have same line of action. Similarly $\vec{F}_2 = -\vec{F}_4$ and $\vec{F}_2$ and $\vec{F}_4$ have same line of action.So, the net force as well as the net torque on the loop PQRS is zero.

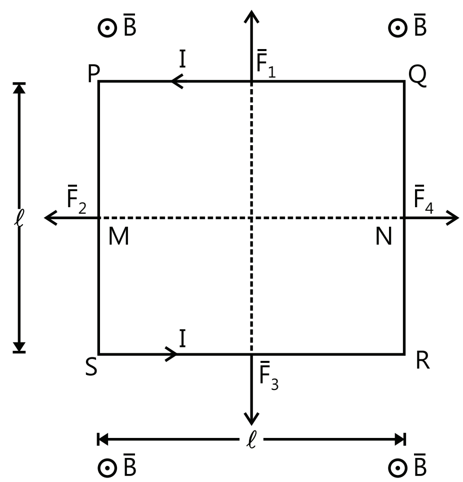

Figure 21.13: Zero torque on closed loop in uniform magnetic field

Now suppose the loop is rotated through an angle θ about the lineMN as shown in Fig. 21.14).So the anglebetween vector $\hat{n}$ and $\vec{B}$ will beθ. In this situation each of the sides Q'R' and S'P' makes an angle 90°-θ with the magnetic field $\vec{B}$ so that $\left|\vec{F}_2\right| = \left|\vec{F}_4\right| = I\ell B\cos\theta$ and again we have $\vec{F}_2 = -\vec{F}_4$ and $\vec{F}_2$ and $\vec{F}_4$ have same line of action. The side PQ shifts to P'Q' and RS shifts to R'S' such that PQ || P'Q' and RS || R'S' so that $\left|\vec{F}_1\right| = \left|\vec{F}_3\right| = I\ell B$ and again we have $\vec{F}_1 = -\vec{F}_3$, but the lines of action of $\vec{F}_1$ and $\vec{F}_3$ are displaced from each other by a distance of l$\sin\theta$. This forms a force couple, and the torque due to it will have magnitude

$$\tau = (I\,\ell\,B)\,\ell\sin\theta = I\,\ell^2\,B\sin\theta = I\,A\,B\sin\theta$$

This torque is directed along the line MN.

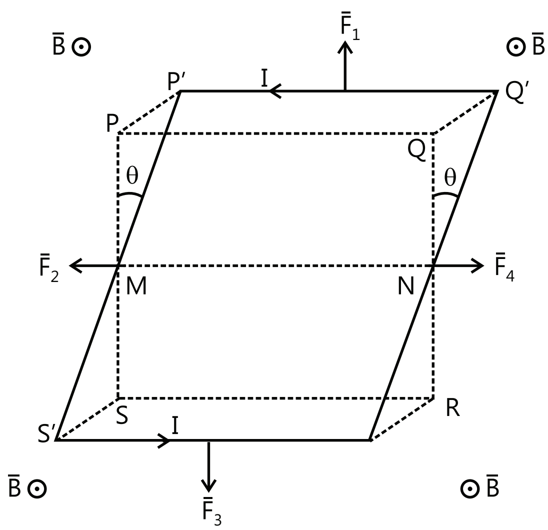

Figure 21.14: 14 Non-zero torque on closed loop in uniform magnetic field

In vector form we can write $\vec{\tau} = I\vec{A}\times\vec{B}$

Defining magnetic dipole moment of the loop as $\vec{M} = I\vec{A} = IA\hat{n}$, we can write torque as $\vec{\tau} = \vec{M}\times\vec{B}$

If the number of turns in the loop is N then we have, $\vec{M} = NI\vec{A} = NIA\hat{n}$

Note that although this formula has been derived for a square loop, it comes out to be true for any shape of the loop.

Illustration 5: A vertical circular coil of radius 0.1 m has moment of inertia as 1 x 10^{-1}kg m^2.It is free to rotate along y-axis coinciding with its diameter. Initially axis of the coil and direction of magnetic field of 1 T are along x-axis. The coil takes a quarter rotation. Find **(JEE ADVANCED)**

(i) Magnetic field strength at the center of the coil. Current of 3.19 A flows through this coil having 200 turns.

(ii) Magnetic moments of the coil.

(iii) Torque at the initial and final positions of the coil.

(iv) Angular speed at the final position.

Sol: The torque on coil is $\vec{\tau} = -\vec{M}\times\vec{B}$ where $\vec{M}$ the magnetic moment of coil is. As torque $\tau = I\alpha = I\frac{d\omega}{dt} = I\frac{d\omega}{d\theta}\omega$, integrating equation of torque we get the angular velocity.

(i) Using $B = \frac{\mu_0 NI}{2R}$, we have $B = \frac{(4\pi x10^{-7})(200)(3.19)}{2x0.1} = 4x10^{-6}T$

(ii) Magnetic moment, m = NIA=NI (πR^2)= 200 x 3.19 x πx$(0.1)^2$ = 20Am^2

(iii) Torqueτ=N_1ABsinθ =m sinθ; initially θ =0 so sinθ =0 and τ=0

Finally, θ =90° so sinθ =sin90°=1 i.e., τ =mB; i.e.τ=20x4x10^{-6}x1=8x10^{-5}Nm

(iv) $\Gamma = I\frac{d\omega}{dt}$ and $\Gamma = mB\sin\theta; I\frac{d\omega}{dt} = mB\sin\theta$, But $\frac{d\omega}{dt} = \frac{d\omega}{d\theta}x\frac{d\theta}{dt} = \frac{d\omega}{d\theta}\omega$ Then, Iωdω=(mB sinθ)dθ

Integrating, we get $I\int_0^{\omega}\omega d\omega = mB\int_0^{\pi/2}\sin\theta d\theta$ i.e, $\frac{I\omega^2}{2} = -mB\cos\theta\Big|_0^{90} = mB$

i.e. $\omega = \left(\frac{2mB}{I}\right)^{1/2} = \left[\frac{2x8x10^{-5}}{0.1}\right]^{1/2}$ =4 x 10^{-2} rad s^{-1}

Note:

(a) Never use Fleming left-hand rule or right hand rule while solving questions. It becomes cumbersome to remember them precisely. Instead always find the direction of force by identifying the directions of motion and the field and then take the cross-product.

(b) Also, torque can be directly calculated by formula $\vec{M}\times\vec{B}$, where M is the magnetic dipolemoment as discussed below.

11. MAGNETIC DIPOLE MOMENT

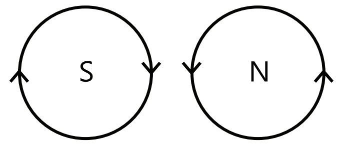

Figure 21.15: North and South Pole of current coil

Every current carrying loop behave like a magnetic dipole. It has two poles, north (N) and south (S) similar to a bar magnet. (see Fig. 21.15) Magnetic field lines are closed pathsdirected from the North Pole to the South Pole in the region outside the magnetic dipole and from South Pole to North Pole inside the magnetic dipole.

Each loop has magnetic dipole moment defined as $\vec{M} = NI\vec{A}$, where N is the number of turns in the loop, I is the current in the loop andA is the area of cross-section of the loop.

For the direction of $\vec{M}$ any one of following methods can be used:

(a) The direction of $\vec{M}$ is from South Pole to North Poles we traverse inside the magnetic dipole. For a current loopthe North and the South Pole can be identified by the sense of current. The side fromwhere the current seems to flow clockwise is the South Pole and the opposite side from where it seems to flow anticlockwise is theNorth Pole.

(b) Vector $\vec{M}$ is along the normal to the plane of the loop. The direction of $\vec{M}$ is related to the direction of current in the loop by the right hand screw rule. Curl the fingers of the right hand around the perimeter of the loop in the direction of current as shown in Fig.21.16. Then thumb extendedperpendicular to the plane of the loop, points in the direction of $\overrightarrow{M}$.

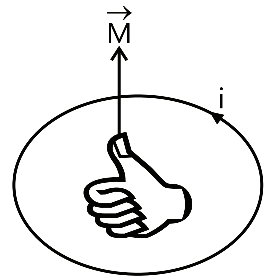

Figure 21.16: Right hand screw rule

The potential energy U of a magnetic dipole placed in a uniform magnetic field is

$$U = -MB\cos\theta$$

or $$U = -\vec{M}.\vec{B}$$

For a bar magnet we define the magnetic dipole moment as

$$\vec{M} = m\vec{\ell}$$

Here m is the pole strength of the bar magnet and vector $\vec{\ell}$ is directed from South Pole to North Pole.

The unit of magnetic dipole moment is A-m^2.

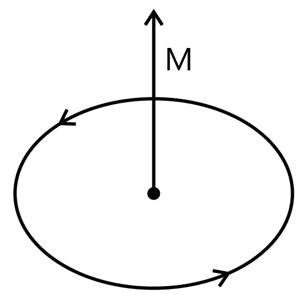

Figure 21.17: Direction of magnetic moment

The magnetic field at a large distance x on the magnetic axis of a bar magnet having magnetic dipole moment $\vec{M}$ is

$$\vec{B} = \frac{\mu_0}{4\pi}\left(\frac{2\vec{M}}{x^3}\right)$$

The magnetic field at a large distance x on the perpendicular bisector of a bar magnet having magnetic dipole moment $\vec{M}$ is

$$\vec{B} = -\frac{\mu_0}{4\pi}\left(\frac{\vec{M}}{x^3}\right)$$

Illustration 6: A square loop OABCO of side ℓ carries a current I. It is placed as shown in Fig. 21.18. Find magnetic moment of the loop. **(JEE MAIN)**

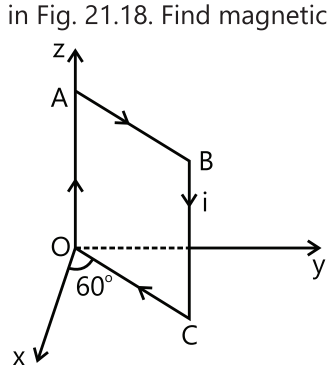

Figure 21.18

Sol: The magnetic moment of the loop is $M = IA$ for single turn. The direction of$\overrightarrow{M}$ is related to the direction of current in the loop by the right hand screw rule.

As discussed earlier, magnetic moments of the loop can be written as,

$$\vec{M} = I\left(\overrightarrow{BC} x \overrightarrow{CO}\right)$$

Here, $\overrightarrow{BC} = \ell\hat{k}$ $\qquad \overrightarrow{CO} = -\ell\cos 60^\circ\,\hat{i} - \ell\sin 60^\circ\,\hat{j} = -\frac{\ell}{2}\hat{i} - \frac{\ell\sqrt{3}}{2}\hat{j}$

$$\therefore\ \vec{M} = I\left[(-\ell\hat{k})x\left(-\frac{\ell}{2}\hat{i} - \frac{\ell\sqrt{3}}{2}\hat{j}\right)\right] \text{ or } \vec{M} = \frac{I\ell^2}{2}(\hat{j} - \sqrt{3}\hat{i})$$

Illustration 7: Find the magnitude of magnetic moment of the current carrying loop ABCDEFA. Each side of the loop is 10 cm long and current in the loop is i=20 A. **(JEE ADVANCED)**

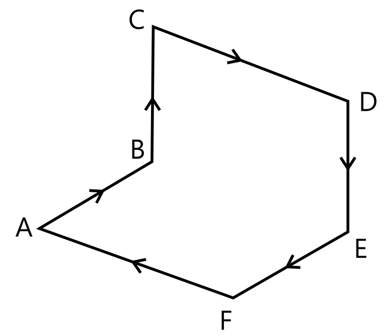

Figure 21.19

Sol: The magnetic moment of the loop is $M = IA$ for single turn. If a loop is divided into different parts, the magnetic moment of entire loop is vector sum of the magnetic moments of its individual parts.

By assuming two equal and opposite currents in BE, two current carrying loops (ABEFA and BCDEB) are formed. Their magnetic moments are equal in magnitude but perpendicular to each other. Hence,

$$M_{net} = \sqrt{M^2 + M^2} = \sqrt{2}M$$

Where M=iA-(2.0)(0.1)(0.1)=0.02 A-m^2 $\qquad M_{net} = (\sqrt{2})(0.02)\ A-m^2$

=0.028 A-m^2

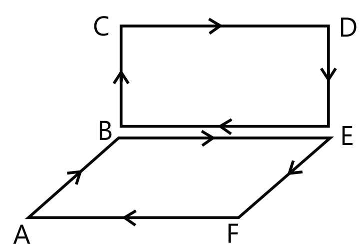

Figure 21.20

12. BIOT-SAVART LAW

Biot-Savart law is gives the strength of the magnetic field at any point due to a current element. If infinitesimalcurrent element of length $d\vec{\ell}$ carries a current I, the magnetic field or magnetic induction $d\vec{B}$ at any point P is given by Biot-Savart law as

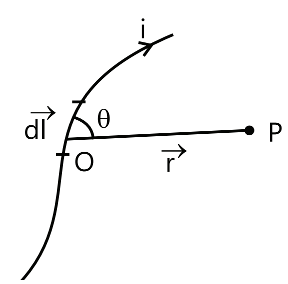

Figure 21.21 : Magnetic field due to current element dℓ

$$d\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\cdot\frac{I d\vec{\ell}\times\vec{r}}{r^3}$$

Here $\vec{r}$ is the position vector from the center of the element of length $d\vec{\ell}$ to the point of observation P. The direction of $d\vec{\ell}$ is along the direction of current I through it. If θ is the angle which $\vec{r}$ makes with the length $d\vec{\ell}$ of the conductor, the magnitude of magnetic induction is given by

$$\left|d\vec{B}\right| = \frac{\mu_o}{4\pi}\frac{I d\ell(r\sin\theta)}{r^3}$$

$$\left|d\vec{B}\right| = \frac{\mu_0}{4\pi}\frac{I d\ell(\sin\theta)}{r^2}$$

Here μ_0 is the permeability of free space and $\frac{\mu_0}{4\pi} = 10^{-7}$ Tesla-meter/ampere.

The direction of $d\vec{B}$ is perpendicular to the plane containing current element $d\vec{\ell}$ and radius vector $\vec{r}$ which joins $d\vec{\ell}$ to P.

The total magnetic induction due to the conductor is given by, $\vec{B} = \int d\vec{B}$.

The magnetic intensity H at any point in the magnetic field is related to the magnetic induction as $H = \frac{B}{\mu}$ or $B = \mu H$ where μ is permeability of the medium. The unit of magnetic intensity H is $A\text{-}m^{-1}$

Maxwell's Cork Screw Rule: If a right handed cork screw is rotated so that its tip moves in the direction of flow of current through the conductor, then the direction of rotation of the head of the screw gives the direction of magnetic field lines around the conductor.

Right Hand Rule: If we hold the conductor in the right hand such that the thumb is stretched in the direction of current, the direction in which the fingers curl gives the direction on the magnetic field.

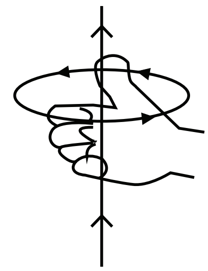

Figure 21.22: Right hand thumb rule

12.1 Application of Biot-Savart Law

Biot-Savart law is used to find the magnetic field due to current carrying conductors.

12.1.1 Magnetic Induction Due to Infinitely Long Straight Current Carrying Conductor

Suppose the current I flows through a long straightcurrent carrying conductor. We intend to find the magnetic field at point P at perpendicular distance r from the conductor. As shown in Fig. 21.23. the magnitude of field $d\vec{B}$ at P due toan infinitesimal element of length $d\ell$, is given by Biot-Savart law as:

$$\left|d\vec{B}\right| = dB = \frac{\mu_0 I d\ell \sin(90+\alpha)}{4\pi x^2}$$

where x is the distance between the current element and point P. The field $d\vec{B}$ is directed into the plane of the figure and perpendicular to it.

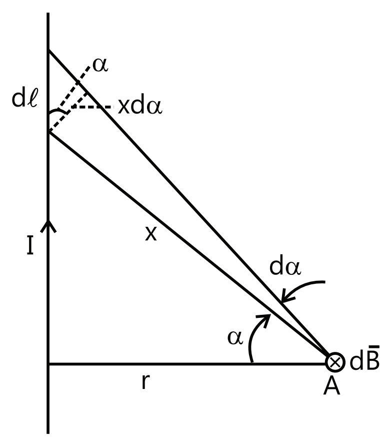

Figure 21.23: Magnetic field due to infinitely long straight wire

Now from Fig. 21.23. it is clear that, $d\ell \cos\alpha = x\, d\alpha$ and $x = \frac{r}{\cos\alpha}$, so we can write,

$$dB = \frac{\mu_0 I}{4\pi} \frac{\cos\alpha\, d\alpha}{r} \quad \text{......(i)}$$

The conductor is infinitely long,so as the angle α varies from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, the infinitesimal element covers the infinite length of the conductor, and for allinfinitesimal elements making-up the conductor the field $d\vec{B}$ is directed into the plane of the figure. Thus we can add the magnitudes of $d\vec{B}$ due to all the infinitesimal elements to get the magnitude of total field as,

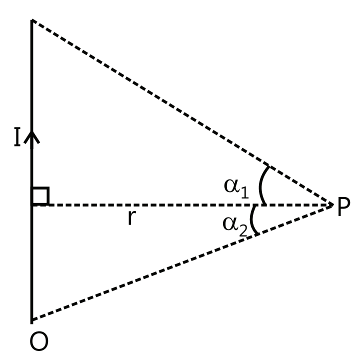

Figure 21.24: Magnetic field due to finite straight wire

$$B = \frac{\mu_o I}{4\pi r}\int_{-\pi/2}^{\pi/2}\cos\alpha\, d\alpha = \frac{\mu_o I}{2\pi r}$$

12.1.2 A Straight Conductor of Finite Length

If a conductor of finite length subtends an angle $\alpha_1$on one side and $\alpha_2$on the other side ofperpendicular from point P as shown in Fig. 21.24 then we can write,

$$B = \frac{\mu_o I}{4\pi r}\int_{-\alpha_2}^{\alpha_1}\cos\alpha\, d\alpha = \frac{\mu_o I}{4\pi r}\left|\sin\alpha\right|_{-\alpha_2}^{\alpha_1} = \frac{\mu_o I}{4\pi r}[\sin\alpha_1 + \sin\alpha_2] \quad ....(ii)$$

12.1.3 At the End of a Straight Conductor of Infinite Length

In this case, the angle α varies from 0 to $\frac{\pi}{2}$, and we can write

$$B = \frac{\mu_o I}{4\pi r}\int_{0}^{\pi/2}\cos\alpha\, d\alpha = \frac{\mu_o I}{4\pi r}$$

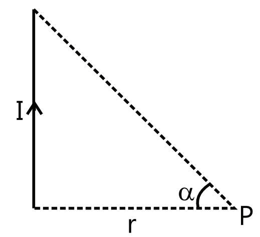

Figure 21.25: Magnetic field at end of straight wire of finite length

12.1.4 At The End of a Straight Conductor of Finite Length

In this case, (see Fig. 21.25) the angle α varies from 0 to α, and we can write

$$B = \frac{\mu_0 I}{4\pi r}\int_{0}^{\alpha}\cos\alpha\, d\alpha = \frac{\mu_o I\sin\alpha}{4\pi r}$$

12.1.5 At a Point Along the Length of the Straight Conductor Near Its End

In this case (see Fig. 21.26) $\alpha_1 = \frac{\pi}{2}$ and $\alpha_2 = -\frac{\pi}{2}$, and thus equation (ii)gives B=0. Actually in this case the value of α does not vary at all i.e. it is constant (at all points of the wire we have $\alpha = \frac{\pi}{2}$), thus $d\alpha = 0$ and thus equation (i) gives $dB = 0$.

Figure 21.26: Magnetic field along length of straight wire

Illustration 8: Calculate the magnetic field at the center of a coil in the form of a square of side 4 cm carrying a current of 5A. **(JEE MAIN)**

Sol: Square loop can be considered as four wires each of length ℓ. Magnetic field due to any one wire, at a the center is calculated as $B_1 = \frac{\mu_0}{4\pi}\frac{1}{x}\left[\sin\theta_1 + \sin\theta_2\right]$

A square coil carrying current is equivalent to four conductors of finite length.

Step 1

Magnetic field at O due to conductor BC is

$$B_1 = \frac{\mu_0}{4\pi}\frac{1}{x}\left[\sin\theta_1 + \sin\theta_2\right]$$

Here $\theta_1 = \theta_2 = 45^0$; I=5A,x=2 cm=2x10^{-2}m

$$\therefore \quad B_1 = \frac{10^{-7}x5}{2x10^{-2}}\left[\sin 45^o + \sin 45^o\right] = \frac{10^{-7}x5x\sqrt{2}}{2x10^{-2}} = 3.54x10^{-5}T$$

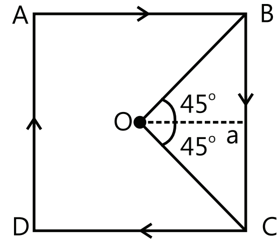

Figure 21.27

By symmetry, magnetic field intensity at O due to each arm will be same.Moreover, the direction of magnetic field at O due to each arm of the square is same

Step 2

∴ Net magnetic field at O due to current carrying square,

$B=4B_1$ $=4 \times 3.54 \times 10^{-5}$T or $B=1.42 \times 10^{-4}$T

12.1.6 Magnetic Field on the Axis of a Current Carrying Circular Arc

If a current I is flowing in a circular arc of radius R lying in the y-z plane with center at origin O and subtending an angle φ at O, then the magneticfield $d\vec{B}$ at a point Pon x-axiswith coordinates (x, 0, 0) due to a small elementary arc of length $|d\vec{\ell}| = R\,d\theta$ at a distance r from P is given by Biot-Savart Law as:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell}\times\vec{r}}{r^3} \quad \text{... (i)}$$

where $\vec{r}$ is a vector from midpoint of $d\vec{\ell}$ to P.

As shown in Fig. 21.28 the coordinates of $d\vec{\ell}$ are (0, R cosθ, R sinθ), where θ is the angle between the radius of the arc through $d\vec{\ell}$ and the y-axis.

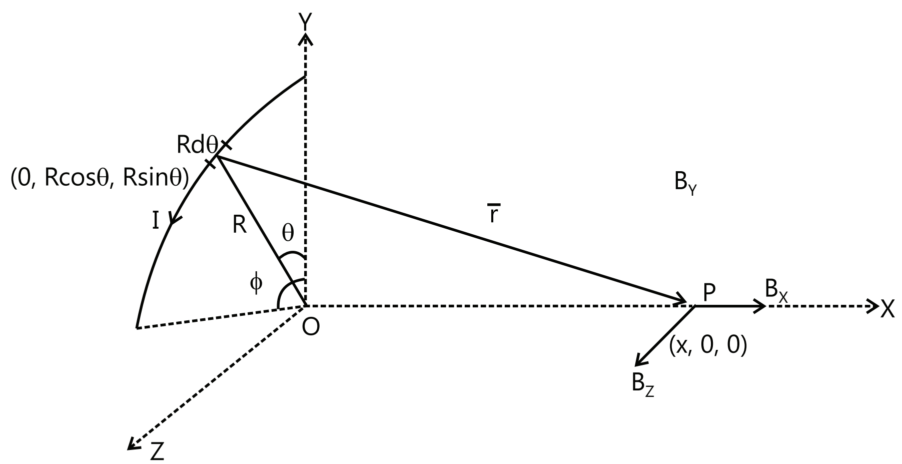

Figure 21.28: Magnetic field at a point on the axis of current carrying arc

So we can write $\vec{r} = x\hat{i} - R\cos\theta\,\hat{j} - R\sin\theta\,\hat{k}$ ... (ii)

Magnitude $r = \sqrt{x^2 + R^2}$ ... (iii)

Let us express $d\vec{\ell}$ in Cartesian coordinates system as shown in Fig. 21.29.

$d\vec{\ell} = -R\sin\theta\,d\theta\,\hat{j} + R\cos\theta\,d\theta\,\hat{k}$ ... (iv)

Put (ii), (iii) and (iv) in (i) to get

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,(-R\sin\theta\,d\theta\,\hat{j} + R\cos\theta\,d\theta\,\hat{k})\times(x\hat{i} - R\cos\theta\,\hat{j} - R\sin\theta\,\hat{k})}{(\sqrt{x^2+R^2}\,)^3}$$

$$\Rightarrow \quad d\vec{B} = \frac{\mu_0 I}{4\pi(x^2+R^2)^{3/2}}(R^2\,d\theta\,\hat{i} + x\,R\cos\theta\,d\theta\,\hat{j} + xR\sin\theta\,d\theta\,\hat{k})$$

Resultant magnetic field at P is

$$\vec{B} = \frac{\mu_0 I}{4\pi(x^2+R^2)^{3/2}}\left(R^2\int_0^{\phi} d\theta\,\hat{i} + xR\int_0^{\phi}\cos\theta\,d\theta\,\hat{j} + xR\int_0^{\phi}\sin\theta\,d\theta\,\hat{k}\right)$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{4\pi(x^2+R^2)^{3/2}}\left[R^2\phi\,\hat{i} + xR\sin\phi\,\hat{j} + xR(1-\cos\phi)\hat{k}\right]$$

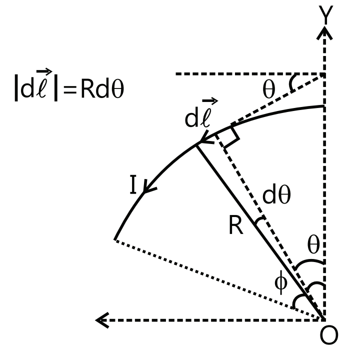

Figure 21.29: Vector is in the YZ plane

Thus $\vec{B}$ can be resolved into components parallel to the x, y and the z axes.

$$B_x = \frac{\mu_0 I R^2 \phi}{4\pi(x^2+R^2)^{3/2}}$$

$$B_y = \frac{\mu_0 I R x \sin\phi}{4\pi(x^2+R^2)^{3/2}}$$

$$B_z = \frac{\mu_0 I R x(1-\cos\phi)}{4\pi(x^2+R^2)^{3/2}}$$

The field at center of the arc: At center x = 0, so

$$B_x = \frac{\mu_0 I \phi}{4\pi R}$$

$$B_y = 0$$

$$B_z = 0$$

Thus at the center the field is normal to the plane of the arc.

For a semicircular loop, the angle subtended at the center is $\phi = \pi$, so $B = \frac{\mu_0 I}{4r}$

12.1.7 Magnetic Field on the Axis of a Current Carrying Circular Loop

The field $\vec{B}$ on the axis of a current carrying circular loop (see Fig. 21.30) can be obtained from the expression of $\vec{B}$ for a current carrying circular arc derived in the previous article by substituting the value of angle φ subtended at the center as 2π.

$$\therefore \quad \vec{B} = \frac{\mu_0 I}{4\pi(x^2+R^2)^{3/2}}\left[R^2(2\pi)\hat{i} + xR\sin 2\pi\,\hat{j} + xR(1-\cos 2\pi)\hat{k}\right]$$

$$\therefore \quad \vec{B} = \frac{\mu_0 I R^2}{2(x^2+R^2)^{3/2}}\hat{i}$$

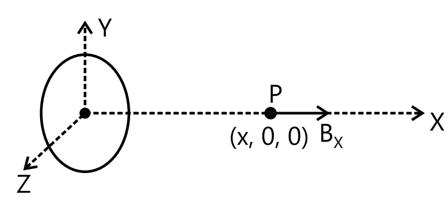

Figure 21.30: Magnetic field at a point on the axis of circular loop

Thus field $\vec{B}$ is directed along the axis of the circular loop.

For a coil havingN circular turns, $\quad B = \frac{\mu_0 N I R^2}{2(R^2+x^2)^{3/2}}$

The field at center of the coil:

At center x = 0, so $B_0 = \frac{\mu_0 N I R^2}{2R^3}$

$$\therefore \quad B_0 = \frac{\mu_0 N I}{2R}$$

The direction of B at the center of circular current carrying arc or closed circular loop can be found as follows:

If we curl the fingers of the right hand in the direction of the current in the arc/loop, then the stretched thumb points in the direction of the field at the center.

If the point P is at a very large distance from the coil,then $x^2 >> R^2$, $B = \frac{\mu_0 NIR^2}{2x^3}$

If A is area of one turn of the coil, $A = \pi R^2$ $B = \frac{\mu_0 NIA}{2\pi x^3}$

Illustration 9: A straight wire carrying a current of 12 A is bent into a semi-circular are of radius 2.0 cm as shown in Fig. 21.31.(i) What is the direction and magnitude of magnetic field (B) at the center of the arc? **(JEE ADVANCED)**

(ii) Would the answer change if wire is bent in the opposite way?

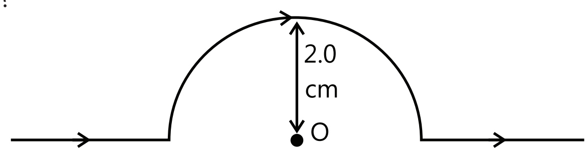

Figure 21.31

Sol: For given arrangement of wire, the magnetic field at the center due to the straight sections will be zero. The magnetic field at center will be due to the semicircular wire. Direction of field depends on direction of current and determined by right hand thumb rule.

(i) The wire is divided into three sections: (a) the straight section to be left (b) the straight section to the right and (c) circular arc.

Step 1. Magnetic field due to a current carrying element at a point is given by $dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$

In the given case, angle between $\vec{dl}$ and r for the straight section is 0°or π .So sin 0 =sin π =0

Hence magnetic field at the center (O) of the arc due to straight sections is ZERO

Step 2. Magnetic field at the center due to current carrying semi-circular section is

$$B = \frac{1}{2}x\frac{\mu_0}{4\pi}\frac{2\pi I}{r} = \frac{\mu_0}{4\pi}\frac{\pi I}{r} = \frac{10^{-7}x3.142x12}{2x10^{-12}} = 1.89x\ 10^{-4}\ T$$

The magnetic field is directed into the plane of the paper.

(ii) Direction of the field will be opposite to the found out in (i).

Illustration 10: A current path shaped as shown in Fig. 21.32 produces a magnetic field at P, the center of the arc. If the arc subtends an angle of 30° and the radius of the arc is 0.6m, what are the magnitude and direction of the field produced at P if the current is 3.0 A **(JEE ADVANCED)**

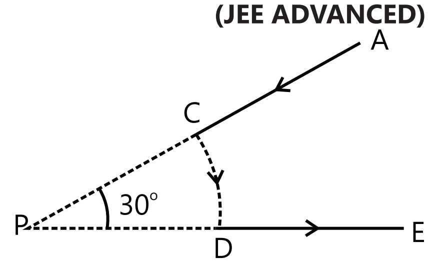

Figure 21.32

Sol: Magnetic field at the center P of arc CD is $B = \frac{\mu_0 I\phi}{4\pi R}$, and due to straight wires AC and DE is zero.

The magnetic field at P due to the straight segment AC and DE is zero, because $\vec{d\ell}$ is parallel to $\vec{r}$ along these paths, this means that $\vec{d\ell} \times \vec{r} = 0$. Each length element $d\vec{\ell}$ along path CD is at the same distance from P, hence B at P is due to segment CD which is given by

$$B = \frac{\mu_0 I}{4\pi r}\phi = \frac{\mu_0 I}{4\pi r} \times \frac{\pi}{6} = \frac{\mu_0 I}{24r}$$

13. FORCE BETWEEN PARALLEL CURRENTS

Consider two long wires kept parallel to each other such that the separation d between them is quite small as compared to their lengths. Suppose currents I_1 and I_2 flow through the wires in the same direction (see Fig. 21.33). Consider a small element $d\ell$ of the wire carrying current I_2. The magnetic field at $d\ell$ due to the wire carrying current

I_1 is $\vec{B} = \frac{\mu_0 I_1}{2\pi d}(-\hat{k})$ ...(i)

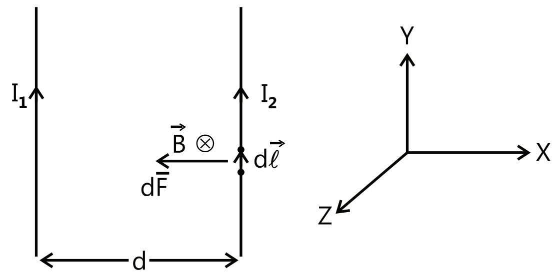

Figure 21.33: Force between parallel currents

($\vec{B}$ is normal to and directed into the plane of the figure)

The magnetic force on this element is $d\vec{F} = I_2 d\vec{\ell} \times \vec{B} = I_2 d\ell(\hat{j}) \times B(-\hat{k})$

or, $d\vec{F} = I_2 d\ell B(-\hat{i}) = \frac{\mu_0 I_1 I_2}{2\pi d} d\ell(-\hat{i})$ (directed towards the wire carrying current I_1)

Thus the wire carrying current I_2 is attracted towards the wire carrying current I_1. By Newton's third law the force acting on wire carrying current I_1 will also be attractive.Thus the two wires are attracted towards each other.

The force per unit length on each of the wires due to the other wire will be,

$$\frac{dF}{d\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Parallel currents attract each other, and antiparallel currents repel each other.

Note: Memorizing various formula of magnetic field due to ring and wire carrying current would easily help in calculating magnetic field due to complicated wire systems. Also, be careful about the direction of field in every problem you solve.

Illustration 11: A current of 10A flows through each two parallel long wires. The wires are 5 cm apart. Calculate the force acting per unit length of each wire. Use the standard values of constants required. **(JEE MAIN)**

Sol: Field of one wire exerts force on other wire and the force per unit length of wire is $\frac{F}{\ell} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d}$.

Force acting per unit length of long conductor due to another long conductor parallel to it and carrying same current.

$\frac{dF}{d\ell} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d}$; $I_1=I_2=10A$, r=5 cm $=5\times10^{-2}$m, $\frac{\mu_0}{4\pi}=10^{-7}TmA^{-1}$; $\frac{dF}{d\ell} = \frac{10^{-7}\times2\times10\times10}{5\times10^{-2}}$ $=4\times10^{-4}\ N\ m^{-1}$

Illustration 12: The wires which connect the battery of an automobile to its starting motor carry a current of 30A (for a short time).What is the force per unit length between the wires, if they are 70 cm long and 1.5 cm apart? Is the force attractive or repulsive? **(JEE ADVANCED)**

Sol: Field of one wire exerts force on other wire and the force per unit length of wire is $\frac{F}{\ell} = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{d}$.

Force depends on direction of current. Parallel currents attract while anti-parallel currents repel.

$\frac{dF}{d\ell} = \frac{\mu_0}{4\pi}\left(\frac{2I_1I_2}{d}\right)$; $I_1=I_2=300A$; r=1.5cm=1.5×10^{-2}m

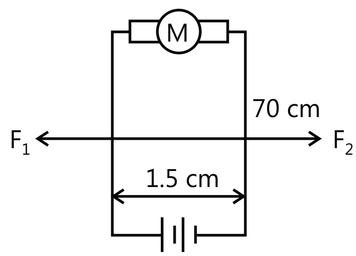

Figure 21.34

$$\therefore \quad \frac{dF}{d\ell} = \frac{10^{-7} \times 2 \times 300 \times 300}{1.5\times10^{-2}} \quad = 1.2\,\text{Nm}^{-1}$$

Since current in both the wires flows in opposite direction, so the force is repulsive.

14. AMPERE'S LAW

This law is also called the 'Theorem on Circulation of Vector B'.

According to this law the line integral or circulation of magnetic field vector $\vec{B}$ around a closed path is equal to μ_0 times the algebraic sum of the currents enclosed by the closed path.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc}$$

The closed path is also called Amperian loop.

I_{enc} is the algebraic sum of all the currents passing through the area enclosed by the closed path. Current is assumed positive if it is along the direction associated with the direction of the circumvention of the closed path through the right-hand screw rule.If we curl the fingers of the right hand around the closed path, in the direction of circumvention, the stretched thumb gives the positive direction of current. The current in the opposite direction is negative.

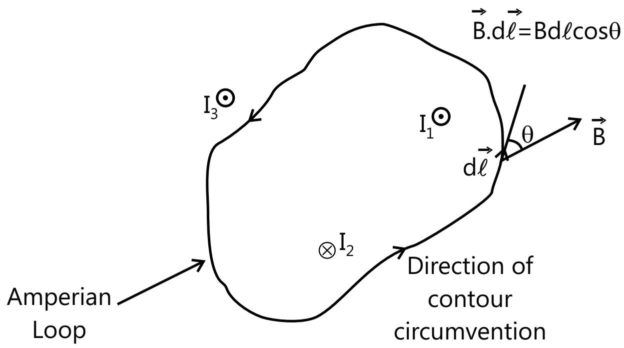

Figure 21.35: Current enclosed by amperian loop

For example in the Fig. 21.35 shown, the current directed out of the plane of the figure is positive, so we have $I_{enc} = I_1 - I_2$; $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \left(I_1 - I_2\right)$

14.1 Limitations of Ampere's Circuital Law

Ampere's law is an important tool in calculating the magnetic field due to a current distribution. However this usefulness is limited to only a few cases where the magnetic field is having a symmetrical distribution in space. The Amperian loop is chosen in such a way that the magnetic field has a constant value along the loopand is directed tangentially at all points of the loop.If such a choice of a loop is not possible, then Ampere's law cannot be used to find out the magnetic field. For example this law can't be used to find the magnetic field at the center of a current carrying loop.

Note: Ampere's circuital law holds good for a closed path of any size and shape around a current carrying conductor.

14.2 Applications of Ampere's Law

14.2.1 Magnetic field due to current carrying circular wire of infinite length

Let R be the radius of the infinite circular wire carrying current I. The magnetic field lines are concentric circles with their centers on the axis of the wire.

(a) Magnetic field intensity at a point outside the wire

We intend to find magnetic field at a distance r> R from the axis of the wire. We choose a circular path of radius r and center at the axis of the wire as the Amperian loop. $\vec{B}$ will be constant and tangential at all points of this loop. Using Ampere's law,

$$\oint \vec{B}.d\vec{\ell} = \mu_0 I \qquad \text{or} \qquad \oint B\,d\ell \cos 0^0 = \mu_0 I$$

or $\quad B\oint d\ell = B(2\pi r) = \mu_0 I$

$\therefore \quad B = \dfrac{\mu_0 I}{2\pi r}$ ...(i)

Thus, the magnetic field intensity at a point outside the wire varies inversely as the distance of the point from the axis of the wire.

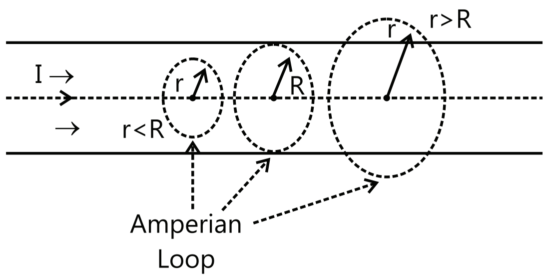

Figure 21.36: Circular cross-section of infinitely long straight wire

That is, $\quad B \propto \dfrac{1}{r}$

At the surface of the wire, r = R, so $\quad B = \dfrac{\mu_0 I}{2\pi R}$ ...(ii)

(b) Magnetic field intensity at a point inside the wire

We intend to find magnetic field at a distance r< R from the axis of the wire. We choose a circular path of radius r and center at the axis of the wire as the Amperian loop. $\vec{B}$ will be constant and tangential at all points of this loop. Using Ampere's law,

$$\oint \vec{B}.d\vec{\ell} = \mu_0 I_{enc} \qquad \text{or} \qquad \oint B\,d\ell \cos 0^0 = \mu_0 I_{enc}$$

or $\quad B\int d\ell = B(2\pi r) = \mu_0 I_{enc}$

If the current is uniformly distributed throughout the cross – section of the wire, then we have

$$I_{enc} = \frac{I}{\pi R^2}(\pi r^2) = \frac{Ir^2}{R^2}$$

$\therefore \quad B(2\pi r) = \mu_0 \dfrac{Ir^2}{R^2}$

$\therefore \quad B = \dfrac{\mu_0}{2\pi}\dfrac{Ir}{R^2}$

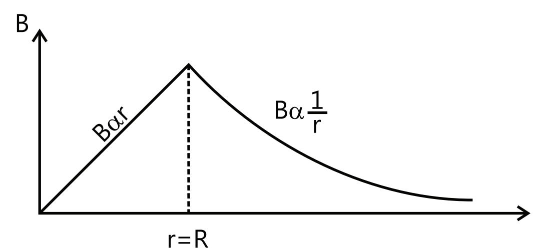

Figure 21.37: Variation of field with radial distance r

Thus, $\quad B \propto r$

The variation of B with distance r from the axis of the wire is shown in Fig.21.37.

Illustration 13: Figure 21.38 shows the cross section of a long conducting cylinder with inner radius a=2.0 cm and outer radius b=4.0 cm.The cylinder carries a current out of the page, and the magnitude of the current density in the cross section is given by $j = cr^2$, with $c=3.0 \times 10^6$ A/m^{4}and r in meters. What is the magnetic field $\vec{B}$ at appoint that is 3.0 cm from the central axis of the cylindrical? **(JEE ADVANCED)**

Sol: The magnetic field in this case is symmetric. The field lines are concentric circles. We choose a circular amperian loop coaxial with the cylinder. First find the current enclosed for region a<x<r where r = 3 cm. Then use $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$ to find $\vec{B}$.

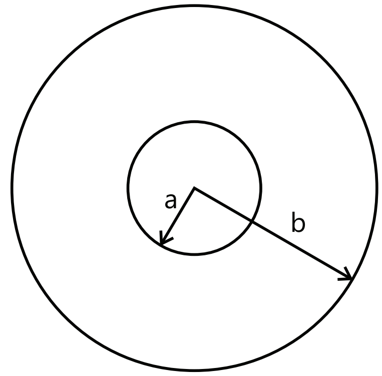

Figure 21.38

We write the integral as $i_{enc} = \int J dA = \int_a^r cr^2 (2\pi r\, dr)$

$$= 2\pi c \int_a^r r^3\, dr = 2\pi c \left[\frac{r^4}{4}\right]_a^r = \frac{\pi c(r^4 - a^4)}{2}$$

The direction of integration indicated in Fig. 21.38 is (arbitrarily) clockwise. Applying the right-hand rule for Ampere's law to that loop, we find that we should take i_{enc} as negative because the current is directed out of the page but our thumb is directed into the page.

We next evaluate the left side of Ampere's law exactly as we did in figure.

Then Ampere's law, $\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$,

Gives us $B(2\pi r) = -\frac{\mu_0 \pi c}{2}(r^4 - a^4)$

Solving for B and substituting known data yield $B = -\frac{\mu_0 \pi c}{4\pi r}(r^4 - a^4)$

$$= -\frac{\left(4\pi x 10^{-7}\ T\cdot m/A\right)\left(3.0 x 10^{6}\ A/m^4\right)}{4(0.030 m)\pi} x \left[\left(0.030\ m\right)^4 - \left(0.020 m\right)^4\right] = -2.0 \text{ x } 10^{-5}\, T$$

Thus, the magnetic field $\vec{B}$ at a point 3.0 cm from the central axis has magnitude B=2.0 x 10^{-5} T and forms magnetic field lines that are directed opposite our direction of integration, hence counterclockwise in figure.

14.2.2 Magnetic Field Inside a Solenoid

A solenoid is an insulated wire wound closely into multiple turnsto form a helix. The length of the solenoid is assumed to be much larger than its diameter. At points very close to a turn, the magnetic field lines are almost concentric circles.The fields due to adjacent turns at points near the axis add-up while fields at points away from the axis cancel each other. If the solenoid is very tightly wound and its length is quite large, then the field inside it is uniform and parallel to its axis, while field outside it will be zero.

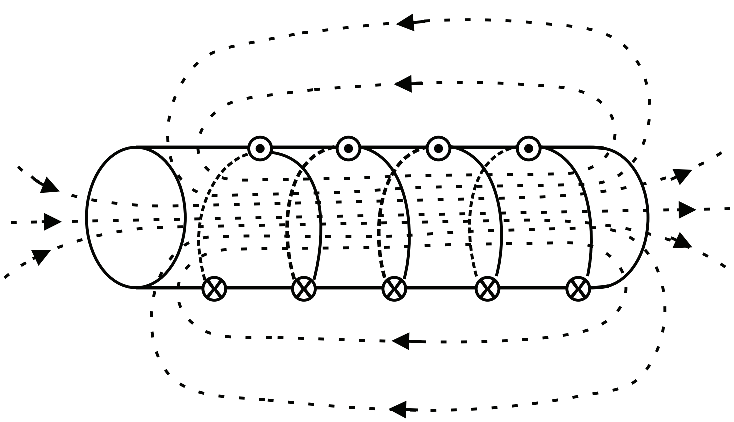

Figure 21.39: Magnetic field lines inside solenoid

We can apply Ampere's law to find the magnetic field inside the solenoid. We choose a rectangular Amperian loop abcd partly inside the solenoid and partly outside it as shown in Fig. 21.40, its length lbeing parallel to the solenoid's axis.

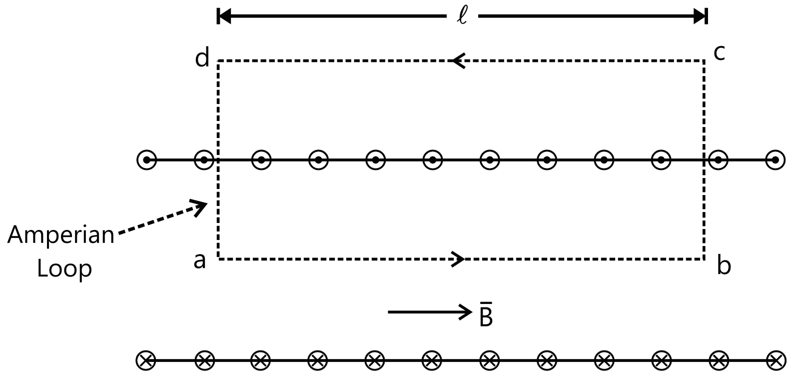

Figure 21.40: Rectangular amperian loop

There are four sides of the rectangle. We write $\oint \vec{B}.d\vec{\ell}$ as the sum of four integrals, one for each side:

$$\oint \vec{B}.d\vec{\ell} = \int_a^b \vec{B}.d\vec{\ell} + \int_b^c \vec{B}.d\vec{\ell} + \int_c^d \vec{B}.d\vec{\ell} + \int_d^a \vec{B}.d\vec{\ell}$$

The sides bc and da do not contribute to the line integral as the magnetic field is perpendicular to these sides at points inside the solenoid and at points outside the solenoid the magnetic field is zero. The side cd is completely outside the solenoid and hence the magnetic field is zero at all its points. So the only side that contributes to the line integral is ab.

Thus, we get $\oint \vec{B}.d\vec{\ell} = B\ell = \mu_0 n\ell I$

Here I is the current through each turn of the solenoid and n is the number of turns per unit length of the solenoid. The net current enclosed by the rectangle is n ℓ I.

$\therefore$ $B = \mu_0 n I$

PLANCESS CONCEPTS

(a) Magnetic field inside a solenoid and coil

(i) Magnetic field is considered uniform throughout the solenoid, while it is not true for coil

(ii) This is because, solenoid is long, while coil is thin.

(iii) Thus, magnetic field lines look very symmetric inside a solenoid, and of nearly equal length, while in a coil, the path are very different, and by Ampere's law, their magnitude is different

(b) Magnetic field on the axis at the end of a long solenoid

(i) Think of an infinite solenoid, if you could take the midpoint at the axis of this solenoid then the magnetic field strength at that point from each side would be $B = \frac{\mu_0 nI}{2}$ the situation you describe is like taking half of this infinite solenoid (as L>>d) and so $B = \frac{\mu_0 nI}{2}$

(μ_0 =permeability of free space, n= number of coils in the solenoid, I= current)

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Illustration 14:A closely wound solenoid 80 cm long has 5 layers of winding of 400 turns each. The diameter of the solenoid is 1.8 cm. if the current carries is 8.0 A, find the magnitude of B inside the solenoid near its center.

(JEE MAIN)

Sol: For solenoid of length ℓ the field at a point inside it is $B = \frac{\mu_0 NI}{\ell}$ where N is the number of turns in solenoid.

Magnetic field induction at a point inside the solenoid is

$$B = \frac{\mu_0 NI}{\ell} = \frac{4\pi 10^{-7} x(400x5)x8}{(80x10^{-2})} = 8\pi x10^{-3} T \approx 2.5\ x10^{-2}\ T$$

Illustration 15: A solenoid is 2 m long and 3 cm in diameter. Ithas 5 layers of winding of 1000 turns each and carries a current of 5A. What is the magnetic field at its center? **(JEE MAIN)**

Sol: For solenoid of length ℓ the field at a point inside it is $B = \frac{\mu_0 NI}{\ell}$ where N is the number of turns in solenoid.

Magnetic field at the center of a solenoid is given by,

$$B = \frac{\mu_0 NI}{l} = (4\pi x10^{-7})\left(\frac{5x1000}{2}\right)x5 = 1.57\ x\ 10^{-2}\ T$$

14.2.3 Magnetic field Inside a Toroid

Toroid is a circular solenoid. An insulated conducting wire is tightly wound on a ring (or torus) made ofnon-conducting material to form a toroid. The magnetic field inside a toroid can be obtained by using Ampere's law. We choose a circularAmperian loop of radius rinside the toroid concentric with it.

$$\oint \vec{B}\cdot d\vec{\ell} = \oint B\,d\ell = B\oint d\ell = B(2\pi r) = \mu_0 I_{enc}$$

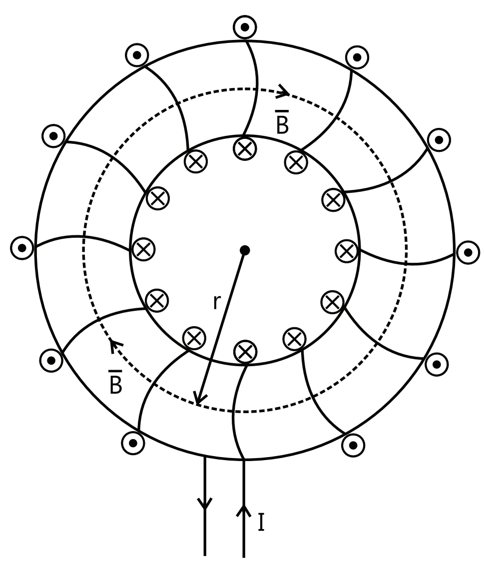


Figure 21.41: Magnetic field inside Toroid

If each turn of the toroid carries current I and the total number of turns in the toroid is N, then current enclosed by the Amperian loop is NI.

So $\quad 2\pi r\ B = \mu_0 N I$ or, $B = \frac{\mu_0 NI}{2\pi r}$

Illustration 16: A toroid of 4000 turns has outer radius of 26 cm and inner radius of 25 cm. If the current in the wire is 10A, calculate the magnetic field of the toroid also in the inner air space of the toroid. **(JEE ADVANCED)**

Sol: For toroid the field at a pointinside it at radial distance r from its center is $B=\frac{\mu_0 NI}{2\pi r}$ where N is the number of turns in toroid.

Radius of toroid $r=\frac{25+26}{2}=25.5\,\text{cm}=25.5 10^{-2}\,\text{m}$

Length of toroid $l=2\pi r=2\pi \times (25.5 \times 10^{-2} = 51 \times 10^{-2}\pi$ m

$\therefore$ Number of turns /unit length, $n=\frac{4000}{51\text{x}10^{-2}\pi}$

Field in a toroid is given by

$B=\mu_0 nI=4\pi\text{x}10^{-2}\left(\frac{4000}{51\text{x}10^{-2}\pi}\right)\text{x}10$;= 3.14×10^{2}T

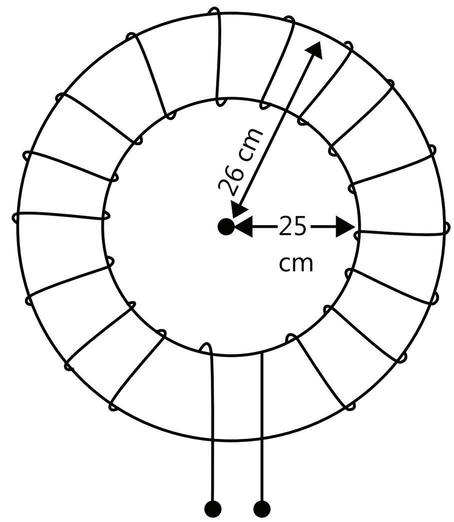


Figure 21.42

Field in the air space bounded by the toroid is zero because the field exists inside the envelope of the winding of the toroid.

15. MOVING COIL GALVANOMETER

Moving Coil Galvanometer is a device used to detect/measure small electric current flowing in an electric circuit.

Principle: When a current carrying loop or coil is placed in the uniform magnetic field, it experiences a torque and thus starts rotating.

Construction: A moving coil galvanometer is shown in Fig. 21.43. It consists of a coil made of insulated copper wire wound on a soft-iron cylinder. The coil is suspended by a spiral spring between two cylindrical shaped poles of a permanent magnet.

The spring exerts a very small restoring torque on the coil.

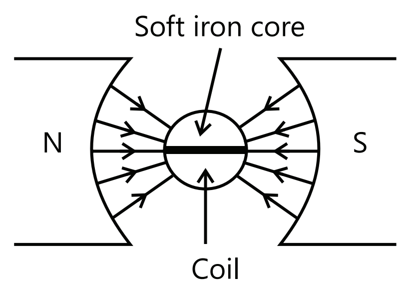


Figure 21.43: Moving coil galvanometer

Theory

Let B = Magnetic field

I = Current flowing through the coil

ℓ = Length of coil

b = Breadth of the coil

$(\ell \text{x} b)$ = A= Area of the coil

N = Number of turns in the coil

When current flows through the coil, it experiences a torque, which is given by

$\tau=NIAB\sin\theta$

where, θ is the angle between the normal to the plane of the coil and the direction of the magnetic field. Initially, $\theta=90^{o}$, so $\tau=NIAB$ ...(i)

This torque is called deflecting torque.As the coil gets deflected, the spring is twisted and a restoring torque is developed in it which is proportional to the angle of deflection ϕ

$\tau_{res}=k\phi$ ...(ii)

Here k is a constant for a particular spring.

For equilibrium of the coil,

Deflecting torque= Restoring torque

i.e. $NIAB = k\phi$ ...(iii)

or $I = \frac{k\phi}{NAB}$ ...(iv)

or $I = G\phi$ ...(v)

where $G = \frac{k}{NAB}$ is Galvanometer constant

$\therefore$ $I \propto \phi$ ...(vi)

Thus, the current flowing through the coil is directly proportional to the deflection of the coil. Hence we can determine the current in the coil by measuring its deflection.

Use of a radial magnetic field in the moving coil galvanometer

A radial magnetic field, produced by cylindrical poles of permanent magnet is always parallel to the plane of the coil of the galvanometer. Thus the angle between the normal to the coil and the magnetic field is always 90°. Thus torque on the coil is $\tau = NIAB = k\phi$ or $I \propto \phi$. Thus, when radial magnetic field is used, the current in the coil is always proportional to the deflection. Hence, a linear scale can be used to determine the currentin the coil.

Use of Galvanometer

(a) It is used to detect electric current in a circuit e.g., Wheatstone Bridge.

(b) It is convertedinto an ammeter by putting a small resistance parallel toit.

(c) It is converted into a voltmeter by putting a high resistance in series with it.

(d) It is used as an ohmmeter.

Sensitivity of a Galvanometer

A galvanometer is said to be sensitive if a small current flowing through its coil produces a large deflection in it.

(a) Current Sensitivity

The current sensitivity of a galvanometer is the deflection produced in the galvanometer per unit current flowing through it.

i.e. Current sensitivity $= \frac{\phi}{I} = \frac{NAB}{k}$

Current sensitivity of galvanometer can be increased either by

(i) Increasing the magnetic field B by using a strong permanent horse-shoe shaped magnet.

(ii) Increasing the number of turns N.

(iii) Increasing the area of the coil A. (but this will make the galvanometer bulky and ultimately less sensitive)

(iv) Using a spring having small value of restoring torque constant k.

(b) Voltage Sensitivity

Voltagesensitivity is the deflection produced in the galvanometer per unit voltage applied to it.

Voltage sensitivity $= \frac{\phi}{V} = \frac{\phi}{IR}$ i.e., voltage sensitivity $= \frac{NBA}{kR}$ (R= resistance of the coil)

Voltage sensitivity can be increased by

(i) Increasing N

(ii) Increasing B

(iii) Increasing A

(iv) Decreasing k and

(v) Decreasing R.

Advantage of a moving coil galvanometer

(a) A minutely small current in the electric circuit can be detected using an extremely sensitively galvanometer.

(b) A linear scale can be used to read the current, since deflection of the coil is directly proportional to the current.

(c) The external magnetic fields (e.g. horizontal component of earth's magnetic field) cannot effect the deflection of the coil of the galvanometer,because the magnetic field of the permanent magnet is very strong. Thus the galvanometer can be placed in any location.

(d) A dead beat type galvanometer is used.(The coil of a dead beat type galvanometer comes to rest quickly after deflecting to its equilibrium position, i.e it does not oscillate)

16. CYCLOTRON

Cyclotron is a device used to accelerate positively charged particles (like protons,α particles, deuteron, ions etc.) to acquire enough energy to carry out nuclear disintegrations.

Principle: It works on the following principle: A positively charged particle is made to accelerate through an electric field and using a strong magnetic field it is circled back to the region of the electric field, to accelerate it again and again to acquire sufficiently large amount of energy.

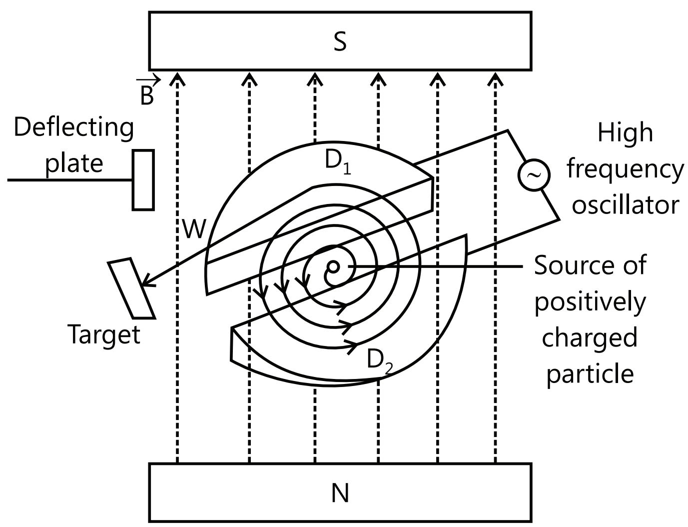


Figure 21.44: Cyclotron

Construction and Working: It consists of two hollow D-shaped metallic chambers D_1 and D_2 called dees. These dees are separated by a small gap where a source of positively charged particles is placed. Dees are connected to high frequency oscillator, which provides high frequency electric field across the gap of the dees which accelerates the particles. The magnetic field inside the dees is perpendicular to the plane of motion of particles and drives theminto a circular path. Suppose the particles start from rest and are accelerated towards chamber D_2. After completing a semicircle, when the particles reach the gap of the dees again, thereversal of the polarity of electric field ensures that the particlesareagain accelerated towards the other chamber D_1 by the electric field. Radius of the circular path increases with increase in speed, thusthe particles follow a spiral path (see Fig. 21.44)

Theory: The magnetic force on the positively charged particle provides the centripetal force to move in a circle of radius r.

$$\therefore \quad qvB = \frac{mv^2}{r} \text{ or } \quad r = \frac{mv}{qB} \qquad \text{... (i)}$$

Time taken by the particle to complete the semi-circle inside the dee,

$$t=\frac{\text{distance}}{\text{speed}}=\frac{\pi r}{v} \text{ or } t=\frac{\pi}{v}\times\frac{mv}{qB} \quad \text{or} \quad t=\frac{\pi m}{qB} \qquad \text{... (ii)}$$

This shows that time taken by the positively charged particle to complete any semi-circle (irrespective of its radius) is same

(a) Time Period: Let T be the period of the high frequency electric field, then the polarities of dees will change after time $\frac{T}{2}$.

The particle will be accelerated if time taken by it to describe the semi-circle is equal to $\frac{T}{2}$.

i.e. $$\frac{T}{2}=t=\frac{\pi m}{qB} \text{ or } T=\frac{2\pi m}{qB} \qquad \text{... (iii)}$$

(b) Cyclotron frequency: $f_c=\frac{1}{T}=\frac{qB}{2\pi m}$ ... (iv)

$\therefore$ Cyclotron angular frequency $\omega=2\pi f_c=\frac{qB}{m}$ ... (v)

(c) Energy gained: Energy gained by the positively charged particle in the cyclotron is given by $E=\frac{1}{2}mv^2$

From eqn.(i), we have $v=\frac{qBr}{m}$, then $E=\frac{1}{2}m \times \left(\frac{qBr}{m}\right)^2$ or $E=\frac{q^2B^2r^2}{2m}$ ... (vi)

Maximum energy gained by the positively charged particle will depend on the maximum value of radius of its path, i.e the radius of the dees.

$$E_{max}=\left(\frac{q^2B^2}{2m}\right)r_{max}^2 \qquad \text{... (vii)}$$

(d) Limitations of Cyclotron: Cyclotron cannot accelerate uncharged particles like neutron.

(e) Cyclotron cannot accelerate electrons because they have very small mass. Electrons start moving at a very high speed when they gain small energy in the cyclotron. The frequency of oscillating electric field required to keep them in phase with the electric field is very high, which is not feasible.

(f) The positively charged particle having large mass (i.e. ions) cannotbe accelerated after a certain speed in the cyclotron. When the speed of ion becomes comparable to the speed of light,the mass of ion increases as per the relation

$$m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}},$$

where m= mass of ion at velocity v, m_0=mass of ion at rest,cis speed of light ($3 \times 10^8\ ms^{-1}$)

Time taken by the ion to describe semi-circular path increases as mass increases.So as the mass increases, the ion does not reach the gap between the two dees exactly at the instant the polarity is reversed and,it is not be accelerated further.

Uses of a Cyclotron

(a) It is used to produce radioactive material for medical purposes.

(b) It is used to synthesize fresh substances.

(c) It is used to improve the quality of solids by adding ions.

(d) It is used to bombard the atomic nuclei with highly accelerated particles to study the nuclear reactions.

Note: Sections after this are not in the syllabus of JEE ADVANCED but they are important for understanding the concepts completely.

Illustration 17:A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons? If the radius of its dees is 60 cm. What is the kinetic energy (in MeV) of the proton beam produced by the acceleration? **(JEE MAIN)**

$\left(e = 1.60 \text{ x } 10^{-19} C, m_0 = 1.67 \text{ x } 10^{-27} \text{ kg}, \ 1\text{MeV} = 1.6 \text{ x } 10^{-13} J\right)$

Sol: The frequency of cyclotron is $f = \frac{Bq}{2\pi m}$ where q is the charge and m is the mass of the charged particle to be accelerated inside the cyclotron. The kinetic energy of the particle is $\left(\frac{mv^2}{2e}\right)$ in eV .

Cyclotron's oscillator frequency should be same as the proton's revolution frequency (in circular path)

$\therefore \quad f = \frac{Bq}{2\pi m}$ or

$B = \frac{2\pi mf}{q}$

Substituting the values in SI units, we have $B = \frac{(2)(22/7)(1.67 \text{x} 10^{-27})(10 \text{x} 10^6)}{1.6 \text{x} 10^{-19}}$ =0.67 T

The emerging beam of proton moves with the velocity

$v = \omega r = 2\pi f r = 2\times\pi\times10^7\times0.60 = 3.77\times10^7 \text{ ms}^{-1}$

Thus the kinetic energy (in MeV) is $\left(\frac{mv^2}{2e}\right) = \frac{1.67\times10^{-27}\times\left(3.77\times10^7\right)^2}{2\times1.6\times10^{-19}}$ eV = 7.42 MeV

17. MAGNETIC POLES AND BAR MAGNET

Two isolated charges of opposite signs are placed near each other, to form an electric dipole characterized by an electric dipole moment $\vec{p}$ .On the other hand in magnetism an isolated 'magnetic charge' does not exist. The simplest magnetic structure is the magnetic dipole, characterized by a magnetic dipole moment $\vec{M}$ .A current loop, a bar magnet and a solenoid of finite length are examples of magnetic dipoles.

When a magnetic dipole is placed in an external magnetic field $\vec{B}$, a torque act on it, given by $\vec{\tau} = \vec{M} \text{x} \vec{B}$

The magnetic field $\vec{B}$ due to a magnetic dipole at a point along its magnetic axis at (large) distance r from its center, is $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{r^3}$

A bar magnet has two poles (North and South) separated by a small distance. However, we cannot separate these poles apart. If a magnet is broken, the fragments prove to be dipoles and not isolated poles. If we break up a magnet into the electrons and nuclei that make up its atom, it will be found that even these elementary particle a re magnetic dipoles.

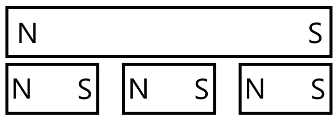


Figure 21.45: Poles of bar magnet

The poles of the bar magnet are modeled as follows:

(a) There are two types of magnetic charges; positive magnetic charge or North Pole and negative magnetic charge or South Pole. Every Pole has a strength m. The unit of Pole strength is A-m.

(b) A magnetic charge placed in a magnetic field experiences a force, $\vec{F} = m\vec{B}$. The force on positive magnetic charge is along the field and force on a negative magnetic charge is opposite to the field.

(c) A magnetic dipole is formed when a negative magnetic charge –m and a positive magnetic charge +m are placed at a small separation d. The magnetic dipole moment is, M=md. The direction of $\vec{M}$ is from –m to +m.

Geometrical Length and magnetic Length

In bar magnet, the poles are located at points which are slightly inside the two ends. The distance between the locations of the poles is called the magnetic length of the magnet. The distance between the ends is called the geometrical length of the magnet.

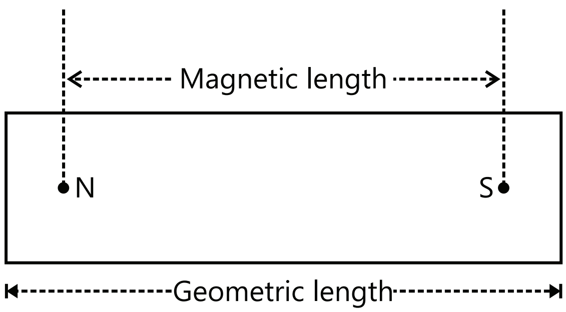


Figure 21.46: Geometric and Magnetic length of a bar magnet

Illustration18: Calculate the magnetic induction at a point 1 Å away from a proton, measured along its axis of spin. The magnetic moment of the proton is 1.4 x 10^{-26} A-m^2. **(JEE MAIN)**

Sol: On the axis of a magnetic dipole, magnetic induction is given by. $B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}$

Substituting the values, we get $B = \frac{\left(10^{-7}\right)\left(2\right)\left(1.4x10^{-26}\right)}{\left(10^{-10}\right)^3}$ = 2.8 x 10^{-3}T= 2.8 mT

18. MAGNETIC SUSCEPTIBILITY

For paramagnetic and diamagnetic materialsthe intensity of magnetization is directly proportional to the magnetic field intensity.

$$\vec{I} = \chi_m \vec{H}$$

The proportionality constant χ_m is called the magnetic susceptibility of the material. I and H have the dimensions of A-m^{-1} and the susceptibility χ_m is a dimensionless constant. For vacuum χ_m =0. For paramagnetic materials χ_m > 0, and for diamagnetic materials χ_m < 0 are diamagnetic.

19. CURIES'S LAW

When the temperature increase, due to thermal agitation the magnetization I decreases for a given magnetic intensity H,which means χ_m decreases as T increases. According to Curie's law, the susceptibility of a paramagnetic substance is inversely proportional to the absolute temperature: $\chi_m = \frac{c}{T}$ where c is a constant called the curie constant.

The magnetization of ferromagnetic material also decreases with increase in temperature, and on reaching a certain temperature,the ferromagnetic properties of the material disappear. This temperature is called Curie point (T_c). At temperatures above T_c ferromagnetic turns into a paramagnetic and its susceptibility varies with temperature as,

$$\chi_m = \frac{C'}{T - T_c}$$

where C′ is a constant.

20. PROPERTIES OF PARA-, DIA- AND FERRO-MAGNETISM

(a) Paramagnetic Substances: Example of such substances are platinum, aluminium, chromium, manganese, $CuSO_4$ solution, etc. They have the following properties:

(i) The substances, when placed in magnetic field, acquire a feeble magnetisation in the same sense as the applied field. Thus, the magnetic inductance inside the substance is slightly greater than outside to it.

(ii) In a uniform magnetic field, these substances rotate until their longest axes are parallel to the field.

(iii) These substances are attracted towards regions of stronger magnetic field when placed in a non-uniform magnetic field.

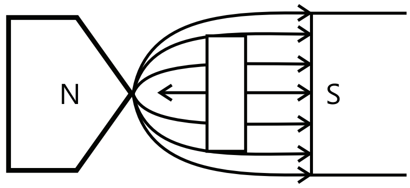


Figure 21.47: Paramagnetic material in strong magnetic field

(iv) Figure 21.47 shows a strong electromagnet in which one of the pole pieces is sharply pointed, while the other is flat. Magnetic field is much stronger near the pointed pole than near flat pole. If a small piece of paramagnetic material is suspended in this region, a force can be observed in the direction of arrow.

(v) If a paramagnetic liquid is filled in a narrow U-tube and one limb is placed in between the pole pieces of an electromagnet such that the level of the liquid is in line with the field, then the liquid will rise in the limb as the field is switched on.

(vi) For paramagnetic substances, the relative permeability μ_r is slightly greater than one.

(vii) At a given temperature the magnetic susceptibility χ_m does not change with the magnetizing field. However it varies inversely as the absolute temperature. As temperature increases χ_m decreases. At some higher temperature χ_m becomes negative and the substance become diamagnetic.

(b) Diamagnetic Substances: Examples of such substances are bismuth, antimony, gold, quartz, water, alcohol, etc. They have the following properties:

(i) These substances, when placed in a magnetic field, acquire feeble magnetization in a direction opposite to that of the applied field. Thus, the lines of induction inside the substance are smaller than those outside to it.

(ii) In a uniform field, these substances rotate until their longest axes are normal to the field.

(iii) In a non-uniform field, these substances move from stronger to weaker parts of the field.

(iv) If a diamagnetic liquid is filled in a narrow U-tube, and one limb is placed in between the pole of an electromagnet, the level depresses when the field is switched on.

(v) The relative permeability μ_r is slightly less than 1.

(vi) The susceptibility χ_m of such substances is always negative. It is constant and does not vary with field or the temperature.

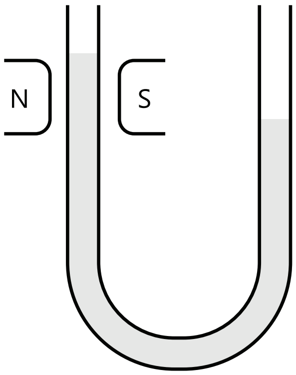


Figure 21.48: Liquid column of paramagnetic substance in strong magnetic field

(c) Ferromagnetic Substances: Examples of such substances are iron, nickel, steel, cobalt and their alloys. These substances resemble to a higher degree the paramagnetic substances with regards to their behaviour. They have the following additional properties:

(i) These substances are strongly magnetized by even a weak magnetic field.

(ii) The relative permeability is very large and is of the order of hundreds and thousands.

(iii) The susceptibility is positive and very large.

(iv) Susceptibility remains constant for very small values of $\vec{H}$, increases for larger values of $\vec{H}$ and then decreases for very large values of $\vec{H}$.

(v) Susceptibility decreases steadily with the rise of the temperature. Above a certain temperature, known as Curie temperature, the ferromagnetic substances become paramagnetic. For iron, it is 1000°C, 770°C for steel, 360°C for nickel, and 1150°C for cobalt.

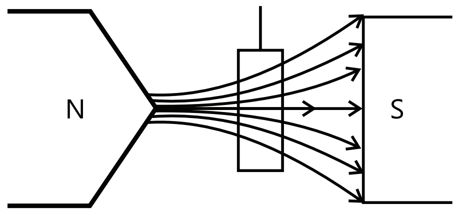


Figure 21.49: Diamagnetic substance in magnetic field

21. HYSTERESIS

Hysteresis is the dependence of the magnetic flux density B in a ferromagnetic material not only on its current magnetizing field H, but also on its history of magnetization or residual magnetization.

When a ferromagnetic material is magnetized in one direction, and then the applied magnetizing field is removed, then its magnetization will not be reduced to zero. It must be driven back to zero by a field in the opposite direction. If an alternating magnetic field intensity is applied to the material, its magnetization will trace out a loop called a hysteresis loop.

The phenomena in which magnetic flux density (B) lags behind the magnetizing field (H) in a ferromagnetic material during cycles of magnetization is called as hysteresis.

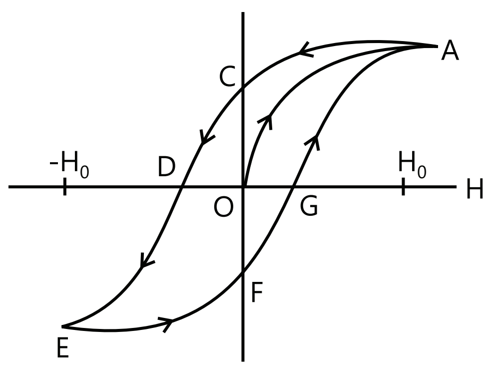


Figure 21.50: Hysteresis loop of I vs H

PROBLEM-SOLVING TACTICS

(a) General advice for this section involves learning of formulae and avoiding silly mistakes. Also it would be better to go by the usual algorithm of noting down known and unknown quantities and linking them.

(b) Much of manipulation and mathematical complexity is involved here which can't be avoided.

FORMULAE SHEET

(a) Magnetic Force on a charge moving with velocity $\vec{v}$ in magnetic field $\vec{B}$ is $\vec{F}_m = q\vec{v} \times \vec{B}$. Magnitude is $F_m = qvB\sin\theta$.

(b) Charged particle moving in uniform magnetic field

(i) Angular velocity $\omega = 2\pi f = \frac{|q|B}{m}$

(ii) Time period $T = \frac{2\pi m}{|q|B}$

(iii) Radius $r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$

(c) Helical Paths: Radius $r = \frac{mv_\perp}{qB}$ Pitch: $p = v_\perp T = v_\perp \frac{2\pi m}{|q|B}$

(d) The cyclotron $|q|B = 2\pi m f_{osc}$

(e) Crossed Fields: Lorentz Force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

(f) Trajectory of a charged particle in electric field $y = \frac{|q|Ex^2}{2mv^2}$

(g) Magnetic force on current element $d\vec{F} = Id\vec{\ell} \times \vec{B}$

(h) Magnetic force on a conductor in uniform field $\vec{F} = I\vec{L} \times \vec{B}$

(i) Magnetic dipole moment of a current coil having N turns $\vec{p}_m = NIA\hat{n}$

(j) Torque on a current coil $\vec{\tau} = \vec{p}_m \times \vec{B}$

(k) Potential energy of current coil $U = -\vec{p}_m . \vec{B}$

(l) Biot-Savart Law $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell} \times \vec{r}}{r^3}$, $dB = \frac{\mu_0}{4\pi}\frac{Id\ell\sin\theta}{r^2}$

(m) Magnetic field at center of an arc subtending angle θ, $B = \left(\frac{\mu_0}{4\pi}\right)\frac{I\theta}{R}$

(n) Magnetic field at a point on the axis of a N turn coil $B = \frac{\mu_0}{2}\frac{NIR^2}{\left(z^2 + R^2\right)^{3/2}}$

(o) Magnetic field at center of N turn coil $B = \frac{\mu_0}{2}\frac{NI}{R}$

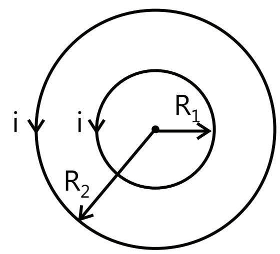


Figure 21.51

(p) Concentric coils with equal turns

(i) Similar currents flowing in the same direction

Net magnetic field, $B = \frac{\mu_0}{2}\frac{NI}{R_1} + \frac{\mu_0}{2}\frac{NI}{R_2} = \frac{\mu_0}{2}NI\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

(ii) Similar currents flowing in the opposite direction

Net magnetic field, $B = \frac{\mu_0}{2}\frac{NI}{R_1} - \frac{\mu_0}{2}\frac{NI}{R_2}$

$= \frac{\mu_0}{2} NI\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

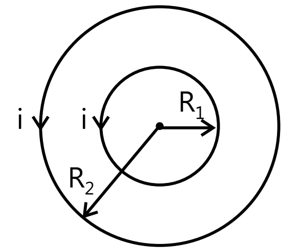


Figure 21.52

(q) Mutually perpendicular coils

Net Magnetic field, $B = \sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{2\pi I}{R}$

(r) Dispatched coils

Net Magnetic Field, $B = \sqrt{2}\frac{\mu_0}{2}\frac{IR^2}{(R^2+x^3)^{3/2}}$

$= \frac{\mu_0 IR^2}{\sqrt{2}(x^2+R^2)^{3/2}}$

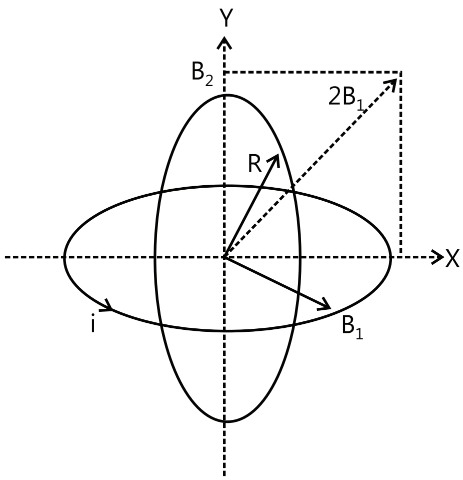


Figure 21.53

(s) Infinite straight wire $B = \frac{\mu_0 I}{2\pi R}$

(t) Semi-infinite straight wire $B = \frac{\mu_0 I}{4\pi R}$

(u) Force per unit length between two parallel currents separated by distance d, $\frac{dF}{d\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$

(v) Ampere's law $\oint \vec{B}.d\vec{\ell} = \mu_0 I_{enc}$

(w) Field inside infinite straight wire of circular cross-section $B = \frac{\mu_0 I}{2\pi R^2} r$

(x) Magnetic Field inside long solenoid having n turns per unit length $B = \mu_0 nI$

(y) Magnetic Field inside toroid having N turns $B = \frac{\mu_0 NI}{2\pi r}$

(z) Magnetic field due to bar magnet at end-on position $B = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$

(aa) Magnetic field due to bar magnet at broadside-on position $B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$

(ab) Moving Coil Galvanometer $I = \frac{k\phi}{NAB}$

(ac) Magnetic field Intensity H, in vacuum is, $H = \frac{B}{\mu_0}$

(ad) Magnetic field Intensity H, in a medium is, $H = \frac{B}{\mu_r \mu_0}$

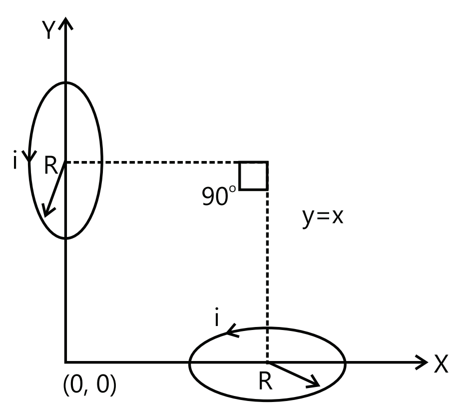


Figure 21.54

Solved Examples

JEE Main/Boards

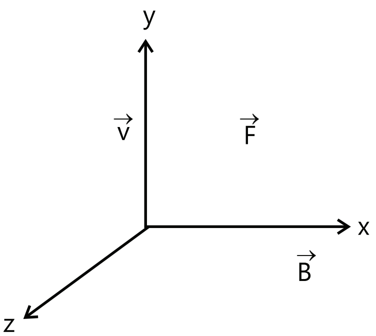
Example 1: A uniform magnetic fields of 30 mT exists in the +X direction. A particle of charge +e and mass1.67 x 10^{-27} kg is projected into the field along the +Y direction with a speed of 4.8 x 10^6 m/s

(i) Find the force on the charged particle in magnitude and direction

(ii) Find the force if the particle were negatively charged.

(iii) Describe the nature of path followed by the particle in both the cases.

Sol: The force on the particle in external magnetic field is $\vec{F} = q(\vec{v} \times \vec{B})$. Take vector product of velocity and magnetic field vector, and solve for force.



(i) Force acting on a charge particle moving in the magnetic field

$\vec{F} = q(\vec{v} \times \vec{B})$ Magnetic field $\vec{B} = 30(\text{mT})\hat{j}$

Velocity of the charge particle $\vec{V} = 4.8 \times 10^6 (m/s)\hat{j}$

$$\vec{F} = 1.6 \times 10^{-19}\left[\left(4.8 \times 10^6 \hat{j}\right) \times \left(30 \times 10^{-3}\right)\left(\hat{i}\right)\right]$$

$\vec{F}$ =230.4 x $10^{-16}(-\hat{k})$N.

(ii) If the particle were negatively charged, the magnitude of the force will be the same but the direction will be along (+z) direction.

(iii) As v $\perp$ B, the path describe is a circle

$$R = \frac{mv}{qB} = (1.67 \text{x} 10^{-27}) \cdot (4.8 \text{x} 10^6) / (1.6 \text{x} 10^{-19}) \cdot (30 \text{x} 10^{-3}) = 1.67 \text{ m.}$$

Example 2: A magnetic field of (4.0 x $10^{-3}\hat{k}$) T exerts a force (4.0 $\hat{i}$ +3.0 $\hat{j}$) x 10^{-10}N on a particle having a charge 10^{-9}C and moving in the x-y plane. Find the velocity of the particle.

Sol: The force on the particle in external magnetic field is $\vec{F} = q(\vec{v} \times \vec{B})$. Take vector product of velocity and magnetic field vector.

Given, $\vec{B} = \left(4 \times 10^{-3}\hat{k}\right)T, q = 10^{-9}C$

and Magnetic force $\vec{F}_m = (4.0\hat{i} + 3.0\hat{j})10^{-10}$ N

Let Velocity of the particle in x-y plane be, $\vec{v} = v_x\hat{i} + v_y\hat{j}$

Then From the relation, $\vec{F}m = q(\vec{v} \times \vec{B})$

We have,

$$(4.0\hat{i} + 3.0\hat{j}) \times 10^{-10} = 10^{-9}\left[\left(v_x\hat{i} + v_y\hat{j}\right) \times \left(4 \times 10^{-3}\hat{k}\right)\right]$$

$$= \left(4v_y X 10^{-12}\hat{j} - 4v_x 10^{-12}\hat{j}\right)$$

Comparing the coefficients of $\hat{i}$ and $\hat{j}$ we have,

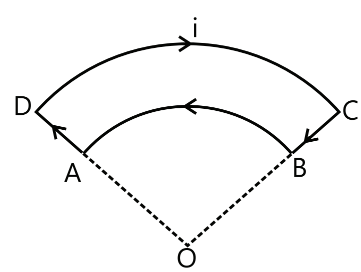
$$4 \times 10^{-10} = 4v_y \times 10^{-12}$$

$\therefore \quad v_y = 10^2 \text{ m/s} = 100 \text{ m/s}$

and $\quad 3.0 \times 10^{-10} = 4v_y \times 10^{-12}$

$\therefore v_x = -75 \text{m/s}; \therefore \vec{V} = -75\hat{i} + 100\hat{j}$

Example 3: Figure shows current loop having two circular arcs joined by two radial lines. Find the magnetic field B at the center O.



Sol: Find magnetic field at the center O of concentric arcs AB and CD by $B = \frac{\mu_0 I\theta}{4\pi R}$ where θ is the angle subtended at the center.

Magnetic field at point O, due to wires CB and AD will be zero. Magnetic field due to wire BA will be, $B_1 = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2a}\right)$ Direction of field $\vec{B}_1$ is coming out of the plane of the figure. Similarly, field at O due to arc

DC will be, $B_2 = \left(\frac{\theta}{2\pi}\right)\left(\frac{\mu_0 i}{2a}\right)$

Direction of field $\vec{B}_2$ is going into the plane of the figure. The resultant field at O is

$$B = B_1 - B_2 = \frac{\mu_0 i\theta(b-a)}{4\pi ab}$$ Coming out of theplane,

Example 4: A current of 2.00 A exist in a square loop of edge 10.0 cm. Find the magnetic field B at the center of the square loop.

Sol: The center of the loop is equidistant from all the sides, and can be considered as a point on the perpendicular bisector of one side.The field at the point due to one side is

$$B = \frac{\mu_o Ia}{2\pi d\sqrt{a^2 + 4d^2}}$$

The magnetic field at the center due to the four sides will be equal in magnitude and direction. The field due to one side will be

$$B_1 = \frac{\mu_0 ia}{2\pi d\sqrt{a^2 + 4d^2}}$$

Here, a=10 cm and d=a/2=5 cm.

$$\text{Thus, } B_1 = \frac{\mu_0(2A)}{2\pi(5\,cm)}\left[\frac{10\,cm}{\sqrt{(10\,cm)^2 + 4(5\,cm)^2}}\right]$$

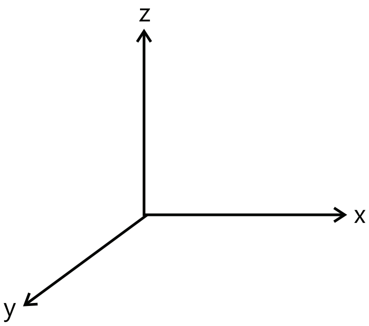
$$= 2 \times 10^{-7}\ T\,mA^{-1} \times 2\ A \times \frac{1}{5\sqrt{2}\,cm} = 5{\cdot}66 \times 10^{-6}\ T$$

Hence, the net field at the center of the loop will be $4 \times 5{\cdot}66 \times 10^{-6}T = 22{\cdot}6 \times 10^{-6}$ T.

Example 5: A particle of mass 1×10^{-26}kg and charge 1.6×10^{-19}C travelling with a velocity $1.28 \times 10^6\ ms^{-1}$ in the +x direction enters a region in which uniform magnetic field of induction B are present such that $E_x=E_y=0$, $E_z = -102.4\ kVm^{-1}$ and $B_x = B_z=0$. $B_y=8 \times 10^{-2}$. The particle enters this region at the origin at time t = 0. Determine the location (x, y and z coordinates) of the particle at t= 5×10^{-6}s. If the electric field is switched off at this instant (with the magnetic field still present), what will be the position of the particle at t = 7.45×10^{-6} s?

Sol: In presence of simultaneous electric and magnetic field, the Lorentz force is $\vec{F} = q\left(\vec{E} + (\vec{v}\times\vec{B})\right)$. Under action of uniform magnetic field only, the particle performs uniform circular motion of radius $r = \frac{mv}{qB}$.

Let $\hat{i}$, $\hat{j}$ and $\hat{k}$ be unit vector along the positive directions of x, y and z axes. Q=charge on the particle=1.6×10^{-19}C, v=velocity of the charged particle



$=(1.28 \times 10^6)ms^{-1}$

$\vec{E}$ = electric field intensity;

$=(-102.4 \times 10^3 Vm^{-1})\hat{k}$ $\vec{B}$ =magnetic induction of the magnetic field = $(8 \times 10^{-2}\ Wbm^{-2})\hat{j}$

$\therefore$ $\vec{F}_e$ =electric force on the charge

$=qE=1.6 \times 10^{-19}(-102.4\times10^3)N\hat{k} = 163.84 \times 10^{-16}N(-\hat{k})$

F_m =magnetic force on the charge = qv x B

$= [1.6\times10^{-19}(1.28\times10^6)(8\times10^{-2})N](\hat{i} \times \hat{j}) = (163.84\times10^{-16}N)(\hat{k})$

The two forces $\vec{F}_e$ and $\vec{F}_m$ are along z-axis and equal, opposite and collinear. The net force on the charge is zero and hence the particle does not get deflection and continues to travel along x-axis. (a) At time $t=5\times10^{-6}$s

$x=(5 \times 10^{-6})(1.28 \times 10^6)=6.4m$$\therefore$ Coordinates of the particle = (6.4 m,0,0)

(b) When the electric field is switched off, the particle is in the uniform magnetic field perpendicular to its velocity only and has a uniform circular motion in the x-z plane (i.e. the plane of velocity and magnetic force), anticlockwise as seen along+ y axis.

Now, $\frac{mv^2}{r}$ =qvB where r is the radius of the circle.

$$\therefore r = \frac{mv}{qB} = \frac{\left(1\times10^{-26}\right)\left(1.28\times10^6\right)}{\left(1.6\times10^{-19}\right)\left(8\times10^{-2}\right)} = 1$$

The length of the arc traced by the particle in [(7.5-5) x 10^{-6}s]

$= (v)(T)=(1.28 \times 10^60)(2.45 \times 10^{-6})=3.136m= \pi\,m= \frac{1}{2}$ circumference

$\therefore$ The particle has the coordinates (6,4,0,2m) as (x,y,z).

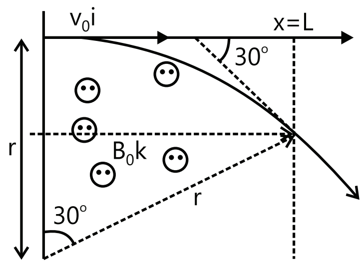
Example 6: The region between x=0 and x=L is filled with uniform, steady magnetic field B_0k. A particle of mass m, positive charge q and velocity $v_0\hat{i}$ travels along x-axis and enters the region of magnetic field. Neglect gravity throughout the question.

(i) Find the value of L if it emerges from the region of magnetic field with its final velocity at an angle 30° to the initial velocity.

(i) Find the final velocity of the particle and the time spent by it in the magnetic field, if the field now extents up to x=2. 1L.

Sol: The particle under action of uniform magnetic field performs uniform circular motion. The magnetic force acting on it provides the centripetal force. The radius of the circular orbit is $r = \dfrac{mv}{qB}$.

(i) As the initial velocity of the particle is perpendicular to the field the particle will move along the arc of a circle as shown.

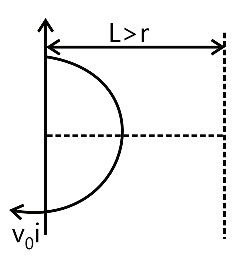


If r is the radius of the circle, then

$\dfrac{mv_0^2}{r} = qv_0B_0$ Also from geometry, L=r sin 30°

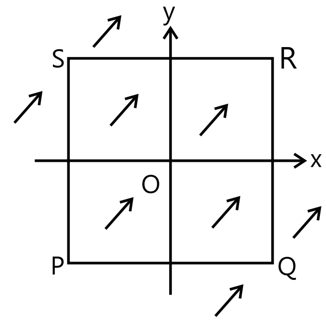
$\Rightarrow r = 2L$ or $L = \dfrac{mv_0}{2qB_0}$

(ii) In this case $L = \dfrac{2.1mv_0}{2qB_0} > r$ Hence the particle will complete a semi-circular path and emerge from the field with velocity $v_0\hat{i}$ as shown. Time spent by the particle in the magnetic field $T = \dfrac{\pi r}{v_0} = \dfrac{\pi m}{qB_0}$



The speed of the particle does not change due to the magnetic field.

Example 7: A uniform, constantmagnetic field $\vec{B}$ is directed at an angle of 45° to the x-axis in the xy-plane. PQRS is a rigid, square wire frame carrying a steady current I_0, with its center at the origin. O. At time t=0, the frame is at rest in the position (shown the Figure) with its sides parallel to the x and y axes. Each side of the frame is of mass M and length L.

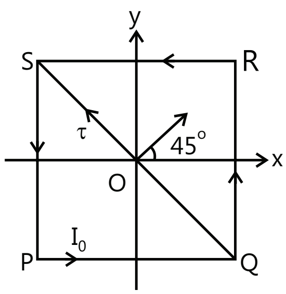


(a) What is the torque τ about O acting on the frame due to the magnetic field?

(b) Find the angle by which the frame rotates under the action of this torque in a short interval of time Δt, and the axis about which this rotation occurs. (Δt is so short that any variation i n the torque during this interval may be neglected). Given moment of any variation in the torque during this interval may be neglected). Given moment of inertia of the frame about an axis through its center perpendicular to its p late is (4/3) ML^2.

Sol: The torque acting on loop is $\vec{\tau} = \vec{M} \times \vec{B}$.

$\alpha = \dfrac{d\omega}{dt} = \dfrac{\tau}{I}$ and



$\theta = \int \omega dt$ (a) As magnetic field B is in x-y plane and subtends an angle of 45° with x-axis.

$B_x = B\cos 45° = B/\sqrt{2}$

and $B_y = B\sin 45° = B/\sqrt{2}$

So in vector from

$\vec{B} = \hat{i}\left(B/\sqrt{2}\right) + j\left(B/\sqrt{2}\right)$

and $\vec{M} = I_0S\hat{k} = I_0L^2\hat{k}$

so, $\hat{\tau} = \vec{M} \times \vec{B} = I_0 L^2 \hat{k} \times \left(\frac{B}{\sqrt{2}} \hat{i} + \frac{B}{\sqrt{2}} \hat{j} \right)$

i.e., $\hat{\tau} = \frac{I_0 L^2 B}{\sqrt{2}} \times \left(-\hat{i} + \hat{j} \right)$

i.e., torque has magnitude $I_0 L^2 B$ and is directed along line QS from Q to S.

(b) As by theorem of perpendicular axes, moment of inertia of the frame about QS,

$$I_{QS} = \frac{1}{2} I_z = \frac{1}{2}\left(\frac{4}{3} ML^2\right) = \frac{2}{3} ML^2$$

And as $\tau = I\alpha$,

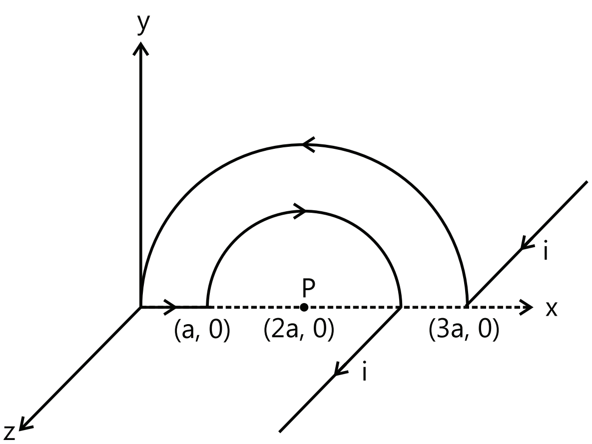
$$\alpha = \frac{\tau}{1} = \frac{I_0 L^2 B \times 3}{2L^2 M} = \frac{3}{2}\frac{I_0 B}{M}$$

As here α is constant, equations of circular motion are valid and hence from

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ with $\omega_0 = 0$ we have

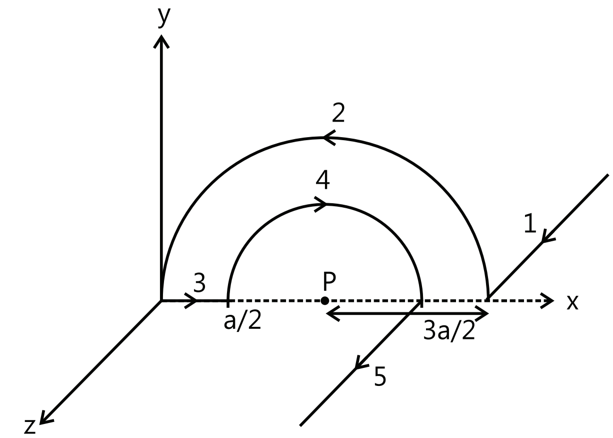
$$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(\frac{3}{2}\frac{I_0 B}{M}\right)(\Delta t)^2 = \frac{3}{4}\frac{I_0 B}{M}\Delta t^2$$

Example 8: In the figure shown the magnetic field at the point P.



Sol: The conductor forms two concentric semicircles and two straight wires. Find magnetic field at the center P due to concentric arcs by formula $B = \frac{\mu_0 I\theta}{4\pi R}$, and fields due to straight wires by formula $B = \frac{\mu_0 I}{4\pi d}$ and then add the fields due to individual parts.

Consider the figure.



$$\vec{B}_P = \left(\vec{B}_1\right)_P + \left(\vec{B}_2\right)_P + \left(\vec{B}_3\right)_P + \left(\vec{B}_4\right)_P + \left(\vec{B}_5\right)_P$$

where $\left(\vec{B}_1\right)_P = \frac{\mu_0 i}{4\pi\left(\frac{3a}{2}\right)}\left(-\hat{j}\right)$

(Semi-infinite wire) $\left(\vec{B}_2\right)_P = \frac{\mu_0 i}{4\left(\frac{3a}{2}\right)}\left(+\hat{k}\right)$ $\left(\vec{B}_3\right)_P = 0$;

$$\left(\vec{B}_4\right)_P = \frac{\mu_0 i}{4\left(\frac{a}{2}\right)}\left(-\hat{k}\right)$$

$$\Rightarrow \vec{B}_P = \frac{\mu_0 i}{2a}\left[-\left(\frac{1}{3\pi} + \frac{1}{\pi}\right)\hat{j} - \left(1 - \frac{1}{3}\right)\hat{k}\right]$$

$$\Rightarrow \vec{B}_P = \frac{2\mu_0 i}{3a}\left[\frac{1}{\pi}\hat{j} - \hat{k}\right] \qquad \Rightarrow \vec{B}_P = \frac{\mu_0 i}{3\pi a}\sqrt{1+\pi^2}$$

Example 9: What is the smallest value of B that can be set up at the equator to permit a portion of speed 10^7 m/s to circulate around the earth?

$\left[R = 6.4 \times 10^6 m, m_p = 1.67 \times 10^{-27} kg\right]$.

Sol: Particle under action of force in uniform magnetic field, moves in circular orbit whose radius is given by $r = \frac{mv}{Bq}$. For charged particle orbiting near earth with high velocity, the magnetic field can be obtained rearranging above formula.

From the relation $r = \frac{mv}{Bq}$

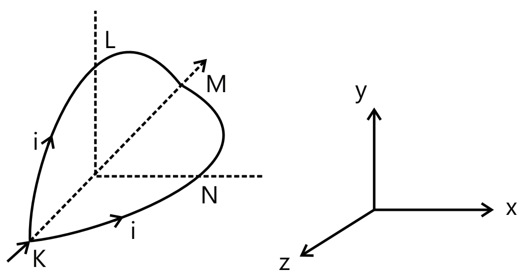
We have $B = \frac{mv}{qr}$

Substituting the values,we have

$$B = \frac{\left(1.67\times10^{-27}\right)\left(10^{7}\right)}{\left(1.6\times10^{-19}\right)\left(6.4\times10^{6}\right)} = 1.6\times10^{8}\,\text{T}$$

JEE Advanced/Boards

Example 1: A circular loop of radius R is bent along a diameter and given a shape as shown in figure. One of the semi-circle (KNM) lies in the x-z plane and the other one (KLM) in the y-z plane with their centers at origin. Current I is flowing through each of the semi-circles as shown in Figure.



A particle of charge q is released at the origin with a velocity $V = -V_0\hat{i}$. Find the instantaneous force F in the particle. Assume that space is gravity free.

Sol: For wire bent as shown the magnetic field at the center is calculated as $B = \frac{\mu_0 I(\pi)}{4\pi R}$, where π is the angle subtended by the wire at center. The Lorentz force acting on particle is $\vec{F} = q(\vec{v}\times\vec{B})$

Magnetic field at the center of a circular wire of radius R carrying a current I is given by $B = \frac{\mu_0 I}{2R}$

In this problem, current are flowing in two semi-circles, KLM in the y-z plane and KNM in the x-z plane. The centers of these semi-circles coincide with the origin of the Cartesian system of axes.

$$\therefore \vec{B}_{KLM} = \frac{1}{2}\left(\frac{\mu_0 I}{2R}\right)\left(-\hat{i}\right) \quad \therefore \vec{B}_{KNM} = \frac{1}{2}\left(\frac{\mu_0 I}{2R}\right)\left(-\hat{j}\right)$$

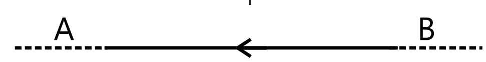
The total magnetic field at the origin is $B_0 = \frac{\mu_0 I}{4R}\left(-\hat{i}+\hat{j}\right)$

It is given that a particle of charge q is released at the origin with a velocity $V = -V_0\hat{i}$. The instantaneous force acting on this particle is given by

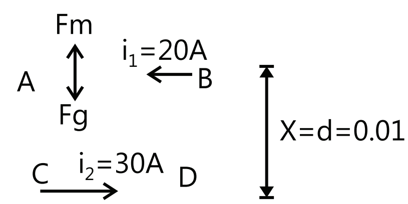
$$f = q\left[V\times B\right] \quad = q\left(-V_0\hat{i}\right)\times\left[\frac{\mu_0 I}{4R}\left(-\hat{i}+\hat{j}\right)\right]$$

$$= \left(\frac{qV_0\mu_0 I}{4R}\right)\left[\left(-\hat{i}\right)\times\left(-\hat{i}+\hat{j}\right)\right] = \frac{qV_0\mu_0 I}{4R}\left(-\hat{k}\right)$$

Example 2: A long horizontal wire AB, which is free to move in a vertical plane and carries a steady current of 20 A, is in equilibrium at a height of 0.01 m over another parallel long wire CD which is fixed in a horizontal plane and carries a steady current of 30 A, as shown in figure Show that when AB is slightlydeed it executes simple harmonic motion. Find the period of oscillation.



Sol: The current carrying wire AB, experiences force due to the magnetic field created by wire CD. Find the equation of motion of wire AB. If the force acting on wire AB is restoring in nature and directly proportional to its displacement from the equilibrium position, then we compare the equation of acceleration with the standard differential equation of SHM. Then time period of oscillation is given by $T = 2\pi\sqrt{\frac{\omega}{g}}$ Let m be the mass per unit length of wire AB. At a height x about the wire AB will be given by



$$F_m = \frac{\mu_0 i_1 i_2}{2\pi x}\ \text{(upwards)} \qquad \text{... (i)}$$

Wt. per unit of wire AB is F_g =mg (downwards) At x=d, wire in equilibrium

$$\text{i.e., } F_m = F_g \Rightarrow \frac{\mu_0}{2\pi}\frac{i_1 i_2}{d} = mg$$

$$\Rightarrow \frac{\mu_0 i_1 i_2}{2\pi d^2} = \frac{mg}{d} \qquad \text{...(ii)}$$

When AB is deed, x decreases therefore, F_m will increase, F_g remains the same. Let

AB is displaced by dx downwards.

Differentiating equation (i) w.r. t.x, we get

$$dF_m = -\frac{\mu_0}{2\pi}\frac{i_1 i_2}{x^2}dx \qquad \text{...(iii)}$$

i.e., restoring force, F=d $F_m \propto -dx$

Hence the motion of wire is simple harmonic. From equation (ii) and (iii), we can write

$$dF_m = -\left(\frac{mg}{d}\right).dx \qquad (x=d)$$

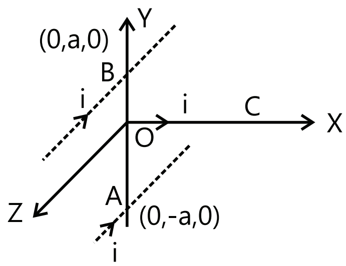
$$\therefore \text{Acceleration of wire, } a = -\left(\frac{g}{d}\right).dx$$

Hence period of oscillations

$$T = 2\pi\sqrt{\frac{dx}{a}} = 2\pi\sqrt{\left|\frac{\text{disp.}}{\text{acc.}}\right|}$$

$$\Rightarrow T = 2\pi\sqrt{d/g} = 2\pi\sqrt{\frac{0.01}{9.8}} \quad \Rightarrow T = 0.2s$$

Example 3: A straight segment OC (of length L meter) of a circuit carrying a current 1 amp is placed along the x-axis. Two infinitely long straight wire A and B, each extending $z = -\infty \text{ to} +\infty$ are fixed at y=-a meter and y=+a meter respectively, as shown in the figure. If the wires A and B each carry a current 1 amp into the plane of the paper, obtain the expression for the force acting on segment OC. What will be the force on OC if the current in the wire B is reversed?



Sol: Find the net field due to wires A and B at any point on the wire OC.Find the force due to this field on a small current element of wire OC at that point. Then integrate this expression to find force on wire OC.

Magnetic field B_A produced at P(x,0, 0) due to wire, $B_A = \mu_0 I/2\pi R$, $B_B = \mu_0 I/2\pi R$.

Components of B_A and B_B along x-axis cancel, while those along y-axis add up to give total field.

$$B = 2\left(\frac{\mu_0 I}{2\pi R}\right)\cos\theta = \frac{2\mu_0 I}{2\pi R}\frac{x}{R} = \frac{\mu_0 I}{\pi}\frac{x}{\left(a^2+x^2\right)}$$

(along – y direction)

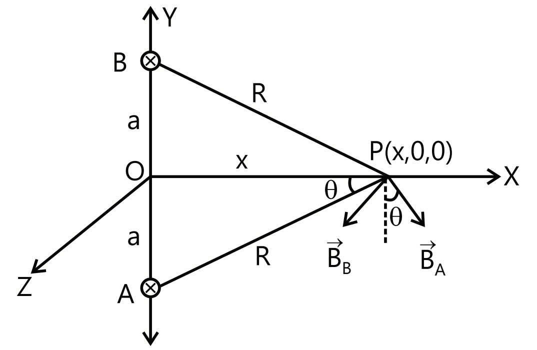
The force dF acting on the current element is

$$dF = I(d\ell \times B)$$

$$dF = \frac{\mu_0 I^2}{\pi}\frac{x\,dx}{a^2+x^2} \quad \left[\therefore \sin 90^\circ = 1\right]$$

$$\Rightarrow F = \frac{\mu_0 I^2}{\pi}\int_0^L \frac{x\,dx}{a^2+x^2} = \frac{\mu_0 I^2}{2\pi}\ln\frac{a^2+L^2}{a^2}$$

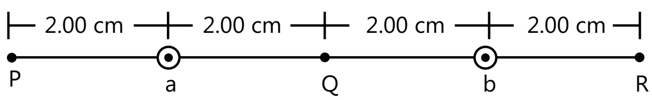
If the current in B is reversed, the magnetic field due to the two wires would be only along



x- direction and the force on the current along

x- direction will be zero.

Example 4: Two long wires a and b, carrying equal currents of 10.0 A, are placed parallel to each other with a separation of 4.00 cm between them as shown in figure. Find the magnetic field B at each of the points P, Q and R.



Sol: Net field at a point will be the vector sum of the fields due to the two wires.

The magnetic field at P due to the wire a has magnitude

$$B_1 = \frac{\mu_0 i}{2\pi d} = \frac{4\pi\times10^{-7}\,\text{TmA}^{-1}\times10\text{A}}{2\pi\times2\times10^{-2}\text{m}} = 1.00\times10^{-4}\,\text{T}.$$

Its direction will be perpendicular to the line shown and will point downward in the figure. The field at this point due to the other wire has magnitude

$$B_2 = \frac{\mu_0 i}{2\pi d} = \frac{4\pi\times10^{-7}\,\text{TmA}^{-1}\times10\text{A}}{2\pi\times6\times10^{-2}\text{m}} = 0.33\times10^{-4}\,\text{T}.$$

Its direction will be the same as that of B_1. Thus, the resultant field will be $1.33\times10^{-4}\,\text{T}$ also along the same direction.

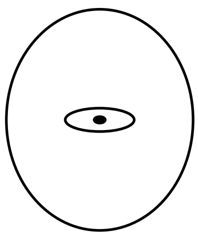
Similarly, the resultant magnetic field at R will be $= 1.33\times10^{-4}\,\text{T}$ along the direction pointing upward in the figure.

The magnetic field at point Q due to the two wires will have equal magnitudes but opposite directions and hence the resultant field will be zero.

Example 5: A coil of radius R carries current I. Another concentric coil of radius (r<<R) carries current i. Planes of two coils are mutually perpendicular and both the coils are free to rotate about a common diameter. Find maximum kinetic energy of smaller coil when both the coils are released, masses of coils are M and m respectively.

Sol: For rotating coils, kinetic energy is $\frac{1}{2}I\,\omega^2$.

Each coil is a magnetic dipole and has a potential energy in magnetic field due to other coil. This potential energy is converted into kinetic energy as the dipole moment of the coil aligns itself with the magnetic field.



If a magnetic dipole having moment M be rotated through angle θ from equilibrium position in a uniform magnetic field B, work done on it is $W = MB(1-\cos\theta)$. This work is stored in the system in the form of energy. When system is release, dipole starts to rotate to occupy equilibrium position and the energy converts into kinetic energy and kinetic energy of the system is maximum when stored energy is completely released.

Magnetic induction, at centers due to current in larger coil $B = \frac{\mu_0 i}{2R}$ Magnetic dipole moment of smaller coil is $i\pi r^2$. Initially planes of two coils are mutually perpendicular, therefore θ is $90°$ or energy of the system is $U = \left(i\pi r^2\right)B\left(1-\cos 90°\right)$

$$U = \frac{\mu_0 I i \pi r^2}{2R}$$

When coils are released, both the coils start to rotate about their common diameter and their kinetic energies are maximum when they become coplanar.

Moment of inertia of larger coil about axis of rotation is $I_1 = \frac{1}{2}mR^2$ and that of smaller coil is $I_2 = \frac{1}{2}mr^2$.

Since, two coils rotate due to their mutual interaction only, therefore, if one coil rotates clockwise then the other rotates anticlockwise.

Let angular velocities of larger and smaller coils be numerically equal to ω_1 and ω_2 respectively when they become coplanar,

According to law of conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$

and according to law of conservation of energy,

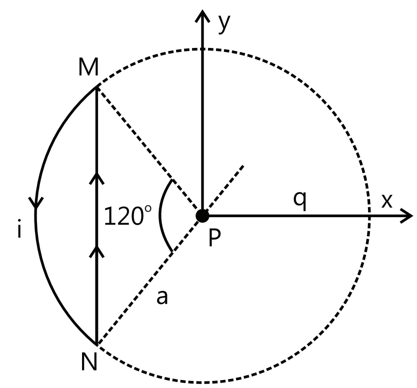
$$\frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 = U$$

From above equations, maximum kinetic energy of smaller coil,

$$\frac{1}{2}I_2\omega_2^2 = \frac{UI_1}{I_1+I_2} = \frac{\mu_0 \pi I i M r^2}{2\left(MR^2+mr^2\right)}$$

Example 6: A wire loop carrying a current I is placed in the x-y plane as shown in Figure.

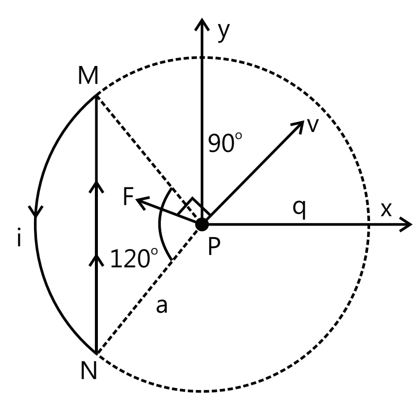
(a) If a particle with charge q and mass m is placed at the centerP and given a velocity v along NP find its instantaneous acceleration.



(b) If an external uniform magnetic induction $B = B\hat{i}$ is applied, find the force and torque acting on the loop.

Sol: Find the net magnetic field at the point P due to the arc and the straight wire and find the magnetic force on q by rules of vector cross product. The magnetic force on a current loop in uniform magnetic field is zero. The toque will be non-zero depending on the angle between field and the area vector of the loop.

(a) As in case of current-carrying straight conductor and arc, the magnitude of B is given by



$B_1 = \frac{\mu_0 i}{4\pi d}(\sin\alpha + \sin\beta)$ and $B_2 = \frac{\mu_0 I\phi}{4\pi r}$

So in accordance with right hand screw rule,

$\left(\vec{B}_W\right) = \frac{\mu_0}{4\pi}\frac{1}{(a\cos 60)} \times 2\sin 60(-\hat{k})$ and due to are

$$\left(\vec{B}\right)_{MN} = \frac{\mu_0}{4\pi}\frac{I}{a} \times \left(\frac{2}{3}\pi\right)\left(+\hat{k}\right)$$

and hence net $\vec{B}$ at P due to the givenloop

$$\vec{B} = \vec{B}_W + \vec{B}_A$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi}\frac{2I}{a}\left[\sqrt{3} - \frac{\pi}{3}\right](-\hat{k}) \qquad \text{...(i)}$$

Now as force on charged particle in a magnetic fields is given by

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$

So here, $\vec{F} = qvB\sin 90°$ along PF

i.e. $\vec{F} = \frac{\mu_0}{4\pi}\frac{2qvI}{a}\left[\sqrt{3} - \frac{\pi}{3}\right]$ along PF

and so $\vec{a} = \frac{\vec{F}}{m} = 10^{-7}\frac{2qvI}{a}\left[\sqrt{3} - \frac{\pi}{3}\right]$ along PF

(b) As $d\vec{F} = Id\vec{L} \times \vec{B}$, so $\vec{F} = \int Id\vec{L} \times \vec{B}$

As here I and $\vec{B}$ are constant

$$F = I\left[\int dL\right] \times B = 0\left[\text{as} \int dL = 0\right]$$

Further as area of coil,

$$\vec{S} = \left[\frac{1}{3}\pi a^2 - \frac{1}{2}.2a\sin 60° \times a\cos 60°\right]\hat{k}$$

$$= a^2\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\hat{k}$$

So $\vec{M} = I\vec{S} = Ia^2\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\hat{k}$

and hence $\vec{\tau} = \vec{M} \times \vec{B} = Ia^2B\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\left(\hat{k} \times \hat{i}\right)$

i.e. $\vec{\tau} = Ia^2B\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\hat{j}N - m$ as $\left(\hat{k} \times \hat{i} = \hat{j}\right)$.

Example 7: A disc of radius R rotates at an angular velocity ω about the axis perpendicular to its surface and passing through its center. If the disc has a uniform charge density σ, find the magnetic induction on the axis of rotation at a

Sol: The disc can be thought as made-up of elementary rings. When disc rotates about axis passing through center and perpendicular to plane of disc, then each elementary ring constitutes a current. The magnetic field along axis of rotation due to each elementary ring is to be considered.

At distance r from the center of disc consider a ring of radius r and width dr.

Charge on the ring, $dq = (2\pi r\,dr)\sigma$

Current due to ring is $dI = \frac{dq}{T} = \frac{\omega dq}{2\pi} = \sigma\omega r\,dr$

Magnetic field due to ring at point P on axis is

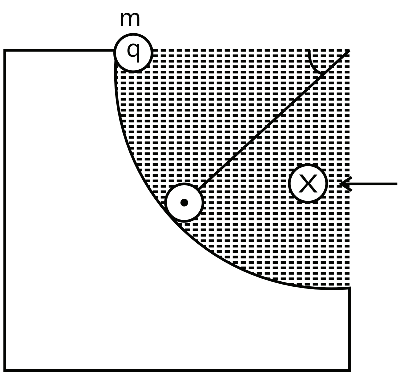
$$dB = \frac{\mu_0 dI r^2}{2\left(r^2 + x^2\right)^{3/2}} \quad \text{or}$$

$$B = \int dB = \frac{\mu_0\sigma\omega}{2}\int_0^R \frac{r^3 dr}{\left(r^2 + x^2\right)^{3/2}} \qquad \text{...(i)}$$

Putting $r^2 + x^2 = t^2$ and 2r dr=2t dt and integrating (i) we get

$$B = \frac{\mu_0\sigma\omega}{2}\left[\frac{R^2 + 2x^2}{\sqrt{R^2 + x^2}} - 2x\right].$$

Example 8: In the figure a charged sphere of mass m and charge q starts sliding from rest on a vertical fixed circular track of radius R from the position shown. There exists a uniform and constant horizontal magnetic field of induction B. The maximum force exerted by the track on the sphere.



Sol: As the sphere moves along the circular track the vector sum of radial component of magnetic force, the normal reaction and the radial component of weight of the sphere provide the necessary centripetal force.

$F_m = qvB$, and directed radially outward.

$$\therefore N - mg\sin\theta + qvB = \frac{mv^2}{R} \Rightarrow N = \frac{mv^2}{R} + mg\sin\theta - qvB$$

Hence at $\theta = \pi/2$

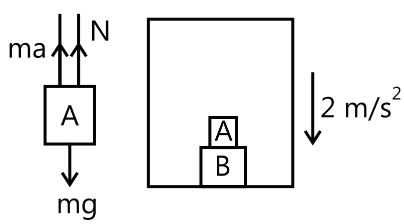
$$\Rightarrow N_{max} = \frac{2mgR}{R} + mg - qB\sqrt{2gR}$$

$$= 3mg - qB\sqrt{2gR}.$$

Example 9: What is the work done in transferring the wire from position (1) to position (2)?

Sol: While transfering wire from position 1 to position 2 find the change in the potential energy of the loop in the field of the wire. This chage in potential energy will be equal to the work done.

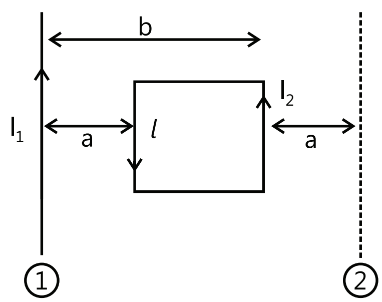
The loop can be considered as the combination of the number of elementary loops. The net current in the dotted wires is 0 as current in the neighboring loops flowing through the same wire opposite in direction. Consider an elementary loop of width dr at a distance r from the wire



The '$d\mu$' magnetic moment of the elemental loop

$= I_2 l dr$

The B at that point due to straight wire $= \mu_0 I_1/2\pi r$.



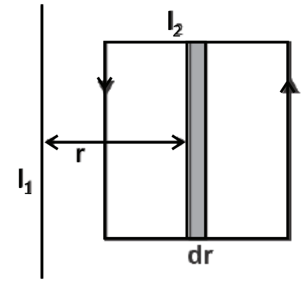
$$dU = -B.d\mu = -\frac{\mu_0 I_1}{2\pi r} I_2 l dr(\cos\pi)$$

[As $d\mu$ is anti-parallel to B.]

$$U_1 = \int du = \frac{\mu_0 I_1 I_2 l}{2\pi}\int_a^b \frac{1}{r}dr = \frac{\mu_0 I_1 I_2 l}{2\pi}\ln\left(\frac{a}{b}\right)$$

By symmetry, $U_2 = -U_1$

$\Rightarrow -\Delta U =$ work done

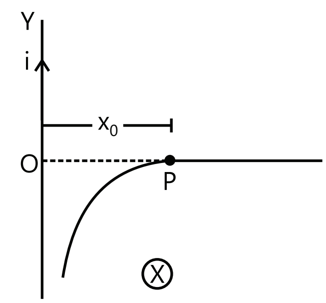


$$= -(U_2 - U_1) = 2\frac{\mu_0 I_1 I_2 l}{2\pi}\ln\frac{b}{a}.$$

The work done in transferring the wire from

Position 1 to 2 $= \dfrac{\mu_0 I_1 I_2 l}{\pi}\ln\dfrac{b}{a}$

Example 10: A long, straight wire carries a current i. A particle having a positive charge q and mass m, kept at a distance x_0 from the wire is projected towards it with a speed v. Find the minimum separation between the wire and the particle.



Sol: At minimum separation the x-component of velocity of the particle will be zero. Find the acceleration of the particle due to the magnetic force and solve to get the expression for velocity and displacement.

Let the particle be initially at P. Take the wire as the y-axis and the foot of perpendicular from P to the wire as the origin. Take the line OP as the x-axis. We have, OP= X_0. The magnetic field B at any point to the right of the wire is along the negative z-axis. The magnetic force on the particle is, therefore, in the x-y plane. As there is no initial velocity along the z-axis, the motion will be in the x-y plane. Also, its speed remains unchanged. As the magnetic field is not uniform, the particle does not go along a circle.

The force at time t is $\vec{F} = q\vec{v}\times\vec{B} = q(\vec{i}v_x + \vec{j}v_y)\times\left(-\frac{\mu_0 i}{2\pi x}\vec{k}\right)$

$$= \vec{j}qv_x\frac{\mu_0 i}{2\pi x} - \vec{i}qv_y\frac{\mu_0 i}{2\pi x}.$$

$$\text{Thus } a_x = \frac{F_x}{m} = -\frac{\mu_0 qi}{2\pi m}\frac{\mu_y}{x} = -\lambda\frac{\mu_y}{x} \quad \text{...(i)}$$

$$\text{Where } \lambda = \frac{\mu_0 qi}{2\pi m}.$$

$$\text{Also, } a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = \frac{v_x dv_x}{dx}. \quad \text{...(ii)}$$

As $v_x^2 + v_y^2 = v^2$,

$$2v_x dv_x + 2v_y dv_y = 0$$

giving $\quad v_x dv_x = -v_y dv_y$. ...(iii)

From (i), (ii) and (iii),

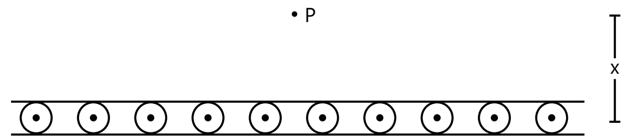
$$\frac{v_y dv_y}{dx} = \frac{\lambda v_y}{x} \quad \text{or} \quad \frac{dx}{x} = \frac{dv_y}{\lambda}.$$

Initially $x = x_0$ and $v_y = 0$. At minimum separation from the wire, $v_x = 0$ so that $v_y = -v$.

$$\text{Thus } \int_{x_0}^{x}\frac{dx}{x} = \int_{0}^{-v}\frac{dv_y}{\lambda} \text{ or, } \ln\frac{x}{x_0} = -\frac{v}{\lambda}$$

$$\text{or, } x = x_0 e^{-v/\lambda} = x_0 e^{-\frac{2\pi mv}{\mu_0 qi}}$$

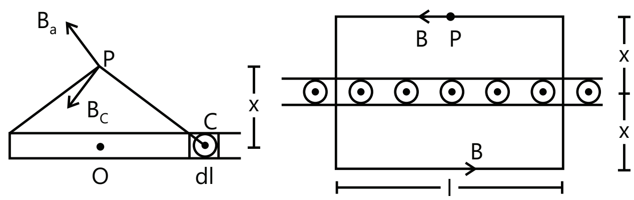
Example 11: Figure shows a cross section of a large metal sheet carrying an electric current along its surface. The current in a strip of width dl is Kdl where K is a constant. Find the magnetic field at a point P at a distance x from the metal sheet.



Sol: Field due to the sheet will be symmetric. Field lines will be parallel to the sheet at points near it. Select a rectangular amperian loop and use Ampere's Law to find the field.

Consider two strips A and C of the sheet situated symmetrically on the two sides of P.The magnetic field at P due to the strip A is B_a perpendicular to AP and that due to the strip C is B_c perpendicular to CP. The resultant of these two is parallel to the width AC of the sheet. The field due to the whole sheet will also be in this direction. Suppose this field has magnitude B.

The field on the opposite side of the sheet at the same distance will also be B but in opposite direction. Applying Ampere's law to the rectangle shown in figure.



(a) (b)

$2Bl = \mu_0 Kl$ or, $B = \frac{1}{2}\mu_0 K$ Note that it is independent of x.

JEE Main/Boards

Exercise 1

Q.1 A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries current of 0.40 A. What is the magnitude of the magnetic field B at the center of the coil?

Q.2 A long straight wire carries a current of 35 A. What is the magnitude of the field B at a point 20 cm from the wire?

Q.3 A long straight wire in the horizontal plane carrier of 50 A in north to south direction. Give the magnitude and direction of Bat a point 2.5 m east of the wire.

Q.4 A horizontal overhead power line carries a current of 90 A in east west direction. What is the magnitude and direction of the magnetic field due to the current 1.5 m below the line?

Q.5 What is the magnitude of a magnetic force per unit length on a wire carrying a current of 8 A and making an angle of $30°$ with the direction of a uniform magnetic field of 0.15 T?

Q.6 In a chamber, a uniform magnetic field of 6.5 G(1G=10^{-4} T) is maintained. An electron is shot into the field with a speed of 4.8×10^6 ms^{-1} normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit.

$\left(e = 1.6 \times 10^{-19} C,\ m_e = 9.1 \times 10^{-31} kg\right)$

Q.7 (i) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of 60° with the normal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.

(ii) Would your answer change, if the circular coil in (a) were replaced by a planner coil of some irregular shape that encloses the same area? (All other particulars are also unaltered.)

Q.8 Two concentric circular coils X and Y radii 16 cm and 10 cm, respectively, lie in the same vertical plane containing the north to south direction. Coil X has 20 turns and carries a current 16 A; coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise, and clockwise in Y, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their center.

Q.9 A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires.

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wire is zero?

(b) What will be the total tension in the wires if the direction of current is reversed keeping the magnetic field same as before?

Q.10 The wires which connect the battery of an automobile to its starting motor carry a current of 300 A (for a short time). What is the force per unit length between its wires if they are 70 cm long and 1.5 cm apart? Is the force attractive of repulsive?

Q.11 A uniform magnetic field of 1.5 T exists in a cylindrical region of radius10.0 cm, its direction parallel to the axis along east to west. A wire carrying current of 7.0 A in the north to south direction passes through this region. What is the magnitude and direction of the force on the wire if,

(a) The wire intersects the axis,

(b) The wire is turned from N-S to northeast-northwest direction,

(c) The wire in the N-S direction is lowered from the axis by a distance of 6.0 cm?

Q.12 A circular coil of N turns and radius R carries a current I. It is unwound and rewound to make another coil of radius R/2. Current I remaining the same. Calculate the ratio of the magnetic moments of the new coil and the original coil.

Q.13 A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the

(a) Total torque on the coil,

(b) Total force on the coil

(c) Average force on each electron is the coil due to the magnetic field?

(The coil is made of copper wire of cross-sectional area $10^{-5}\ m^2$, and the free electron density in copper is given to be about $10^{29} m^{-3}$.)

Q.14 State the Biot-Savart law for the magnetic field due to a current-carrying element. Use this law to obtain a formula for magnetic field at the center of a circular loop of radius a carrying a, steady current I.

Q.15 Give the formula for the magnetic field produced by a straight infinitely long current-carrying wire. Describe the lines of field B in this case.

Q.16 How much is the density B at the center of a long solenoid?

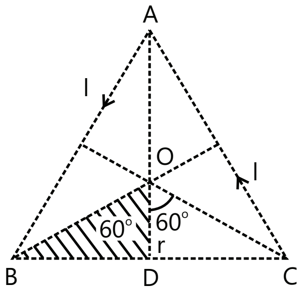
Q.17 A proton shot at normal to magnetic field describe a circular path of radius R. If a deuteron $\left({}_1H^2\right)$ is to move on the same path, what should be the ratio of the velocity of proton and the velocity of deuteron?

Q.18 State the principle of cyclotron.

Q.19 A charge q is moving in a region where both the magnetic field B and electric field E are simultaneously present. What is the Lorentz force acting on the charge?

Q.20 A charged particle moving in a straight line enters a uniform magnetic field at an angle of $45°$. What will be its path?

Q.21 A current of 1A is flowing in the sides of an equilateral triangle of side $4.5 \times 10^{-2} m$. Find the magnetic field at the centroid of the triangle.



Q.22 The radius of the first electron orbit of a hydrogen atom is 0.5 Å. The electron moves in this orbit with a uniform speed of $2.2\times10^{6}ms^{-1}$. What is the magnetic field produced at the center of the nucleus due to the motion of this electron?

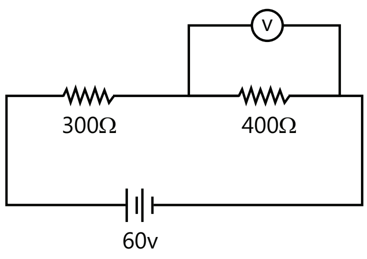
Q.23 A solenoid is 2 m long and 3 cm in diameter. It has 5 layers of windings of 1000 turns each and carries a current of 5 A. What is the magnetic field at itscenter? Use the standard value of μ_0.

Q.24 A proton entersa magnetic field of flux density 2.5 T with a velocity of $1.5\times10^{7}ms^{-1}$ at an angle of 30° with the field. Find the force on the proton.

Q.25 Two parallel wires one meter apart carry currents of 1A and 3 respectively in opposite directions. Calculate the force per unit length acting between these wires.

Q.26 A solenoid of length 0.4, and having 400 turns of wire carries a current of 3 A. A thin coil having 10 turns of wire and radius 0.01 m carries a current 0.4 A. Calculate the torque required to hold the coil in the middle of the solenoid with its axis perpendicularto the axis of the solenoid.

Q.27 In a circuit shown in figure a voltmeter reads 30 V, when it is connected across 400ohm resistance. Calculate what the same voltmeter will read when connected across the 300 Ω resistance?



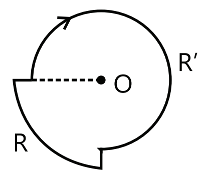
Q.28 Two long straight parallel wires are 2m apart, perpendicular to the plane of the paper. The wire A carries a current of 9.6 ampere directed into the plane of the paper. The wire B carries a current such that the magnetic field induction at the point P, at a distance of $\frac{10}{11}$ m from the wire B, is zero. Calculate

(i) the magnitude and direction of current in B (ii) the magnitude of magnetic field induction at S

(ii) the force per unit length of the wire B.

Exercise 2

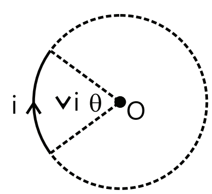
Q.1 A current 1 ampere is flowing through each of the bent wires as shown figure. The magnitude and direction of magnetic field at O is



(A) $\frac{\mu_0 i}{4}\left(\frac{1}{R}+\frac{2}{R'}\right)$ (B) $\frac{\mu_0 i}{4}\left(\frac{1}{R}+\frac{3}{R'}\right)$

(C) $\frac{\mu_0 i}{8}\left(\frac{1}{R}+\frac{3}{2R'}\right)$ (D) $\frac{\mu_0 i}{8}\left(\frac{1}{R}+\frac{3}{R'}\right)$

Q.2 Net magnetic field at the center of the circle O due to a current carrying loop as shown in figure is $(\theta<180^\circ)$



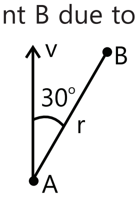
(A) Zero

(B) Perpendicular to paper inwards

(C) Perpendicular to paper outwards

(D) Is perpendicular to paper inwards if $\theta\le 90^\circ$ and perpendicular to paper outwards if $90^\circ\le\theta<180^\circ$

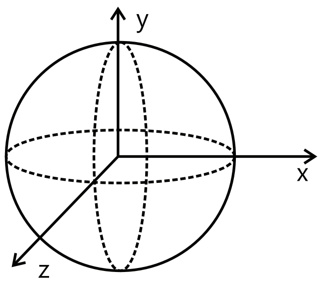
Q.3 A charge particle A of charge q=2C has velocity v=100 m/s. When it passes through point A and has velocity in the direction shown. The strength of magnetic field at point B due to this moving charge is (r=2 m).



(A) 2.5 μT (B) 5.0 μT

(C) 2.0 μT (D) None

Q.4 Three rings, each having equal radius R, are placed mutually perpendicular to each other and each having its center at the origin of co-ordinates system. If current is flowing through each ring then the magnitude of the magnetic field at the common center is



(A) $\sqrt{3}\frac{\mu_0 I}{2R}$ (B) Zero

(C) $\left(\sqrt{2}-1\right)\frac{\mu_0 I}{2R}$ (D) $\left(\sqrt{3}-\sqrt{2}\right)\frac{\mu_0 I}{2R}$

Q.5 Two concentric coils X and Y of radii 16 cm and 10 cm lie in the same vertical plane containing N-S direction. X has 20 turns and carries 16 A. Y has 25 turns & carries 18 A. X has current in anticlockwise direction and Y has current in clockwise direction for an observer, looking at the coils facing the west. The magnitude of net magnetic field at their common center is

(A) $5\pi\times10^{-4}$ T towards west

(B) $13\pi\times10^{-4}$ T towards east

(C) $13\pi\times10^{-4}$ T towards west

(D) $5\pi\times10^{-4}$ T towards east

Q.6 Equal current i is flowing in three infinitely long wires along positive x, y and z directions. The magnetic field at a point (0, 0, -a) would be:

(A) $\frac{\mu_0 i}{2\pi a}\left(\hat{j}-\hat{i}\right)$ (B) $\frac{\mu_0 i}{2\pi a}\left(\hat{i}+\hat{j}\right)$

(C) $\frac{\mu_0 i}{2\pi a}\left(\hat{i}-\hat{j}\right)$ (D) $\frac{\mu_0 i}{2\pi a}\left(\hat{i}+\hat{j}+\hat{k}\right)$

Q.7 An electron is moving along positive x-axis. A uniform electric field exists towards negatively y-axis. What should be the direction of magnetic field of suitable magnitude so that net force of electron is zero.

(A) Positive z-axis (B) Negative z-axis

(C) Positive y-axis (D) Negative y-axis

Q.8 A particle of charge q and mass m starts moving from the origin under the action of an electric field $E=E_0\hat{i}$ and $B=B_0\hat{i}$ with velocity $\vec{v}=v_0\hat{j}$. The speed of the particle will become $2v_0$ after a time

(A) $t=\frac{2mv_0}{qE}$ (B) $t=\frac{2Bq}{mv_0}$

(C) $t=\frac{3Bq}{mv_0}$ (D) $t=\frac{\sqrt{3}mv_0}{qE}$

Q.9 An electron is projected with velocity v_0 in a uniform electric field E perpendicular to the field. Again it is projected with velocity v_0 perpendicular to a uniform magnetic field B. If r_1 is initial radius of curvature just after entering in the electric field and r_2 in initial radius of curvature just after entering in magnetic field then the ratio r_1/r_2 is equal to

(A) $\frac{Bv^2 0}{E}$ (B) $\frac{B}{E}$ (C) $\frac{Ev_0}{B}$ (D) $\frac{Bv_0}{E}$

Q.10 A uniform magnetic field $B=B_0\hat{j}$ exists in a space. A particle of mass m and charge q is projected towards negative x-axis with speed v from the point (d, 0, 0). The maximum value v for which the particle does not hit y-z plane is

(A) $\frac{2B_0q}{dm}$ (B) $\frac{B_0q}{m}$ (C) $\frac{B_0q}{2dm}$ (D) $\frac{B_0qd}{2m}$

Q.11 Two protons move parallel to each other, keeping distance r between them, both moving with same velocity v. Then the ratio of the electric and magnetic force of interaction between them is.

(A) c^2/v^2 (B) $2c^2/v^2$

(C) $c^2/2v^2$ (D) None

Q.12 Three ions H^+, He^+ and O^{+2} having same kinetic energy pass through a region in which there width is a uniform magnetic field perpendicular to their velocity, then:

(A) H^+ will be least deflected.

(B) He^+ and O^{+2} will be deflected equally.

(C) O^{+2} will be deflected most.

(D) all will be deflected equally.

Q.13 An electron having kinetic energy T is moving in a circular orbit of radius R perpendicular to a uniform magnetic induction B. If kinetic energy is doubled and magnetic induction tripled, the radius will become.

(A) $\frac{3R}{2}$ (B) $\sqrt{\frac{3}{2}}R$

(C) $\sqrt{\frac{2}{9}}R$ (D) $\sqrt{\frac{4}{3}}R$

Q.14 A charged particle moves in magnetic field $B = 10\hat{i}$ with initial velocity $\vec{u} = 5\hat{i} + 4\hat{j}$.

The path of the particle will be.

(A) Straight line (B) Circle

(C) Helical (D) None

Q.15 A electron experiences a force $(4.0\hat{i} + 3.0\hat{j})\,x10^{-13}N$ in a uniform magnetic field when its velocity is $2.5\hat{k} \times 10^{7}ms^{-1}$. When the velocity is redirected and becomes $(1.5\hat{i} - 2.0\hat{j})x10^{7}ms^{-1}$, the magnetic force of the electron is zero. The magnetic field vector B is :

(A) $-0.075\hat{i} + 0.1\hat{j}$ (B) $0.1\hat{i} + 0.075\hat{j}$

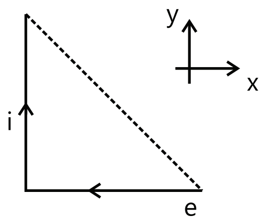
(C) $0.075\hat{i} + 0.1\hat{j} + \hat{k}$ (D) $0.075\hat{i} + 0.1\hat{j}$

Q.16 An electron moving with a velocity $V_1 = 2\hat{i}\,m/s$ at a point in a magnetic field experiences a force $F_1 = -2\hat{j}\,N$. If the electron is moving with a velocity $V_2 = 2\hat{j}\,m/s$ at the same point, it experiences a force $F_2 = +2\hat{i}\,N$. The force the electron would experience if it were moving with a velocity $V_3 = 2\hat{k}\,m/s$ at the same point is

(A) Zero (B) $2\hat{k}$ N

(C) $-2\hat{k}$ N (D) Information is insufficient

Q.17 The direction of magnetic force on the electron as shown in the diagram is along



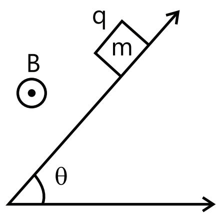
(A) y-axis (B) –y-axis (C) z-axis (D) –z-axis

Q.18 A block of mass m & charge q is released on a long smooth inclined plane magnetic field B is constant, uniform, horizontal and parallel to surface as shown. Find the time from start when block loses contact with the surface.

(A) $\frac{m\cos\theta}{qB}$ (B) $\frac{m\,\text{cosec}\,\theta}{qB}$

(C) $\frac{m\cot\theta}{qB}$ (D) None

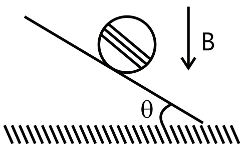
Q.19 A metal ring of radius r=0.5m with its plane normal to a uniform magnetic field B of induction 0.2T carries a current I=100A. The tension in Newton developed in the ring is:



(A) 100 (B) 50

(C) 25 (D) 10

Q.20 In the shown a coil of single turn is wound on a sphere of radius R and mass m. The plane of the coil is parallel to the plane and lies in the equatorial plane of the sphere. Current in the coil is i. The value of B if the sphere is in equilibrium is



(A) $\frac{mg\cos\theta}{\pi iR}$ (B) $\frac{mg}{\pi iR}$

(C) $\frac{mg\tan\theta}{\pi iR}$ (D) $\frac{mg\sin\theta}{\pi iR}$

Q.21 The magnetic moment of a circular orbit of radius 'r' carrying a charge 'q' and rotating with velocity v is given by

(A) $\frac{qvr}{2\pi}$ (B) $\frac{qvr}{2}$

(C) $qv\pi r$ (D) $qv\pi r^2$

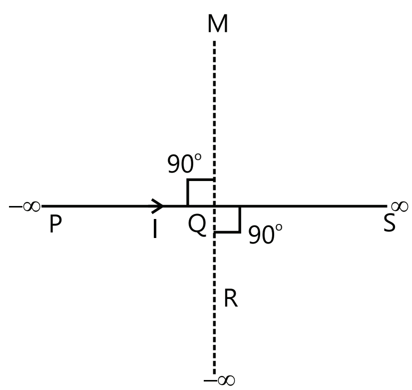
Previous Years' Questions

Q.1 Two very long straight parallel wires carry steady currents I and –I respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires in the plane of the wires. Its instantaneous velocity $\vec{V}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is ***(1998)***

(A) $\frac{\mu_0 Iqv}{2\pi d}$ (B) $\frac{\mu_0 Iqv}{\pi d}$

(C) $\frac{2\mu_0 Iqv}{\pi d}$ (D) Zero

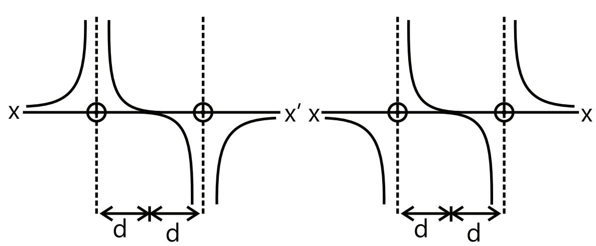
Q.2 An infinitely long conductor PQR is bent to form a right angle as shown in Figure. A current I flows through PQR. The magnetic field due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q, so that current is I/2 in QR as well as in QS, the current in PQ remaining uncharged. The magnetic field at M is now H_2. The ratio H_1/H_2 is given by ***(2000)***



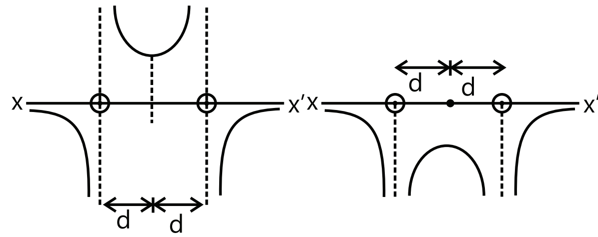
(A) 1/2 (B) 1

(C) 2/3 (D) 2

Q.3 Two long parallel wire are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX′ is given by ***(2000)***

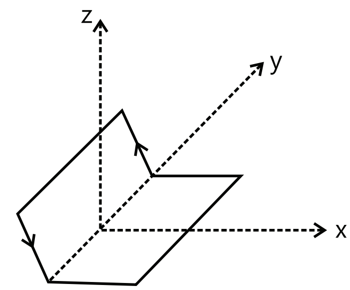


(a) (b)



(a) (b)

Q.4 A non-planar loop of conducting wire carrying a current I is placed as shown in the figure. Each of the straight section of the loop is of length 2a. The magnetic field due tothis loop at the point P(a,0,a) points in the direction ***(2001)***



(A) $\frac{1}{\sqrt{2}}\left(-\hat{j}+\hat{k}\right)$ (B) $\frac{1}{\sqrt{3}}\left(-\hat{j}+\hat{k}+\hat{i}\right)$

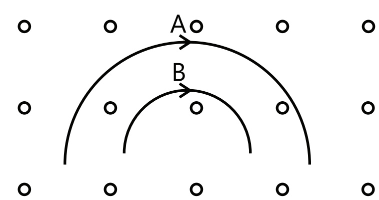
(C) $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$ (D) $\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{k}\right)$

Q.5 A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic field at the center is ***(2001)***

(A) $\frac{\mu_0 NI}{b}$ (B) $\frac{2\mu_0 NI}{a}$

(C) $\frac{\mu_0 NI}{2(b-a)}\log\frac{b}{a}$ (D) $\frac{\mu_0 I^N}{2(b-a)}\log\left(\frac{b}{a}\right)$

Q.6 Two particles A and B of masses m_A and m_B respectively and having the same charge are moving in a plane. A uniform magnetic field exists perpendicular to this plane. The speeds of the particles are V_A and V_B respectively and the trajectories are as shown in the figure. Then ***(2001)***



(A) $m_A v_A < m_B v_B$ (B) $m_A v_A > m_B v_B$

(C) $m_A < m_B$ and $v_A < v_B$ (D) $m_A = m_B$ and $v_A = v_B$

Q.7 A long straight wire along the z-axis carries a current I in the negative z-direction. The magnetic vector field $\vec{B}$ at a point having coordinate (x,y) on the z=0 plane is ***(2002)***

(A) $\frac{\mu_0 I(y\hat{i} - x\hat{j})}{2\pi(x^2+y^2)}$ (B) $\frac{\mu_0 I(x\hat{i} - y\hat{j})}{2\pi(x^2+y^2)}$

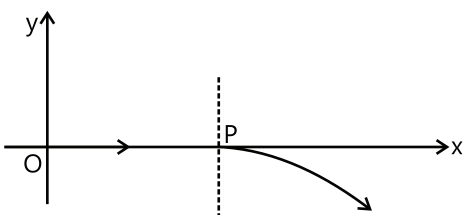
(C) $\frac{\mu_0 I(x\hat{j} - y\hat{i})}{2\pi(x^2+y^2)}$ (D) $\frac{\mu_0 I(x\hat{i} - y\hat{j})}{2\pi(x^2+y^2)}$

Q.8 A particle of mass m and charge q moves with a constant velocity v along the positive x-direction. It enters a region containing a uniform field B directed along the negative z-direction, extending from x=a to x=b. the minimum value of v required so that the particle can just enter the region x>b is ***(2002)***

(A) $\frac{qbB}{m}$ (B) $\frac{q(b-a)B}{m}$ (C) $\frac{qaB}{m}$ (D) $\frac{q(b+a)B}{2m}$

Q.9 For a positively charged particle moving in a x-y plane initially along x-axis, there is a sudden change in its path due to presence of electric and/or magnetic fields beyond P. The curved path is shown in the x-y plane and is found to be non-circular.

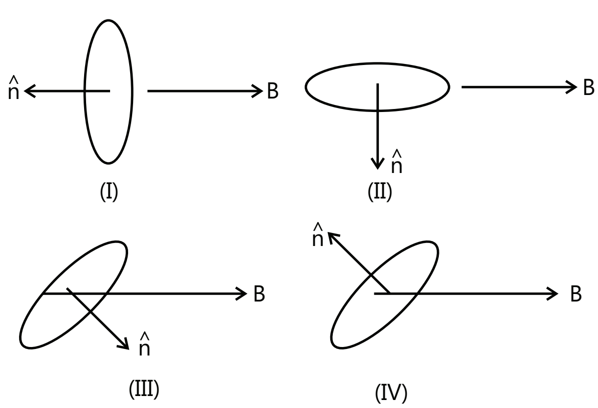
Which one of the following combinations is possible? ***(2003)***



(A) $\vec{E}=0; \vec{B}=b\hat{j}+c\hat{k}$ (B) $\vec{E}=a\hat{i}; \vec{B}=c\hat{k}+a\hat{i}$

(C) $\vec{E}=0; \vec{B}=c\hat{j}+b\hat{k}$ (D) $\vec{E}=a\hat{i}; \vec{B}=c\hat{k}+b\hat{j}$

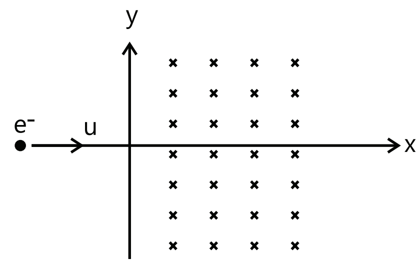
Q.10 A current carrying loop is placed in a uniform magnetic field in four different orientations, I, II, III and IV, arrange them in the decreasing order of potential energy ***(2003)***



(A) I > III > II > IV (B) I > II > III > IV

(C) I > IV > II > III (D) III > IV > I > II

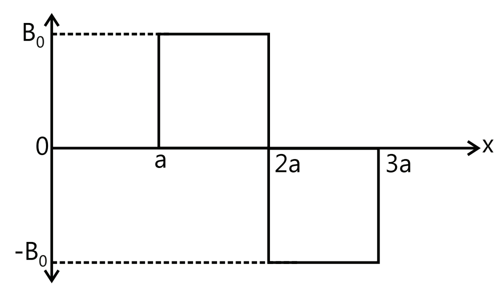
Q.11 An electron moving with a speed u along the position x-axis at y=0 enters a region of uniform magnetic field $\vec{B} = -B_0\hat{k}$ which exists to the right of y-axis. The electron exits from the region after sometime with the speed v at coordinate y, then ***(2004)***



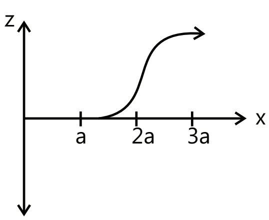
(A) v>u, y<0 (B) v=u, y>0

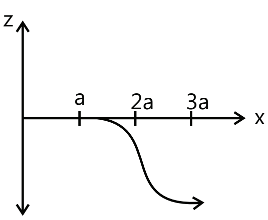
(C) v>u, y>0 (D) v=u, v<0

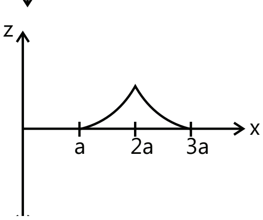
Q.12 A magnetic field $\vec{B} = -B_0\hat{j}$ exists in the region a<x<2a and $\vec{B} = -B_0\hat{j}$, in the region 2a<x<3a, where B_0 is a positive constant. A positive point charge moving with a velocity $\vec{v} = -v_0\hat{i}$, where v_0 is a positive constant, enters the magnetic field at x=a.

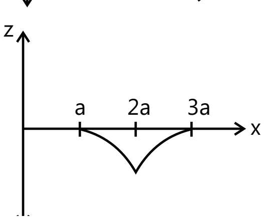


The trajectory of the charge in this region can be like ***(2007)***

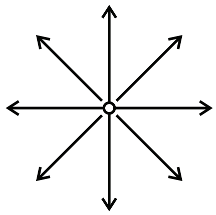
(A)


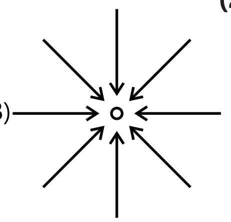
(B)


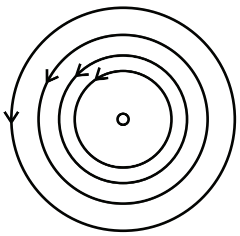
(C)


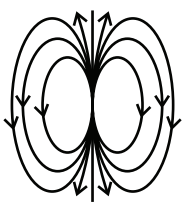
(D)


Q.13 Which of the field patterns given in the figure is valid for electric field as well as for magnetic field? ***(2011)***

(A) 

(B) 

(C) 

(D) 

Q.14 A long insulated copper wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. The spiral lies in the X-Y plane and a steady current I flows through the wire. The Z-component of the magnetic field at the center of the spiral is ***(2011)***

(A) $\frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$ (B) $\frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b+a}{b-a}\right)$

(C) $\frac{\mu_0 NI}{2b}\ln\left(\frac{b}{a}\right)$ (D) $\frac{\mu_0 NI}{2b}\ln\left(\frac{b+a}{b-a}\right)$

Q.15 Proton, Deuteron and alpha particle of the same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α. Which one of the following relations is correct? ***(2012)***

(A) $r_\alpha = r_p = r_d$ (B) $r_\alpha = r_p < r_d$

(C) $r_\alpha > r_d > r_p$ (D) $r_\alpha = r_d > r_p$

Q.16 Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid - point O of the line joining their centres is close to (Horizontal component of earth's magnetic induction is $3.6\times10^{-5}\,Wb/m^2$) ***(2013)***

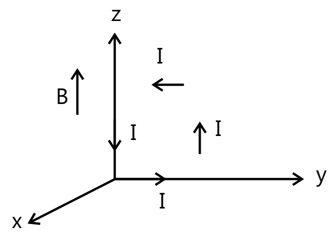
(A) $2.56\times10^{-4}\,Wb/m^2$ (B) $3.50\times10^{-4}\,Wb/m^2$

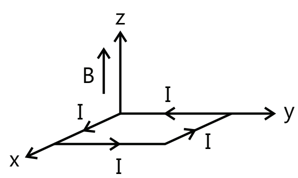
(C) $5.80\times10^{-4}\,Wb/m^2$ (D) $3.6\times10^{-5}\,Wb/m^2$

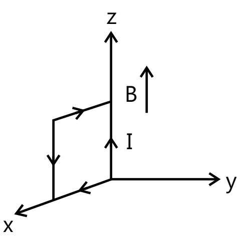
Q.17 The coercivity of a small magnet where the ferromagnet gets demagnetized is $3\times10^3\,Am^{-1}$. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is: ***(2014)***

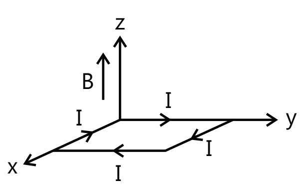
(A) 3A (B) 6A (C) 30 mA (D) 60 mA

Q.18 A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below:

(A) 

(B) 

(C) 

(D) 

If there is a uniform magnetic field of 0.3 T in the positive z direction , in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium? ***(2015)***

(A) (a) and (c), respectively

(B) (b) and (d), respectively

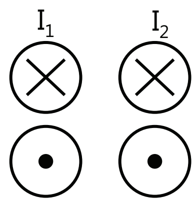
(C) (b) and (c), respectively

(D) (a) and (b), respectively

JEE Advanced/Boards

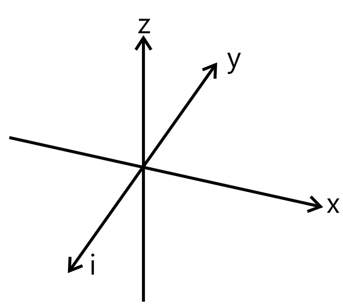
Exercise 1

Q.1 A system of long four parallel conductors whose sections with the plane of the drawing lie at the vertices of a square there flow four equal currents. The directions of these currents are as follows:



Those marked ⊗ point away from the reader, while those marked with a dot point towards the reader. How is the vector of magnetic induction directed at the center of the square?

Q.2 A long straight wire carriers a current of 10A directed along the negative y-axis as shown in figure. A uniform magnetic field B_0 of magnitude 10^{-6}T is directed parallel to the x-axis. What is the resultant magnetic field at the following points?

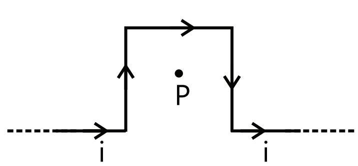


(a) x=0, z=2m ;

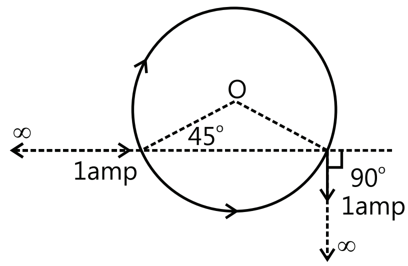
(b) x=2m, z=0

(c) x=0, z=-0.5m

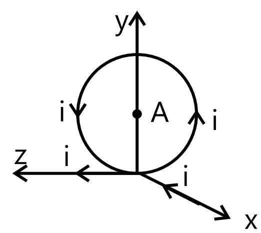
Q.3 Find the magnetic field at the center P of square of side a shown in figure.



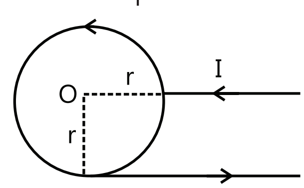
Q.4 What is the magnitude of magnetic field at the center 'O' of loop of radius $\sqrt{2}$ m made of uniform wire when a current of 1amp enters in the loop and taken out of it by two long wires as shown in the figure.



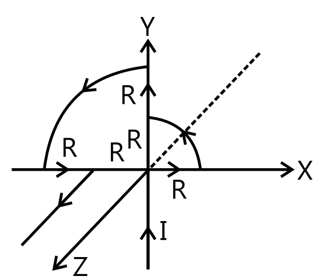
Q.5 Find the magnetic induction at the origin in the figure shown.



Q.6 Find themagnetic induction at point O, if the current carrying wire is in the shape shown in the figure.

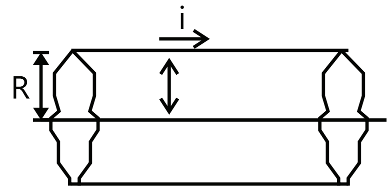


Q.7 Find the magnitude of the magnetic induction B of a magnetic field generated by a system of thin conductors along which a current I is flowing at a point A(O,R,O), that is the center of a circular conductor of radius R. The ring is in the yz plane.



Q.8 A cylindrical conductor of radius R carriers a current along its length. The current density J, however, is not uniform over the cross section of the conductor but is a function of the radius according to J=br, where b is a constant. Find an expression for the magnetic field B.

(a) at $r_1 < R$ (b) at distance $r_2 < R$, measured from the axis



Q.9 Electric charge q is uniformly distributed over a rod of length L. The rod is placed parallel to a long wire carrying a current I. The separation between the rod and the wire is a. Find the force needed to move the rod along its lengths with a uniform velocity V.

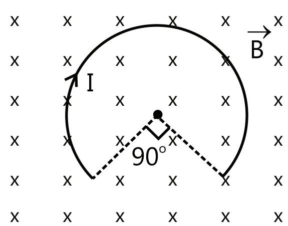
Q.10 An electron moving with a velocity $5x10^6 ms^{-1}\hat{j}$ in the uniform electric field of $5x10^7 Vm^{-1}\hat{j}$. Find the magnitude and direction of a minimum uniform magnetic field in tesla that will cause the electron to move undeviated along it original path.

Q.11 A charged particle (charge q, mass m) has velocity V_0 at origin in +x direction. In space there is a uniform magnetic field B in –z direction. Find the y coordinate of the particle when it crosses y axis.

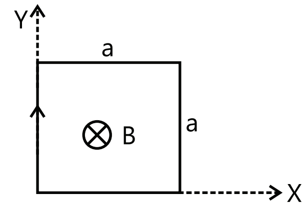
Q.12 A proton beam passes without deviation through a region of space where there are uniform transverse mutually perpendicular electric and magnetic field with E and B. Then the beam strikes a grounded target. Find the force imparted by a beam on the target if the beam current is equal to I.

Q.13 A conducting circular loop of radius r carriers a constant current i. It is placed in a uniform magnetic field B_0 such that B_0 is perpendicular to the plane of the loop. Find the magnetic force acting on the loop.

Q.14 An arc of a circular loop of radius R is kept in the horizontal plane and a constant magnetic field B is applied in the vertical direction as shown in the figure. If the carries current I then find the force on the arc.



Q.15 A rectangular loop of wire is oriented with the left corner at the origin, one edge along X-axis and the other edge along. Y-axis as shown in the figure. A magnetic field is into the page and has a magnitude that is given by $\beta = \alpha y$ where α is constant. Find the total magnetic force on the loop if it carries current i.



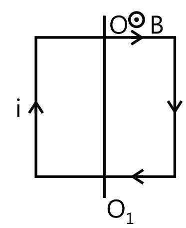
Q.16 A particle of charge +q and mass m moving under the influence of a uniform electric field $E\hat{i}$ and a magnetic field $B\hat{k}$ enters in I quadrant of a coordinate system at a point (0, a) with initial velocity $v\,\hat{i}$ and leaves the quadrant at a point (2a, 0) with velocity $-2v\,\hat{j}$. Find Magnitude of electric field

(a) Rate of work done by the electric field at point

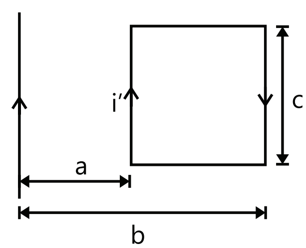
(b) (0, a) Rate of work done by both the fields at.

(c) (2a, 0).

Q.17 A square current carrying loop made of thin wire and having a mass m=10g can rotate without friction with respect to the vertical axis OO_I, passing through the center of the loop at right angles to two opposite sides of the loop. The loop is placed in a uniform magnetic field with an induction $B=10^{-1}$T directed at right angles to the plane of the drawing. A current I=2A is flowing in the loop. Find the period of small oscillations that the loop performs about its position of stable equilibrium.



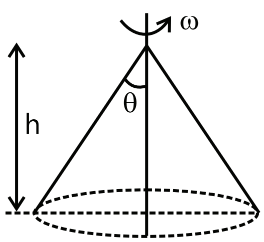
Q.18 An infinitely long straight wire carries a conventional current I as shown in the figure. The rectangular loop carries a conventional current I' in the clockwise direction. Find the net force on the rectangular loop.



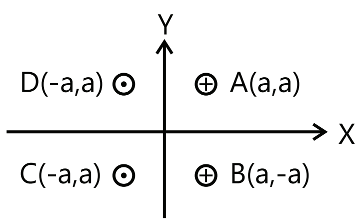
Q.19 3 Infinitely long thin wires each carrying current i in the same direction, are in the x-y plane of a gravity free space. The central wire is along the y-axis while the other two are along $x = \pm d$. (i) Find the locus of the points for which the magnetic field B is zero.

(ii) If the central wire is displaced along the Z-direction by a small amount & released, show that it will execute simple harmonic motion. If the linear density of the wires is λ, find the frequency of oscillation.

Q.20 Q charge isuniformly distributed over the same surface of a right circular cone of semi-vertical angle θ and height h. The cone is uniformly rotated about its axis at angular velocity ω. Calculated associated magnetic dipole moment.



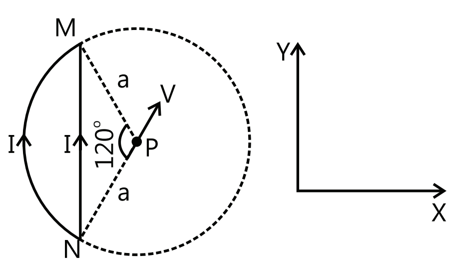
Q.21 Four long wires each carrying current I as shown in the figure are placed at the point A, B, C and D. Find the magnitude and direction of



(i) Magnetic field at the center of the square.

(ii) Force per metre acting on wire at point D.

Q.22 A wire loop carrying current I is placed in the X-Y plane as shown in the figure.



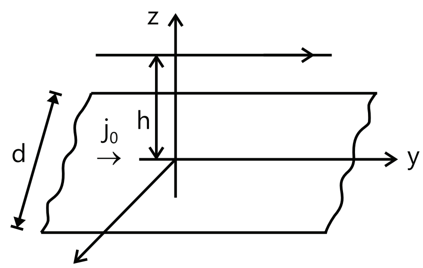
(a) If a particle with charge +Q and mass m is placed at the center P and given a velocity along NP (see figure). Find its instantaneous acceleration.

(b) If an external uniformmagnetic induction field $B = B\hat{i}$ is applied, find the torque acting on the loop due to the field.

Q.23 (a) A rigid circular loop of radius r & mass m lies in the xy plane on a flat table and has a current I flowing in it. At this particular place, the earth's magnetic field is $B = B_x\hat{i} + B_y\hat{j}$. How large must I be before one edge of the loop will lift from table?

(b) Repeat if, $B = B_x\hat{i} + B_z\hat{k}$.

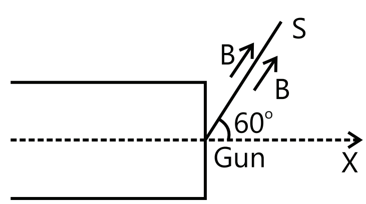
Q.24 A conductor carrying a current is placed parallel a current per unit width j_0 and width d, as shown in the Figure.



Find the force per unit length on the conductor.

Q.25 The figure shows a conductor of weight 1.0N and length L= 0.5m placed on a rough inclined plane making an angle 30^0 with the horizontal so that conductor is perpendicular to a uniform horizontal magnetic field of induction B=0.10 T. The coefficient of static friction between the conductor and the plane is 0.1. A current of I=10A flows through the conductor inside the plane of this paper as shown. What is the force that should be applied parallel to the inclined plane for sustaining the conductor at rest?

Q.26 An electron gun G emits electron of energy 2kev traveling in the (+) ve x-direction. The electron are required to hit the spot S where GS=0.1m & line GS makes an angle of 60^0 with the x-axis, as shown in the figure. A uniform magnetic field B parallel to GS exists in the region outside to the electron gun. Find the minimum value of B needed to make the electron hit S.



Q.27 Two coils each of 100 turns are held such that one lies in the vertical plane with their centers coinciding. The radius of the vertical coil is 20cm and that of the horizontal coil is 30cm. How would you neutralize the magnetic field of the earth at their common center? What is the current to be passed through each coil? Horizontal component of earth's magnetic induction = 3.49×10^{-5}T and angle of dip=30^0.

Q.28 An infinite wire, placed along z-axis, has current i_1 in positive z-direction. A conducting rod placed in xy plane parallel to y-axis has current i_2 in positive y-direction. The ends of the rod subtend $+30^0$ and -60^0 at the origin with positive x-direction. The rod is at a distance a from the origin. Find net force on the rod.

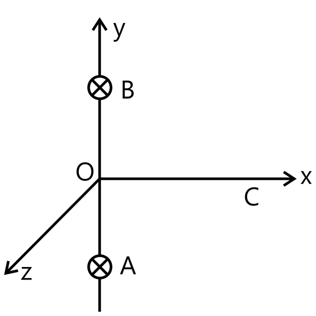
Q.29 A square loop of wire of edge a carries a current i.

(a) Show that B for a point on the axis of the loop and a distance x from its center is given by,

$$B = \frac{4\mu_0 i a^2}{\pi\left(4x^2 + a^2\right)\left(4x^2 + 2a^2\right)^{1/2}}$$

(b) Can the result of the above problem be reduced to give field at x=0?

Q.30 A straight segment OC (of length L meter) of a circuit carrying a current I amp is placed along the x-axis. Two infinitely line straight wires A and B, each extending from $z = -\infty$ to $+\infty$, are fixed by $y = -a$ meter and $y = +a$ meter respectively, as shown in the Figure.

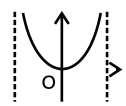


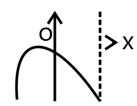
If the wires A and B each carry a current I amp into plane of the paper. Obtain the expression for the force acting on the segment OC. What will be the force OC if current in the wire B is reversed?

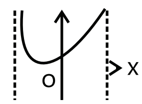
Exercise 2

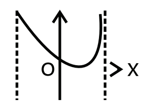
Single Correct Choice Type

Q.1 Two very long straight parallel wires, parallel to -y direction, respectively. The wire are passes through the x-axis at the point (d, 0, 0) and (-d, 0, 0)respectively. The graph of magnetic field z-component as one moves along the x-axis from x=-d to x=+d, is best given by

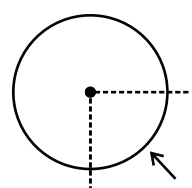
(A) 

(B) 

(C) 

(D) 

Q.2 A long thin walled pipe of radius R carries a current I along its length. The current density is uniform over the circumference of the pipe. The magnetic field at the center of the pipe due to quarter portion of the pipe shown, is



(A) $\frac{\mu_0 I\sqrt{2}}{4\pi^2 R}$ (B) $\frac{\mu_0 I}{\pi^2 R}$

(C) $\frac{2\mu_0 I\sqrt{2}}{\pi^2 R}$ (D) None

Q.3 An electron (mass=9.1×10^{-31}; charge=-1.6×10^{-19}C) experiences no deflection if subjected to an electric field of 3.2×10^5 V/m and a magnetic field of 2.0×10^{-3} Wb/m^2. Both the fields are normal to the path of electron and to each other. If the electric field is removed, then the electron will revolve in an orbit of radius:

(A) 45m (B) 4.5m (C) 0.45m (D) 0.045m

Q.4 A particle of specific charge (charge/mass) α starts moving from the origin under the action of an electric field $E = E_0\hat{i}$ and magnetic field $B = B_0\hat{k}$. Its velocity at $\left(x_0, y_0, 0\right)$ is $\left(4\hat{i} - 3\hat{j}\right)$. The value of x_0 is:

(A) $\frac{13}{2}\frac{\alpha E_0}{B_0}$ (B) $\frac{16\alpha B_0}{E_0}$

(C) $\frac{25}{2\alpha E_0}$ (D) $\frac{5\alpha}{2B_0}$

Q.5 A particle of specific charge (q/m) is projected from the origin of coordinates with initial velocity [ui-vj]. Uniform electric magnetic field exist in the region along the +y direction, of magnitude E and B. The particle will definitely return to the origin once if

(A) $\left[vB/2\pi E\right]$ is an integer

(B) $\left(u^2+v^2\right)^{1/2}\left[B/\pi E\right]$ is an integer

(C) $\left[vB/\pi E\right]$ in an integer

(D) $\left[uB/\pi E\right]$ is an integer.

Q.6 Two particles of charges +Q and –Q are projected from the same point with a velocity v in a region of uniform magnetic field B such that the velocity vector makes an angle θ with the magnetic field. Their masses are M and 2M, respectively. Then, they will meet again for the first time at a point whose distance from the point of projection is

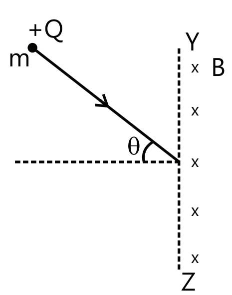
(A) $2\pi Mv\cos\theta/QB$ (B) $8\pi Mv\cos\theta/QB$

(C) $\pi Mv\cos\theta QB$ (D) $4\pi Mv\cos\theta/QB$

Q.7 A particle with charge +Q and mass m enters a magnetic field of magnitude B, existing only to the right of the boundary YZ. The direction of the motion of the particle is perpendicular to the direction of B. Let $T=\dfrac{2\pi M}{QB}$. The time spent by the particle in the field will be

(A) $T\theta$ (B) $2T\theta$

(C) $T\left(\dfrac{\pi+2\theta}{2\pi}\right)$ (D) $T\left(\dfrac{\pi-2\theta}{2\pi}\right)$

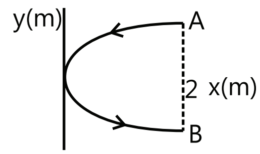


Q.8 In the previous question, if the particle has-Q charge, the time spend by the particle in the field will be

(A) $T\theta$ (B) $2T\theta$

(C) $T\left(\dfrac{\pi+2\theta}{2\pi}\right)$ (D) $T\left(\dfrac{\pi-2\theta}{2\pi}\right)$

Q.9 A conducting wire bent in the form of a parabola $y^2=2x$ carriers a current i=2A as shown in figure. This wire is placed in a uniform magnetic field $B=-4\hat{k}$ Tesla. The magnetic force on the wire is (in newton).

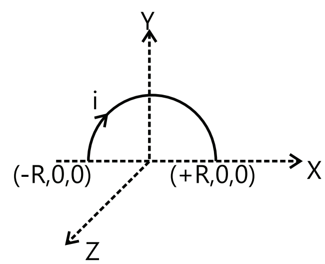


(A) $-16\hat{i}$ (B) $32\hat{i}$

(C) $-32\hat{i}$ (D) $16\hat{i}$

Q.10 A semicircular current carrying wire having radius R is placed in x-y plane with its center at origin 'O'. There is non-uniform magnetic field

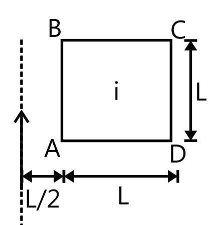
$\vec{B}=\dfrac{B_o x}{2R}\hat{k}$ (here B_o is +ve constant) is existing in the region. The magnetic force acting on semicircular wire will be along



(A) –x-axis (B) +y-axis

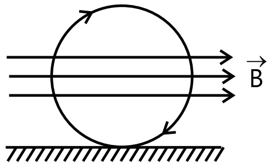
(C) –y-axis (D) +x-axis

Q.11 A square loop ABCD, carrying a current I, is placed near and coplanar with a long straight conductor XY carrying a current I, the net force on the loop will be



(A) $\dfrac{2\mu_0 Ii}{3\pi}$ (B) $\dfrac{\mu_0 Ii}{2\pi}$ (C) $\dfrac{2\mu_0 Iil}{3\pi}$ (D) $\dfrac{\mu_0 Iil}{2\pi}$

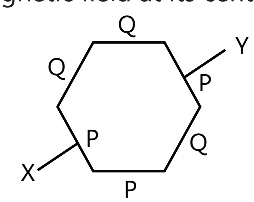
Q.12 A conducting ring of mass 2kg and radius 0.5m is placed on a smooth horizontal plane. The ring carries a current i=4A. A horizontal magnetic field B=10T is switched on at time t=0 as shown in figure. The initial angular acceleration of the ring will be

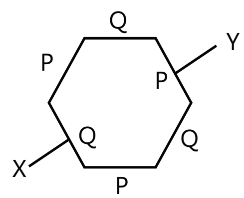


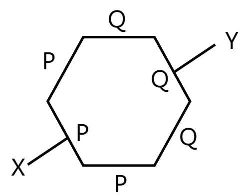
(A) $40\pi rad / s^2$ (B) $20\pi rad / s^2$

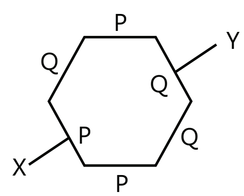
(C) $5\pi rad / s^2$ (D) $15\pi rad / s^2$

Q.13 In the following hexagons, made up of two different material P and Q, current enters and leaves from points X and Y respectively. In which case the magnetic field at its center is not zero.

(A)


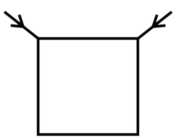
(B)


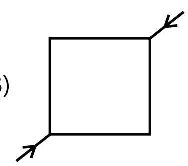
(C)


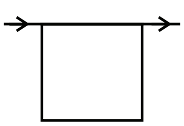
(D)


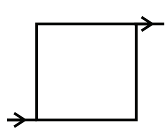
Q.14 Current flows through uniform, square frames as shown.

In which case is the magnetic field at the center of the frame not zero?

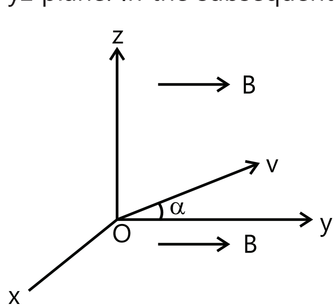
(A)


(B)


(C)


(D)


Q.15 In a region of space, a uniform magnetic field B exists in the y-direction. A proton is fired from the origin, with initial velocity v making a small angle α with the y-direction in the yz plane. In the subsequent motion of the proton,



(A) Its x-coordinate can never be positive

(B) Its x- and z-coordinates cannot both be zero at the same time.

(C) Its z-coordinate can never be negative.

(D) Its y-coordinate will be proportional to the square of its time of flight.

Multiple Correct Choice Type

Q.16 Which of the following statements is correct:

(A) A charged particle enters a region of uniform magnetic field at an angle 85^0 to magnetic lines of force. The path of the particle is a circle.

(B) An electron and proton are moving with the same kinetic energy along the same direction. When they pass through uniform magnetic field perpendicular to their direction of motion, they describe circular path.

(C) There is no change in the energy of a charged particle moving in a magnetic field although magnetic force acts on it.

(D) Two electrons enter with the same speed but in opposite direction in a uniform transverse magnetic field. Then the two describe circle of the same radius and these move in the same direction.

Q.17 Consider the magnetic field produced by a finitely long current carrying wire.

(A) The lines of field will be concentric circles with centers on the wire.

(B) There can be two points in the same plane where magnetic fields are same.

(C) There can be large number of points where the magnetic field is same.

(D) The magnetic field at a point is inversely proportional to the distance of the point from the wire.

Q.18 A long straight wire carriers a current along the x-axis. Consider the points A(0,1,0), B(0,1,1), C(1,0,1) and D(1,1,1). Which of the following pairs of points will have magnetic field of the same magnitude?

(A) A and B (B) A and C (C) B and C (D) B and D

Q.19 Consider three quantities x=E/B, $y = \sqrt{1/\mu_0\varepsilon_0}$ and $z = \frac{1}{CR}$. Here, I is the length of a wire, C is a capacitance and R is a resistance. All other symbols have standard meanings.

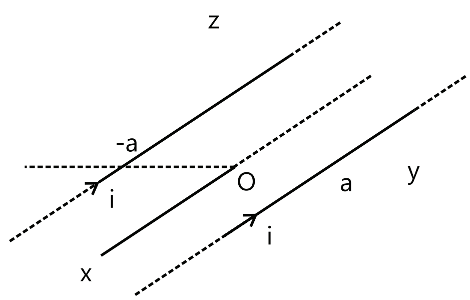
(A) x,y have the same dimensions

(B) y, z have the same dimension

(C) z, x have the same dimensions

(D) None of the three pairs have the same dimensions.

Q.20 Two long thin, parallel conductors carrying equal currents in the same direction are fixed parallel to the x-axis, one passing through y=a and the other through y=-a. The resultant magnetic field due to the two conductors at any point is B. Which of the following are correct?



(A) B=0 for all points on the x-axis

(B) At all points on the y-axis, excluding the origin, B has only a z-component.

(C) At all points on the z-axis, excluding the origin, B has only an x-component.

Q.21 An electron is moving along the positive X-axis. You want to apply a magnetic field for a short time so that the electron may reverse its direction and move parallel to the negative X-axis. This can be done by applying the magnetic field along.

(A) Y-axis (B) Z-axis

(C) Y-axis only (D) Z-axis only

Q.22 Two identical charged particles enter a uniform magnetic field with same speed but at angles 30^0 and 60^0 with field. Let a, b and c be the ratio of their time periods, radii and pitches of the helical paths then

(A) abc=1 (B) abc > 1

(C) abc < 1 (D) a=bc

Q.23 Consider the following statements regarding a charged particle in a magnetic field. Which of the statement are true :

(A) Starting with zero velocity, it accelerates in a direction perpendicular to the magnetic field.

(B) While deflecting in magnetic field its energy gradually increases.

(C) Only the component of magnetic field perpendicular to the direction of motion

of the charged particle is effective in deflecting it.

(D) Direction of deflecting force on the moving charged particle is perpendicular to its velocity.

Assertion Reasoning Type

(A) Statement-I is true, statement-II is true and Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Q.24 Statement-I: A charged particle can never move along a magnetic field line in absence of any other force.

Statement-II: Force due to magnetic field is given by $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$.

Q.25 Statement-I : It is not possible for a charged particle to move in a circular path around a long straight uncharged conductor carrying current under the influence of its magnetic field alone.

Statement-II: The magnetic force (if nonzero) on a moving charged particle is normal to its velocity.

Q.26 Statement-I: For a charged particle to pass through a uniform electro-magnetic field without change in velocity, its velocity vector must be perpendicular to the magnetic field.

Statement-II: Net Lorentz force on the particle is given by $F = q\left[E + \vec{v} \times B\right]$

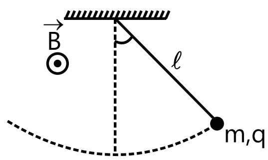
Q.27 Statement-I: Two long parallel conductors carrying current in the same direction experience a force of attraction.

Statement-II: The magnetic fields produced in the space between the conductors are in the same direction.

Q.28 Statement-I: Ampere law can be used to find magnetic field due to finite length of a straight current carrying wire.

Statement-II: The magnetic field due to finite length of a straight current carrying wire is symmetric about the wire.

Q.29 Statement-I: A pendulum made of a non-conducting rigid massless rod of length ℓ is attached to a small sphere of a mass m and charge q. The pendulum is undergoing oscillations of small amplitude having time period T. Now a uniform horizontal magnetic field out of plane of page is switched on. As a result of this change, the time period of oscillations will change.

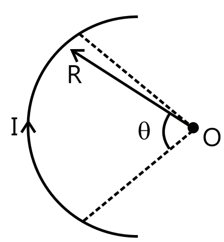


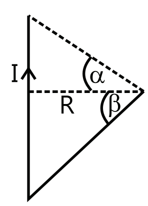
Statement-II: In the situation of statement-I, after the magnetic field is switched on the tension in string will change (except when the bob is at extreme position).

Comprehension Type

Paragraph 1: Magnetic field intensity (B) due to current carrying conductor can be calculated by use of Biot-Savart law. Which is

$B = \frac{\mu_0}{4\pi}\frac{Idl x r}{r^3}$,





where dB is magnetic field due current element Idl at a position r from current element. For straight wire carrying current magnetic field at a distance R from wire is

$B = \frac{\mu_0}{4\pi}\frac{I}{R}\left(\sin\alpha + \sin\beta\right)$

And magnetic field due to a circular arc at its center is

$B = \frac{\mu_0 I}{4\pi R}.\theta$

where θ angle of circular arc at center, R is radius of circular arc.

Q.30 The magnetic field at C due to curved part is

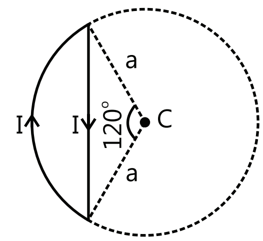
(A) $\frac{\mu_0 I}{6\alpha}$, directed into the plane of the paper

(B) $\frac{\mu_0 I}{6\alpha}$, directed towards you

(C) $\frac{\mu_0 I}{3\alpha}$, directed towards you

(D) $\frac{\mu_0 I}{3\alpha}$, directed up the plane of the paper

Q.31 A wire loop carrying a current I is shown in figure. The magnetic field induction at C due to straight part is



(A) $\frac{\sqrt{3}\mu_0 I}{2\pi\alpha}$, directed up the plane of the paper

(B) $\frac{\mu_0 I}{6\alpha}$, directed into the plane of the paper

(C) $\frac{\mu_0 I}{6\alpha}$, directed towards you

(D) $\frac{\mu_0 I}{2\alpha}\left(\frac{\sqrt{3}}{\pi} - \frac{1}{3}\right)$ towards you

Q.32 The net magnetic field at C due to the current carrying loop is directed into the plane of the paper

(A) Zero (B) $\frac{\mu_0 I}{\alpha}$

(C) $\frac{\mu_0 I}{9\alpha}$ (D) $B = -\frac{\mu_0 I}{6a} + \frac{\sqrt{3}\mu_0 I}{2\pi a}$,

Paragraph 2: A current carrying coil behave like short magnet whose magnetic dipole moment M=nIA. Where direction of M is taking along the direction of magnetic fields on its axis and n is no of turns A is area of coil and I is current flowing through coil. When such a coil is put in magnetic field (B) magnetic torque (τ) acts on it as $\tau = -MxB$ and potential energy of the current loop in the magnetic field is u=–M.B.

Q.33 A current of 3A is flowing in a plane circular coil of radius 1cm and having 20 turns. The coil is placed in a uniform magnetic field of 0.5 Wbm^{-2}. Then, the dipole moment of the coil is

(A) $3000 Am^2$ (B) $0.3 Am^2$

(C) $75 Am^2$ (D) $1.88 x 10^{-2} Am^2$

Q.34 A current of 3A is flowing in a plane circular coil of radius 1cm and having 20 turns. The coil is placed in a uniform magnetic field of 0.5 Wbm^{-2}. Then, the P.E. of the magnetic dipole in the position of stable equilibrium is

(A) -1500 J (B) -9.4 mJ

(C) +0.15 J (D) +1500 J

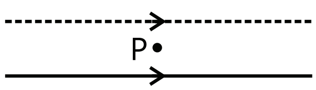
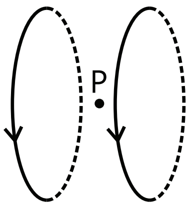
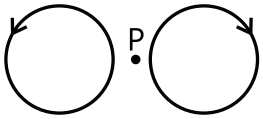
Q.35 In above question, to hold the current-carrying coil with the normal to its plane making an angle of 90^0 with the direction of magnetic induction, the necessary torque is

(A) 1500 Nm (B) 9.4×10^{-3} Nm

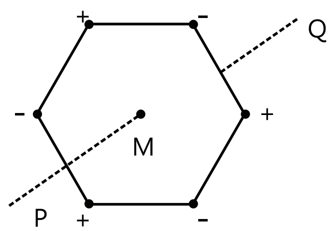
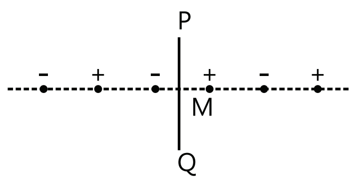
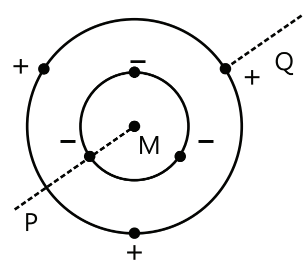
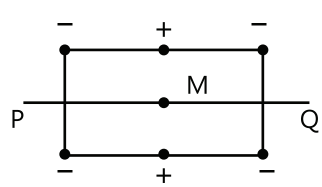
(C) 15 Nm (D) 150 Nm

Match the Column

Q.36 Two wires each carrying a steady current I are shown in four configuration in column I. Some of the resulting effects are described in column II. Match the statement in column I with the statements in column II and indicate your answer by darkening appropriate bubbles in the 4 x 4 matrix given in the ORS.

Column I	Image	Column II
(A) Point P is situated midway between the wires		(p) The magnetic fields (B) at P due to the currents in the wires are in the same direction.
(B) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii.		(q) The magnetic fields (B) at P due to the current in the wires are in opposite directions.
(C) Point P is situated at the mid-point of the line joining the centers of the circular wires, which have same radii.		(r) There is no magnetic field at P.

Q.37 Six point charges, each of the same magnitude q, are arranged in different manners as shown in column II. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M (potential at infinity is zero) due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and μ be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current.

Column-I	Image	Column-II
(A) $E=0$		Charges are at the corners of a regular hexagon. M is the center of the hexagon. PQ is perpendicular to the plane of the hexagon.
(B) $V \neq 0$		Charges are on a line perpendicular to PQ at equal intervals. M is the midpoint between the two innermost charges.
(C) $B=0$		Charges are placed at the corners of a rectangle of sides a and 2a and at the mid points of the longer sides. M is at the center of the rectangle. PQ is parallel to the longer sides.
(D) $\mu \neq 0$		Charges are placed at the corners of a rectangle of sides a and 2a and at the mid points of the longer sides. M is at the center of the rectangle. PQ is parallel to the longer sides.

Previous Years' Questions

Q.1 Statement I: The sensitivity of a moving coli galvanometer is increased by placing a suitable magnetic material as a core inside the coil. ***(2008)***

Statement II: Soft iron has a high magnetic permeability and cannot be easily magnetized or demagnetized.

(A) Statement-I is true, statement-II is true and Statement-II is correct explanation for Statement-I.

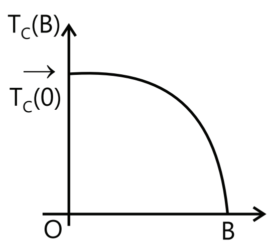
(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

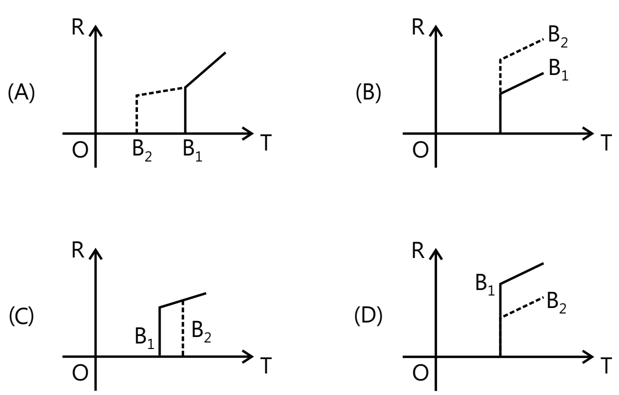
(D) Statement-I is false, statement-II is true.

Passage: (Q.2-Q.3)

Electrical resistance of certain material, known as superconductors, changes abruptly from a non-zero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in a magnetic field i.e., the critical temperature $T_C(B)$ is a function of the magnetic strength B. The dependence of $T_C(B)$ on B is shown in the Figure.



Q.2 In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields? ***(2010)***



Q.3 A superconductor has $T_C(0)=100K$. When a magnetic field of 7.5 Tesla is applied, its T_C decreases to 75K. For this material one can definitely say that when (Note: T=Tesla) ***(1987)***

(A) B=5T, $T_C(B)=80K$

(B) B=5T, $75K < T_C(B) < 100K$

(C) B=10T, $75K < T_C(B) < 100K$

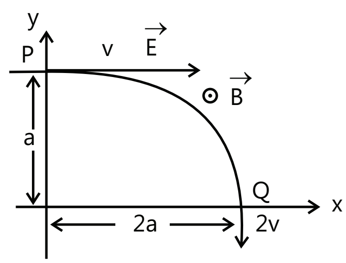
(D) B=10T, $T_C(B)=70K$

Q.4 A proton moving with a constant velocity passes through a region of space without any change in its velocity. If E and B represent the electric and magnetic fields respectively. Then, this region of space may have ***(1985)***

(A) E=0, B=0 (B) E=0, B≠0

(C) E≠0, B=0 (D) E≠0, B≠0

Q.5 A particle of charge +q and mass m moving under the influence of a uniform electric field $E\hat{i}$ and uniform magnetic field $B\hat{k}$ follows a trajectory from P to Q as shown in Figure. The velocities at P and Q are $v\hat{i}$ and $-2\hat{j}$. Which of the following statement (s) is/are correct? ***(1991)***



(A) $E=\frac{3}{4}\left[\frac{mv^2}{qa}\right]$

(B) Rate of work done by the electric field at P is $\frac{3}{4}\left[\frac{mv^2}{a}\right]$

(C) Rate of work done by the electric field at P is zero

(D) Rate of work done by both the fields at Q is zero

Q.6 H^+, He^+ and O^{2+} all having the same kinetic energy pass through a region in which there is a uniform magnetic field perpendicular to their velocity. The masses of H^+, He^+ and O^{2+} are 1 amu, 4 amu and 16 amu respectively. Then ***(1994)***

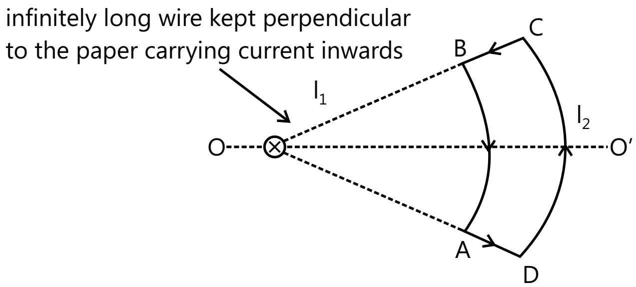
(A) H^+ will, be deflected most

(B) O^{2+} will be deflected most

(C) He^+ and O^{2+} will be deflected equally

(D) All will be deflected equally

Q.7 Which of the following statement is (are) correct in the given Figure? ***(2006)***



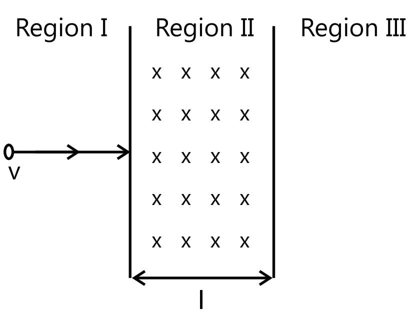
(A) Net force on the loop is zero.

(B) Net torque on the loop is zero.

(C) Loop will rotate clockwise about axis OO' when seen from O

(D) Loop will rotate anticlockwise about OO′ when seen from O

Q.8 A particle of mass m and charge q. moving with velocity v enters Region II normal to the boundary as shown in the Figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is l. Choose the correct choice (s). ***(2008)***



(A) The particle enters Region III only if its velocity$>\dfrac{qlB}{m}$.

(B) The particle enters Region III only if its velocity $v<\dfrac{qlB}{m}$.

(C)Path length of the particle in Region II is maximum when velocity $v=\dfrac{qlB}{m}$.

(D) Time spent in Region II is same for any velocity v as long as the particle returns to Region I.

Q.9 An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true? ***(2011)***

(A) They will never come out of the magnetic field region

(B) They will come out travelling along parallel axis

(C) They will come out at the same time

(D) They will come out at different times.

Q.10 Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E}=E_0\hat{j}$ and $\vec{B}=B_0\hat{j}$. At time t = 0, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is(are) correct for time t > 0 ? ***(2012)***

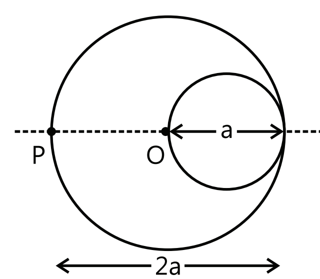
(A) If $\theta=0^o$, the charge moves in a circular path in the x-z plane.

(B) If $\theta=0^o$, the charge undergoes helical motion with constant pitch along the y-axis.

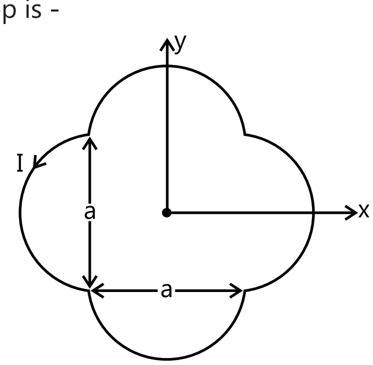
(C) If $\theta=10^o$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis

(D) $\theta=90^o$, the charge undergoes linear but accelerated motion along the y-axis.

Q.11 A cylindrical cavity of diameter a exists inside a cylinder of diameter 2a as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\dfrac{N}{12}\mu_0 aJ$, then the value of N is ***(2012)***



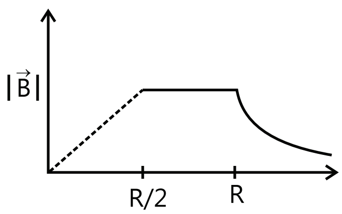
Q.12 A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is - ***(2012)***

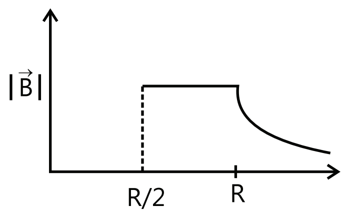


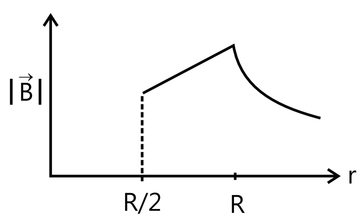
(A) $a^2I\,\hat{k}$ (B) $\left(\frac{\pi}{2}+1\right)a^2I\,\hat{k}$

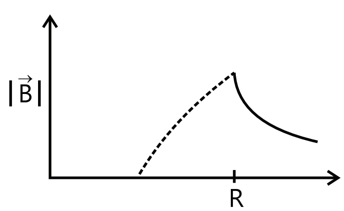
(C) $-\left(\frac{\pi}{2}+1\right)a^2I\,\hat{k}$ (D) $(2\pi+1)a^2I\,\hat{k}$

Q.13 An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by ***(2012)***

(A) 

(B) 

(C) 

(D) 

Q.14 A particle of mass M and positive charge Q, moving with a constant velocity $u_1 = 4\hat{i}\,ms^{-1}$, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from x = 0 to x = L for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\vec{u}_2 = 2(\sqrt{3}\hat{i}+\hat{j})m/s^{-1}$. The correct statement(s) is (are) ***(2013)***

(A) The direction of the magnetic field is –z direction.

(B) The direction of the magnetic field is +z direction.

(C) The magnitude of the magnetic field $\frac{50\pi M}{3Q}$ units.

(D) The magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units

Q.15 Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are) ***(2013)***

(A) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$

(B) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$

(C) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

(D) The energy of the mass m remains constant.

Q.16 Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2.

If $\frac{X_0}{X_1}=3$, the value of $\frac{R_1}{R_2}$ is ***(2014)***

Q.17 When d $\approx$ a but wires are not touching the loop, it is found that the net magnetic field on the axis of the loop is zero at a height h above the loop. In that case ***(2014)***

(A) Current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx a$

(B) Current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx a$

(C) Current in wire 1 and wire 2 is the direction PQ and SR, respectively and $h \approx 1.2a$

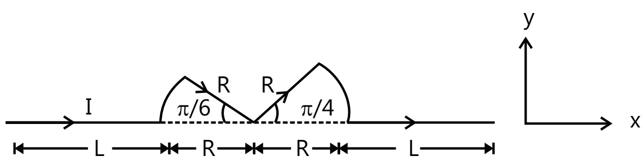
(D) Current in wire 1 and wire 2 is the direction PQ and RS, respectively and $h \approx 1.2a$

Q.18 Consider d >> a, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop) ***(2014)***

(A) $\frac{\mu_0 I^2 a^2}{d}$ (B) $\frac{\mu_0 I^2 a^2}{2d}$

(C) $\frac{\sqrt{3}\mu_0 I^2 a^2}{d}$ (D) $\frac{\sqrt{3}\mu_0 I^2 a^2}{2d}$

Q.19 A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is (are). **(2015)**



(A) If $\vec{B}$ is along $\hat{z}$, $F \propto (L+R)$

(B) If $\vec{B}$ is along $\hat{x}$, $F = 0$

(C) If $\vec{B}$ is along $\hat{y}$, $F \propto (L+R)$

(D) If $\vec{B}$ is along $\hat{z}$, $F = 0$

Q.20 Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is(are) ***(2015)***

(A) If $w_1 = w_2$ and $d_1 = 2d$, then $V_2 = 2V_1$

(B) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$

(C) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$

(D) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

Q.21 Consider two different metallic strips (1 and 2) of same dimensions (lengths ℓ, with w and thickness d) with carrier densities n1 and n2, respectively. Strip 1 is placed in magnetic field B1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is(are) ***(2015)***

(A) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

(B) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$

(C) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$

(D) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q. 7 Q.8 Q.12

Q.20 Q.25

Q.26 Q.27

Exercise 2

Q.2 Q.5 Q.20

JEE Advanced/Boards

Exercise 1

Q.4 Q.5 Q.16 Q.19

Q.22 Q.25 Q.30

Exercise 2

Q.1 Q.3 Q.11 Q.13

Q.15 Q.19 Q.20 Q.22

Q.40 Q.42 Q.44 Q.45

Q.48 Q.50

Answer Key

JEE Main/Boards

Exercise 1

Q.1 $\pi \times 10^{-4}\,T \approx 3.1 \times 10^{-4}\,T$

Q.2 $3.5 \times 10^{-5}\,T$

Q.3 $4 \times 10^{-6}\,T$, vertically up

Q.4 $1.2 \times 10^{-5}\,T$, towards south

Q.5 $0.6N\ m^{-1}$

Q.6 4.2cm

Q.7 (i) 3.1 Nm, (ii) No

Q.8 $5\pi \times 10^{-4}\,T = 1.6 \times 10^{-3}\,T$ towards west

Q.9 (a) A horizontal magnetic field to magnitude 0.26T normal to the conductor in such a direction that Fleming's left-hand rule gives a magnetic force upward. (b) 1.176N

Q.10 $1.22N\ m^{-1}$

Q.11 (a) 2.1 N vertically downwards (b) 2.1N vertically downwards (c) 1.68N vertically downwards

Q.12 2:1

Q.13 (a) Zero (b) Zero (c) Force on each electron in evB=IB(nA)$= 5 \times 10^{-25}$N.

Note: Answer (c) Denotes only the magnetic force.

Q.14 $B = \dfrac{\mu_0 I}{2R}$

Q.15 $B = \dfrac{\mu_0 I}{2\pi R}$

Q.16 $B = \mu_0 IN$ where N is the number of turns per unit length and I is the current flowing through the solenoid.

Q.17 2:1

Q.19 $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$

Q.20 Circle

Q.21 $4 \times 10^{-5}\,T$

Q.22 B=14.1 Wb

Q.23 $1.57 \times 10^{-2}\,T$

Q.24 3×10^{-12}

Q.25 $6 \times 10^{-7}\,Nm^{-1}$

Q.26 $5.9 \times 10^{-6}\,N\ m$

Q.27 22.5V

Q.28 (i) 8A (ii) $3 \times 10^{-7}\,T$ (iii) $7.68 \times 10^{-6}\,Nm^{-1}$

Exercise 2

Q.1 D	**Q.2** C	**Q.3** A	**Q.4** A	**Q.5** A	**Q.6** A
Q.7 B	**Q.8** D	**Q.9** D	**Q.10** B	**Q.11** A	**Q.12** B
Q.13 C	**Q.14** C	**Q.15** A	**Q.16** A	**Q.17** A	**Q.18** C
Q.19 D	**Q.20** B	**Q.21** B			

Previous Years' Questions

Q.1 D	**Q.2** C	**Q.3** B	**Q.4** D	**Q.5** C	**Q.6** B
Q.7 A	**Q.8** B	**Q.9** B	**Q.10** C	**Q.11** D	**Q.12** A
Q.13 C	**Q.14** A	**Q.15** B	**Q.16** A	**Q.17** A	**Q.18** B

JEE Advanced/Boards

Exercise 1

Q.1 In the plane of the drawing from right to left

Q.2 (a) 0 (b) $1.41\times10^{-6}\,T, 45°$ in xz plane, (c) $5\times10^{-6}\,T$, +x-direction

Q.3 $\frac{(1-2\sqrt{2})\mu_0 I}{\pi a}\hat{k}$

Q.4 zero

Q.5 $\frac{\mu_0 I}{4R}\left(\frac{3}{4}\hat{k}+\frac{1}{\pi}\hat{j}\right)$

Q.6 $\frac{\mu_0 i}{4\pi R}\left[\frac{3}{2}\pi+1\right]$

Q.7 $B=\frac{\mu_0 i}{4\pi R}\sqrt{2\left(2\pi^2-2\pi+1\right)}$

Q.8 $B_1=\frac{\mu_0 b r_1^2}{3}$, $B_2=\frac{\mu_0 b r^3}{3r_2}$

Q.9 $\frac{\mu_o iqv}{2\pi a}$

Q.10 $10\hat{k}$

Q.11 $\frac{2mv_0}{qB}$

Q.12 $\frac{mEI}{Be}$

Q.13 Zero

Q.14 $\sqrt{2}\,IRB\,\hat{j}$

Q.15 $F=\alpha a^2\hat{i}\hat{j}$

Q.16 (a) $\frac{3mv^2}{4qa}$, (b) $\frac{3mv^3}{4a}$, (c) zero

Q.17 $T_0=2\pi\sqrt{\frac{m}{6IB}}=0.57s$

Q.18 $\frac{\mu_0 II'c}{2\pi}\left[\frac{1}{a}-\frac{1}{b}\right]$ to the left

Q.19 (i) z=0, $x=\pm\frac{d}{\sqrt{3}}$ (ii) $\frac{I}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$

Q.20 $\frac{Q_\omega}{4}h^2\tan^2\theta$

Q.21 (i) $\frac{\mu_0}{4\pi}\left(\frac{4I}{a}\right)$ along Y-axis, (ii) $\frac{\mu_0}{4\pi}\left(\frac{I^2}{2a}\right)\sqrt{10}, \tan^4\left(\frac{1}{3}\right)+\pi$ with positive axis

Q.22(a) $\frac{Qv\mu_0 I}{m\,6a}\left(\frac{3\sqrt{3}}{\pi}-1\right)$ (b) $\vec{\tau}=BI\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)a^2\hat{j}$

Q.23 (a) $I=\frac{mg}{\pi r\left(B_x^2+B_y^2\right)^{1/2}}$ (b) $I=\frac{mg}{\pi r B_x}$

Q.24 $\frac{\mu_0 iJ_0}{\pi}\tan^{-1}\left(\frac{d}{2h}\right)\left(-\hat{k}\right)$

Q.25 0.62N<F<0.88N

Q.26 $B_{min}=4.7\times10^3\,T$

Q.27 $i_1=0.1110A, i_2=0.096A$

Q.28 $\frac{\mu_0 I_1 I_2}{4\pi}$ In (3) along –ve z direction

Q.29 (b) Yes

Q.30 $F=\frac{\mu_0 I^2}{2\pi}\text{In}\left(\frac{a^2}{L^2+a^2}\right)$, zero

Exercise 2

Single Correct Choice Type

Q.1 C **Q.2** A **Q.3** C **Q.4** C **Q.5** C **Q.6** D

Q.7 C **Q.8** D **Q.9** B **Q.10** A **Q.11** A **Q.12** A
Q.13 A **Q.14** C **Q.15** C

Multiple Correct Choice Type

Q.16 B, C **Q.17** A **Q.18** B, D **Q.19** A, B, C **Q.20** A, B, C, D **Q.21** A, B
Q.22 A, D **Q.23** C, D

Assertion Reasoning Type

Q.24 D **Q.25** B **Q.26** D **Q.27** C **Q.28** D **Q.29** D

Comprehension Type

Paragraph 1:

Q.30 A **Q.31** A **Q.32**D

Paragraph 2:

Q.33D **Q.34** B **Q.35** B

Matric Match Type or Match the Column

Q.36 A $\rightarrow$ q, r; B$\rightarrow$ p; C $\rightarrow$q, r; D $\rightarrow$q, or; A $\rightarrow$q, r; B$\rightarrow$ p; C $\rightarrow$q, r; D$\rightarrow$ q, s

Q.37 A $\rightarrow$p, r, s; B$\rightarrow$ r, s; C $\rightarrow$p, q; D$\rightarrow$ r,s

Previous Years' Questions

Q.1 C **Q.2** C **Q.3** B **Q.4** A, B, D **Q.5** A, B, D **Q.6** A, C
Q.7 A, C **Q.8** A, C, D **Q.9** B, D **Q.10** C, D **Q.11** 5 **Q.12** B
Q.13 D **Q.14** A, C **Q.15** B **Q.16** 3 **Q.17** C **Q.18** B
Q.19 A, B, C **Q.20** A, D **Q.21** A, C

Solutions

JEE Main/Boards

Exercise 1

Sol 1: $B = \dfrac{\mu_0 NI}{2R} = \dfrac{\mu_0 \times 100 \times 0.4}{2\times 8\times 10^{-2}} = 3.1 \times 10^{-4}T$

Sol 2: $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{\mu_0 \times 35}{2\pi \times \frac{1}{5}} = 3.5 \times 10^{-5}T$

Sol 3: $B = \dfrac{\mu_0 I}{2\pi r}\hat{k} = \dfrac{\mu_0 \times 50}{2\pi\times\frac{5}{2}}\hat{k} = 4 \times 10^{-6}T$,

vertically upward.

Sol 4: $B = \dfrac{\mu_0 \times 90}{2\pi\times\frac{3}{2}} = \dfrac{\mu_0}{\pi}\times 30 = 1.2 \times 10^{-5}T$,

towards south.

Sol 5: $F = I\ell \times B = 8 \times 1 \times 0.15 \times \dfrac{1}{2} = 0.6\ Nm^{-1}$

Sol 6: $F = \dfrac{\mu_0 I}{4\pi\sqrt{x^2+\frac{a^2}{4}}}\cdot\dfrac{a}{\sqrt{x^2+\frac{a^2}{2}}}$

Since force is always perpendicular to velocity so path will be a circle

$R = \dfrac{mv}{qB} = \dfrac{9.1\times 10^{-31}\times 4.8\times 10^{6}}{1.6\times 10^{-19}\times 6.5\times 10^{-4}} = 4.2\ cm$

Sol 7: (i) $\tau = M \times B$; $M = INA$

$\tau = 6 \times 30 \times \pi(0.08)^2 \times 1 \times \sin60^\circ = 3.1$ Nm

(ii) No

Sol 8: $B = \dfrac{-\mu_0 \times 20 \times 16}{2 \times \dfrac{16}{100}} + \dfrac{\mu_0 \times 25 \times 18}{2 \times \dfrac{10}{100}}$

$= 5\pi \times 10^{-4}$T toward west

Sol 9: For tension to be zero

(a) $F = I\ell \times B = mg$

$= 5 \times 0.45 \times B = \dfrac{\sqrt{2mKE}}{qB}$

$B = \dfrac{0.6}{5 \times 0.45} = 0.26$ T

(b) By force equilibrium

$2T = Mg + F$

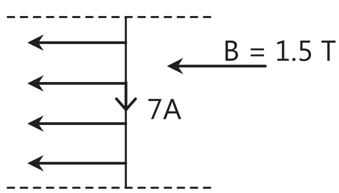
$= 0.06 \times 9.8 + 0.06 \times 9.8 = 1.176$ N

Sol 10: I = 300A; Force per unit length = F

$F = \dfrac{\mu_0 i_1 i_2}{2\pi d} = \dfrac{\mu_0 \times (300)^2}{2\pi \times \dfrac{3}{2} \times 10^{-2}} = 1.2\ \text{Nm}^{-1}$

Since the direction of the current is in opposite direction in the wire, the force will be repulsive in nature.

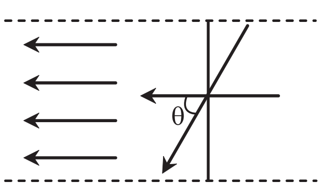
Sol 11: B = 1.5T; r = 0.1m



(a) $F = i\ \ell \times \vec{B}$

$= 7 \times 0.2 \times 1.5 = 2.1$ N vertically downwards.

(b)



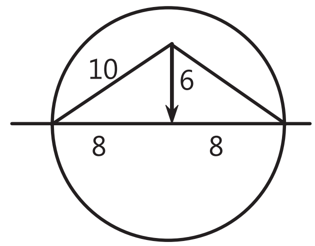
$F = i\ \ell_1 \times \vec{B}$

$= i\ell_1\vec{B}\sin\theta$

$= i\ B \times \left(\dfrac{20}{100}\right)$

$= 7 \times 0.2 \times 1.5 = 2.1$ N vertically downwards.

(c)



Effective length of wire is 16 cmin the magnetic field so

$F = i\ell \times \vec{B}$

$= 7 \times 0.16 \times 1.5$

$= 1.68$ N downwards

Sol 12: Length of wire = $N \times 2\pi R$

Final no. of turns = $\dfrac{N \times 2\pi R}{2\pi \dfrac{R}{2}} = 2N$

Magnetic moment $\mu = INA$

$\dfrac{\mu_1}{\mu_2} = \dfrac{N_1A_1}{N_2A_2} = \dfrac{N \times \pi R^2}{2N \times \pi\left(\dfrac{R}{2}\right)^2} = \dfrac{2}{1}$

Sol 13: N = 20

r = 0.1 m

B = 0.1 T

I = 5 A

(a) $\tau = M \times \vec{B} = MB\sin0^\circ = 0$

(b) $F = i\ell \times \vec{B}$

F = Total force is zero as $\vec{\ell}$ is zero for a closed loop

(c) Force on each electron = $q\vec{v} \times \vec{B}$

$= eVB = \dfrac{IB}{nA} = 5 \times 10^{-25}$ N

Sol 14: Refer page 21.11 to 21.14

Sol 15: Refer page 21.11 and 21.12

Sol 16: B at the centre is $B = \mu_0 NI$

N - number of turns

I - current

Sol 17: $R = \dfrac{mv}{qB}$

$v = \dfrac{RqB}{m}$

$\dfrac{v_p}{v_d} = \dfrac{RqB}{m_p}\dfrac{m_d}{RqB} = 2$

Sol 18: Refer page 21.25 to 21.26

Sol 19: $F = q\vec{E} + q\vec{v} \times \vec{B}$

Sol 20: Its path will be a circle.

Sol 21: Magnetic field due to side BC is B_{BC}

$B_{BC} = \dfrac{\mu_0 I}{4\pi R}\left(\dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{3}}{2}\right)$; $B_{BC} = \dfrac{3\sqrt{3}\mu_0 I}{4\pi R}$

Magnetic field due to all sides will be equal so

$B_{net} = \dfrac{3\sqrt{3}\mu_0 I}{4\pi R}$

Sol 22: Electron moving in a circle will act like a loop carrying current I.

So, $I = \dfrac{q}{t} = \dfrac{q\omega}{2\pi} = \dfrac{qv}{2\pi R}$

So magnetic field at centre $= B = \dfrac{\mu_0 I}{2R} = \dfrac{\mu_0 qv}{4\pi R^2}$

Thus $B = \dfrac{9\times 10^9 \times 1.6\times 10^{-19} \times 2.2\times 10^6}{\left(0.5\times 10^{-10}\right)^2} = 14.1$ Wb

Sol 23: $B = \mu_0 ni$

$= 4\pi \times 10^{-7} \times \dfrac{5000}{2} \times 5 = 1.57 \times 10^{-2}$T

Sol 24: B = 2.5 T

$v = 1.5 \times 10^7$ m/s

$F = q\vec{v} \times \vec{B} = q \times 1.5 \times 10^7 \times 2.5 \times \dfrac{1}{2}$

$= 1.6 \times 10^{-19} \times 1.5 \times 10^7 \times \dfrac{2.5}{2} \times 3 \times 10^{-12}$

Sol 25: Force per unit length

$F = \dfrac{\mu_0 i^2}{2\pi d} = 6 \times 10^{-7}$ N/m

Sol 26: Magnetic field inside the solenoid is

$B = \mu_0 NI = \mu_0 \times \dfrac{400}{0.4} \times 3$

$= 4\pi \times 10^{-7} \times 3 \times 1000 = 12\pi \times 10^{-4}$ T

Torque on the coil is $\tau = M \times \vec{B} = MB$

$= 0.4 \times 10 \times \pi (0.01)^2 \times 12\pi \times 10^{-4} = 5.9 \times 10^{-6}$ Nm

Sol 27: Let the resistance of the voltmeter be R

Voltage across $300\,\Omega$ = 60 – 30 V= 30 V

$I = \dfrac{30}{300} = 0.1$ A

Let equivalent resistance of voltmeter and 400Ω be R_{eq}

$IR_{eq} = 30$V

$0.10\ R_{eq} = 30$ V

$R_{eq} = 300\,\Omega = \dfrac{R \times 400}{R + 400}$

3R + 1200 = 4R

Resistance of voltmeter = R = $1200\,\Omega$

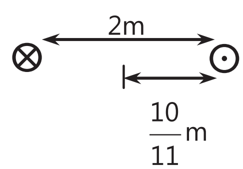
When voltmeter is connected to $300\,\Omega$

$R_{eq} = \dfrac{1200 \times 300}{1500} = 240\,\Omega$

$i = \dfrac{60}{640}A = \dfrac{6}{64}A$

Voltage measured $= \dfrac{6}{64} \times 240 = 22.5$ V

Sol 28: (i) i = 9.6A



B at $\dfrac{10}{11}$ m from wire B is

$B = \dfrac{\mu_0 \times 9.6}{2\pi\left(\dfrac{12}{11}\right)} - \dfrac{\mu_0 I}{2\pi \times \dfrac{10}{11}} = 0$

$I = \dfrac{9.6 \times 10}{12} = 8A$

(ii) Force per unit length $F = \dfrac{\mu_0 i_1 i_2}{2\pi \times 2}$

$= \dfrac{\mu_0 \times 9.6 \times 8}{4\pi} = 7.68 \times 10^{-6}\ Nm^{-1}$

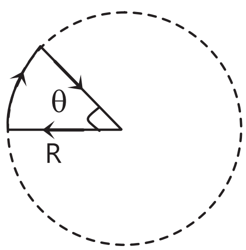
Exercise 2

Sol 1: (D) Total magnetic field at point O

$B = \dfrac{\mu_0 \ell}{2R'}\dfrac{3}{4} + \dfrac{\mu_0 \ell}{2R}\dfrac{1}{4}$

$= \dfrac{\mu_0 \ell}{8}\left[\dfrac{3}{R'} + \dfrac{1}{R}\right]$

Sol 2: (C)



Magnetic field $B = \dfrac{\mu_0 I}{2R}\left(\dfrac{\theta}{2\pi}\right)(-\hat{k})$

Sol 3: (A) $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\,\vec{V} \times \vec{r}}{r^3} = \dfrac{\mu_0}{4\pi}\dfrac{qV\sin\theta}{r^2}$

$= \dfrac{\mu_0}{4\pi} \times \dfrac{2 \times 100 \times \sin 30^o}{4} = 10^{-7} \times 25 = 2.5\mu T$

Sol 4: (A) Magnetic field at the common centre is

$\dfrac{\mu_0 I}{2R}\hat{i} + \dfrac{\mu_0 I}{2R}\hat{j} + \dfrac{\mu_0 I}{2R}\hat{k}$

$= \dfrac{\mu_0 I}{2R}\sqrt{3}$

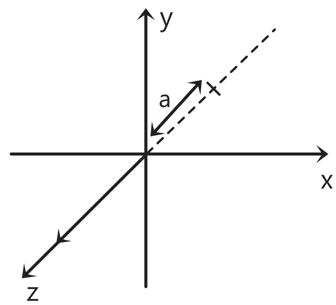
Sol 5: (A) Magnitude of magnetic field at the centre

$= +\dfrac{\mu_0 \times 20 \times 16}{2 \times 16 \times 10^{-2}} - \dfrac{\mu_0 \times 25 \times 18}{2 \times 10 \times 10^{-2}}$

$= -\mu_0 \times 10^3 + \mu_0 \times 2250$

$= \mu_0 \times 1250 = \dfrac{\mu_0}{4\pi} \times 5000\pi = 5\pi \times 10^{-4}\,T$

Sol 6: (A)



Magnetic field $= B = -\dfrac{\mu_0 i}{2\pi a}\hat{i} + \dfrac{\mu_0 i}{2\pi a}\hat{j}$

Sol 7: (B) $\vec{E} = -K_1\hat{j}$;

K_1 is some constant

$\vec{V} = K_2\hat{i}$

$F = q\vec{V} \times \vec{B} + q\vec{E} = 0$

$\Rightarrow \vec{V} \times \vec{B} = -\vec{E} \Rightarrow \vec{B} = -\hat{k}$

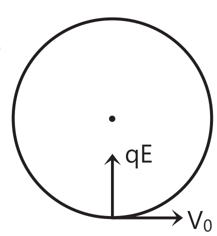
Sol 8: (D) Final velocity of the particle

$= v = \sqrt{v_0^2 + \left(\dfrac{qEt}{m}\right)^2} = 2v_0$

$v_0^2 + \left(\dfrac{qEt}{m}\right)^2 = 4v_0^2$

$\left(\dfrac{qEt}{m}\right)^2 = 3v_0^2 \Rightarrow t = \dfrac{\sqrt{3}mv_0}{qE}$

Sol 9: (D)



When electric field is applied

$\dfrac{mv_0^2}{R_1} = qE$

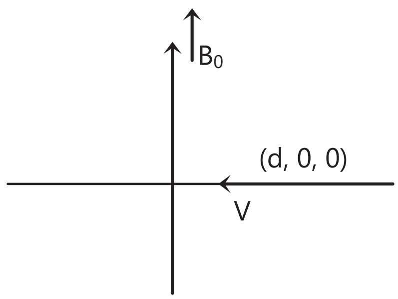
$R_1 = \dfrac{mv_0^2}{qE}$

When magnetic field is applied

$$R_2 = \frac{mv_0}{qB}$$

$$\frac{R_1}{R_2} = \frac{mv_0^2\, qB}{qEmv_0} = \frac{v_0\, B}{E}$$

Sol 10: (B)



For the particle to not hit y-z plane radius of the particle should be less than equal to d

$$R = \frac{mv}{qB_0} \le d$$

$$v_{max} = \frac{qB_0 d}{m}$$

Sol 11: (A) Electric force Fe$= \frac{kq_1q_2}{r^2} = \frac{kq^2}{r^2}$

$$\text{Magnetic force} = qv\left(\frac{\mu_0}{4\pi}\times\frac{qv}{r^2}\right)$$

$$F_m = q^2v^2\left(\frac{\mu_0}{4\pi}\right)\frac{1}{r^2}$$

$$\frac{Fe}{F_m} = \frac{k}{v^2\left(\frac{\mu_0}{4\pi}\right)} = \frac{1}{v^2\varepsilon_0\mu_0} = \frac{c^2}{v^2}$$

Sol 12: (B) $R = \frac{mv}{qB} = \frac{\sqrt{2mKE}}{qB}$

$$R_{H^+} = \sqrt{\frac{2KE\times 1\times m_p}{eB}} \quad (m_p = \text{mass of proton})$$

$$R_{He^+} = \sqrt{\frac{2KE\times 4\times m_p}{eB}}$$

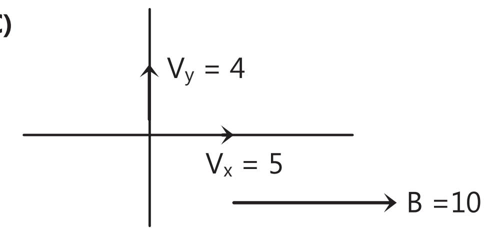
$$R_{O^{+2}} = \sqrt{\frac{2KE\times 16\times m_p}{2eB}}$$

So, $R_{He^+} = R_{O^{+2}}$

Sol 13: (C) $R = \frac{mv}{qB} = \frac{\sqrt{2mKE}}{qB}$

$$R' = \frac{\sqrt{2m(2KE)}}{q(3B)} = R\sqrt{\frac{2}{9}}$$

Sol 14: (C)



Y-component of velocity will make the particle to move in circle whereas x-component of velocity will make particle move along x-axis.

So motion is helical.

Sol 15: (A) Force on a particle moving in magnetic field is $q\vec{v}\times\vec{B}$.

$(4\hat{i}+3\hat{j})\times 10^{-13} = 1.6\times 10^{-19}\times 2.5\times 10^{7}\,(\vec{K}\times\vec{B})$

Force will be zero if direction of magnetic field and velocity is same.

So $\vec{B} = (0.6\,\hat{i}\times -0.8\,\hat{j})\,B$

$\Rightarrow (4\,\hat{i} + 3\,\hat{j}) = 1.6\times 25\,(\hat{k}\times(0.6\,\hat{i} - 0.8\,\hat{j})\,B$

$\Rightarrow \vec{B} = -0.075\,\hat{i} + 0.1\,\hat{j}$

Sol 16: (A) Force acting on particle $= q.\vec{v}\times\vec{B}$

$\Rightarrow q.2\,\hat{i}\times\vec{B} = -2\,\hat{j}$

$\Rightarrow \vec{B}$ is in +ve z direction ($\hat{k}$)

Electric force on the particle is zero.

So when $v_3 = 2\hat{k}$, force is zero.

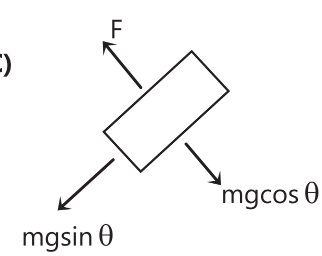
Sol 17: (A) Magnetic field is in ($-\hat{k}$) direction

So direction of force

$\vec{F} = q\,\vec{v}\times\vec{B}$

$\hat{F} = -[-\,\hat{i}\times(-\hat{k})] = \hat{j}$

Sol 18: (C)



$F = qVB$

Particle will leave the inclined plane when

$F = mg\cos\theta \Rightarrow qvB = mg\cos\theta$

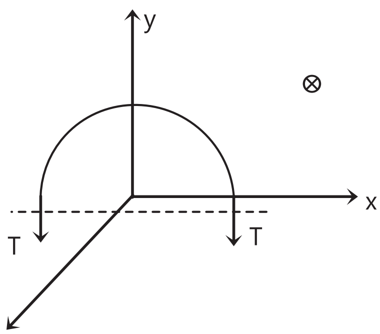
$$v = \frac{mg\cos\theta}{qB}$$

Time taken to reach v is t

$v = g\sin\theta\, t$

$$t = \frac{v}{g\sin\theta} = \frac{mg\cot\theta}{qgB} = \frac{m\cot\theta}{qB}$$

Sol 19: (D)



$F = I\int d\vec{\ell} \times \vec{B}$

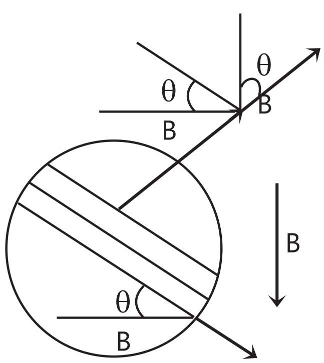
$= I\,(2r\hat{i}) \times (-0.2\hat{k}) = 20\,\hat{j}$

Magnetic force is in +ve y direction

So balancing force on semi-circular ring we get

$2T = 20 \Rightarrow T = 10N$

Sol 20: (B)



Torque due to magnetic field will be balanced by gravity.

$mg\sin\theta\, R = I \times \pi R^2 \times B\sin\theta$

$$B = \frac{mg}{\pi i R}$$

Sol 21: (B) Magnetic field = I × A

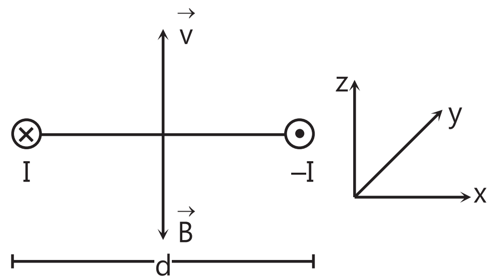
$$M = \frac{q.\pi r^2}{t}$$

$$t = \frac{2\pi}{\omega} = \frac{2\pi r}{v}$$

$$M = \frac{q.\pi r^2 v}{2\pi r} = \frac{qrv}{2}$$

Previous Years' Questions

Sol 1: (D) Net magnetic field due to both the wires will be downward as shown in the figure.



Since, angle between $\vec{v}$ and $\vec{B}$ is 180°.

Therefore, magnetic force

$\vec{F}_m = q(\vec{v} \times \vec{B}) = 0$

Sol 2: (C) H_1 = Magnetic field at M Due to PQ + magnetic field at M due to QR

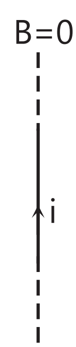
But magnetic field at M due to QR = 0

∴ Magnetic field at M due to PQ (or due to current I in PQ)= H_1

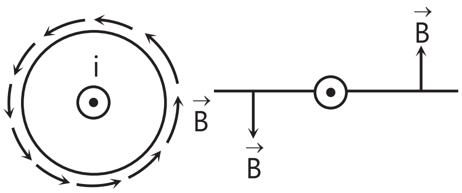
Now H_2 = Magnetic field at M due to PQ (current I) + magnetic field at M due to QS (current I/2) + magnetic field at M due to QR

$= H_1 + \frac{H_1}{2} + 0 = \frac{3}{2}H_1\,;\ \frac{H_1}{H_2} = \frac{2}{3}$

Note: Magnetic field at any point lying on the current carrying straight conductor is zero.



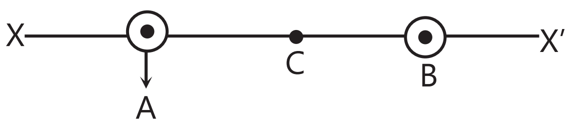
Sol 3: (B) If the current flows out of the paper, the magnetic field at points to the right of the wire will be upwards and to the left will be downwards as shown in figure.



Now, let us come to the problem.

Magnetic field at C = 0

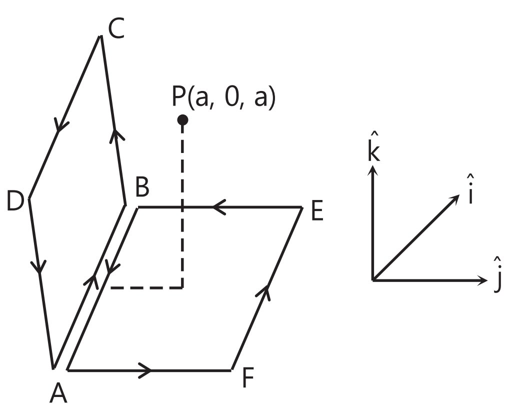
Magnetic field in region BX′ will be upwards (+ve) because all points lying in this region are to the right of both the wires.



Magnetic field in region AC will be upwards (+ve),because points are closer to A, compared to B. Similarly magnetic field in region BC will be downwards (–ve).

Graph (B) satisfies all these conditions. Therefore, correct answer is (B).

Sol 4: (D) The magnetic field at P(a, 0, a) due to the loop is equal to the vector sum of the magnetic fields produced by loops ABCDA and AFEBA as shown in the figure.



Magnetic field due to loop ABCDA will be along $\hat{i}$ and due to loop AFE BA, along $\hat{k}$. Magnitude of magnetic field due to both the loops will be equal. Therefore, direction of resultant magnetic field at P will be $\frac{1}{\sqrt{2}}$ $(\hat{i}+\hat{k})$.

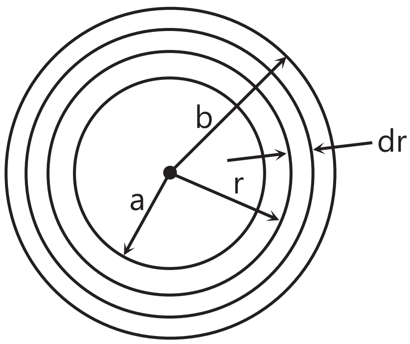
Note: This is a common practice, when by assuming equal currents in opposite directions in an imaginary wire (here AB) loops are completed and solution becomes easy.

Sol 5: (C) Consider an element of thickness dr at a distance r from the centre. The number of turns in this element,$dN = \left(\frac{N}{b-a}\right)dr$

Magnetic field due to this element at the centre of the coil will be

$$dB = \frac{\mu_0(dN)I}{2r} = \frac{\mu_0 I}{2}\frac{N}{b-a}\cdot\frac{dr}{r}$$

$$\therefore B = \int_{r=a}^{r=b} dB = \frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$$



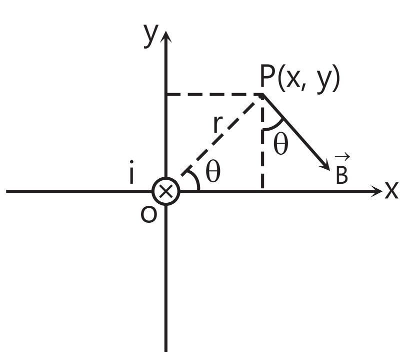
Note: The idea of this question is taken from question number 3.245 of IE Irodov.

Sol 6: (B) Radius of the circle = $\frac{mv}{Bq}$

or radius ∝ mv if B and q are same.

$(\text{Radius})_A > (\text{Radius})_B$; $\therefore m_A v_A > m_B v_B$

Sol 7: (A) Magnetic field at P is $\vec{B}$, perpendicular to OP in the direction shown in figure.



So, $\vec{B} = B\sin\theta\,\hat{i} - B\cos\theta\,\hat{j}$

Here, $B = \dfrac{\mu_0 I}{2\pi r}$

$\sin\theta = \dfrac{y}{r}$ and $\cos\theta = \dfrac{x}{r}$

$\therefore \vec{B} = \dfrac{\mu_0 I}{2\pi}\cdot\dfrac{1}{r^2}\,(y\hat{i} - x\hat{j}) = \dfrac{\mu_0 I(y\hat{i} - x\hat{j})}{2\pi(x^2+y^2)}$

(as $r^2 = x^2 + y^2$)

Sol 8: (B) If $(b - a) \geq r$

(r = radius of circular path of particle)

The particle cannot enter the region $x > b$.

So, to enter in the region $x > b$

$r > (b - a)$ or $\dfrac{mv}{Bq} > (b - a)$ or $v > \dfrac{q(b-a)B}{m}$

Sol 9: (B) Electric field can deviate the path of the particle in the shown direction only when it is along negative y-direction. In the given options $\vec{E}$ is either zero or along x-direction. Hence, it is the magnetic field which is really responsible for its curved path. Options (a) and (c) cannot be accepted as the path will be circular in that case. Option (d) is wrong because in that case component of net force on the particle also comes in $\hat{k}$ direction which is not acceptable as the particle is moving in x-y plane. Only in option (b) the particle can move in x-y plane.

In option (d) $\vec{F}_{net} = q\vec{E} + q(\vec{v} \times \vec{B})$

Initial velocity is along x-direction. So, let

$\vec{v} = v\hat{i}$

$\vec{F}_{net} = qa\hat{i} + q\,[(v\hat{i}) \times (c\hat{k} + b\hat{j})]$

$= qa\hat{i} - qvc\hat{j} + qvb\hat{k}$

In option (b) $\vec{F}_{net} = q\,(a\hat{i}) + q[(v\hat{i}) \times$

$(c\hat{k} + a\hat{i})] = qa\hat{i} - qvc\hat{j}$

Sol 10: (C) $\vec{U} = -\vec{M}\vec{B} = -MB\cos q$

Here, $\vec{M}$ = magnetic moment of the loop

θ = angle between $\vec{M}$ and $\vec{B}$

U is maximum when $\theta = 180°$ and minimum when $\theta = 0°$. So, as θ decreases from 180° to 0° its PE also decrease.

Sol 11: (D) Magnetic force does not change the speed of charged particle. Hence, $v = u$. Further magnetic field on the electron in the given condition is along negative y-axis in the starting. Or it describes a circular path in clockwise direction. Hence, when it exits from the field, $y < 0$.

Therefore, the correct option is (D)

Sol 12: (A) $\vec{F}_m = q\,(\vec{v} \times \vec{B})$

$\therefore$ Correct option is (A)

Sol 13: (C) Correct answer is (C), because induced electric field lines (produced by change in magnetic field) and magnetic field lines form closed loops.

Sol 14: (A) If we take a small strip of dr at distance r from centre, then number of turns in this strip would be, $dN = \left(\dfrac{N}{b-a}\right)dr$

Magnetic field due to this element at the centre of the coil will be

$dB = \dfrac{\mu_0 (dN) I}{2r} = \dfrac{\mu_0 NI}{(b-a)}\dfrac{dr}{r}$

$\therefore B = \displaystyle\int_{r=a}^{r=b} dB = \dfrac{\mu_0 NI}{2(b-a)}\ln\left(\dfrac{b}{a}\right)$

Sol 15: (B)

$r = \dfrac{\sqrt{2mK}}{Bq} \Rightarrow r \propto \dfrac{\sqrt{m}}{q}$

$r_\alpha = r_p < r_d$

Sol 16: (A) $B_{net} = B_{M_1} + B_{M_2} + B_H$

$= \dfrac{\mu_0 M_1}{4\pi x^3} + \dfrac{\mu_0 M_2}{4\pi x^3} + B_H$

$= \dfrac{\mu_0}{4\pi x^3}(M_1 + M_2) + B_H$

$= \dfrac{10^{-7}}{10^{-3}} \times 2.2 + 3.6 \times 10^{-5}$

$= 2.56 \times 10^{-4}\ \text{Wb}/\text{m}^2$

Sol 17: (A) $\mu_0 H = \mu_0 ni$

$$3\times10^3 = \frac{100}{0.1}\times i \Rightarrow i = 3A$$

Sol 18: (B) Since $\vec{B}$ is uniform, only torque acts on a current carrying loop. $\vec{\tau} = (I\vec{A})\times\vec{B}$

$\vec{A} = A\hat{k}$ for (b) and $\vec{A} = -A\hat{k}$ for (d).

$\therefore\ \ \vec{\tau} = 0$ for both these cases.

The energy of the loop in the $\vec{B}$ field is: $U = -I\vec{A}\cdot\vec{B}$, which is minimum for (b).

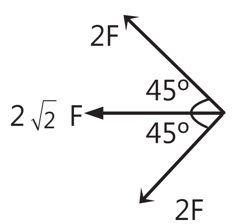
JEE Advanced /Boards

Exercise 1

Sol 1: $I_1 = I_2 = I_3 = I_4$

$\Rightarrow\ F_1 = F_2 = F_3 = F_4 = F$

$\Rightarrow\ 2F$



Resultant force will be $2\sqrt{2}$ F from right to left

Sol 2: Let magnetic field due to wire be B_w

(a)x = 0, z = 2m;

$$B = B_0 + B_w = -\frac{\mu_0 I\hat{i}}{2\pi\times2} + 10^{-6}\hat{i}$$

$$= -10^{-7}\times10\,\hat{i} + 10^{-6}\hat{i}$$

$$= 0$$

(b)x = 2m, z = 0

$$B = B_0 + B_w = \frac{\mu_0 I}{2\pi\times2}\hat{k} + 10^{-6}\hat{i}$$

$$B = 10^{-6}\hat{k} + 10^{-6}\hat{i} = \sqrt{2}\times10^{-6}\text{ T}$$

(c)x=0, z=–0.5m

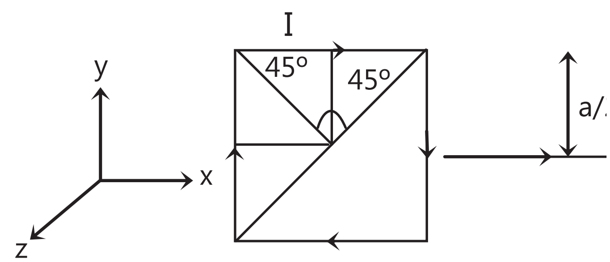
$$B = B_0 + B_w$$

$$= 10^{-6}\hat{i} + \frac{\mu_0\times10}{2\pi\times\frac{1}{2}}$$

$$= 10^{-6}\hat{i} + 4\times10^{-7}\times10\,\hat{i}$$

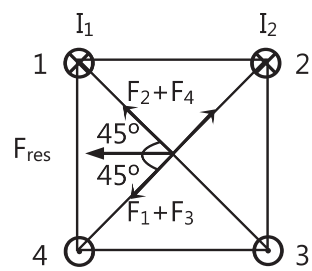
$$= 5\times10^{-6}\hat{i}\text{ T}$$

Sol 3: Magnetic field can be found as the super position of both given below.



Magnetic field due to loop = B_l

$$= -\left[\frac{\mu_0 I}{4\pi\left(\frac{a}{2}\right)}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)\right]\times4\hat{k}$$



$$= -\frac{\mu_0 I}{2\pi a}\times\sqrt{2}\times4\hat{k}$$

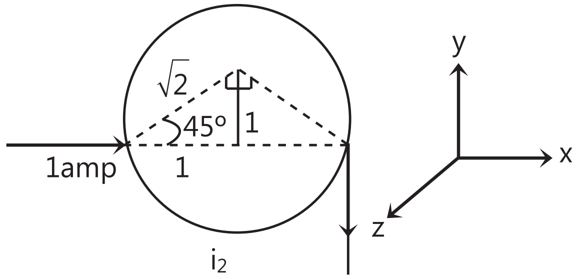
$$= -\frac{2\sqrt{2}\mu_0 I}{\pi a}\hat{k}$$

Magnetic field due to infinite length wire = $B_w = \dfrac{\mu_0 I\hat{k}}{2\pi\left(\frac{a}{2}\right)}$

$$= \frac{\mu_0 I}{\pi a}\hat{k}$$

Net magnetic field = $\dfrac{(1-2\sqrt{2})\mu_0 I}{\pi a}\hat{k}$

Sol 4:



$$i_1 = \frac{\frac{\pi}{2}}{2\pi}\times 1 = \frac{1}{4}\text{amp}$$

$$i_2 = 1-\frac{1}{4} = \frac{3}{4}\ \text{amp}$$

Magnetic field due to $i_1 = B_1 = -\dfrac{\mu_0\left(\frac{1}{4}\right)}{2\sqrt{2}}\left(\dfrac{\frac{3\pi}{2}}{2\pi}\right)\hat{k}$

$$= -\frac{\mu_0}{8\sqrt{2}}\times\frac{3}{4}\hat{k}$$

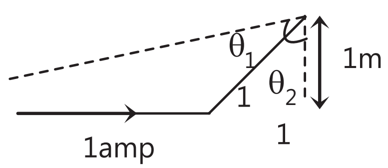
Magnetic field due to $i_2 = B_2 = \dfrac{\mu_0\left(\frac{3}{4}\right)}{2\sqrt{2}}\dfrac{\left(\frac{\pi}{2}\right)}{2\pi}\hat{k}$

$$= \frac{3\mu_0}{8\sqrt{2}}\times\frac{1}{4}\hat{k}$$

Magnetic field due to wire in x-direction = B_3

$$B_3 = \frac{\mu_0\times 1}{4\pi\times 1}(\sin(-45^\circ)+\sin 90^\circ)\,\hat{k}$$

$$B_3 = \frac{\mu_0}{4\pi}\left(1-\frac{1}{\sqrt{2}}\right)\hat{k}$$



Magnetic field due to wire in negative y-direction $=B_y$

$$B_y = -\left(\frac{\mu_0\times 1}{4\pi\times 1}(\sin(-45^\circ)+\sin 90^\circ)\right)\ \hat{k}$$

$$= -\frac{\mu_0}{4\pi}\left(1-\frac{1}{\sqrt{2}}\right)\hat{k}$$

Net magnetic field = $B = B_1 + B_2 + B_3 + B_4 = 0$

Sol 5: Magnetic Induction

$$\vec{B} = \frac{\mu_0 I}{2(2R)}\left(\frac{1}{4}\right)\hat{k} + \frac{\mu_0 I}{2R}\left(\frac{1}{4}\right)\hat{k} + \frac{\mu_0 I}{4\pi R}\hat{j}$$

$$= \frac{\mu_0 I}{4R}\left[\frac{3}{4}\hat{k}+\frac{1}{\pi}\hat{j}\right]$$

Sol 6: Magnetic Induction

$$\vec{B} = \frac{\mu_0 I}{2R}\left(\frac{3\frac{\pi}{2}}{2\pi}\right)\hat{k} + \frac{\mu_0 I}{4\pi R}\hat{k}$$

$$= \frac{\mu_0 I}{2R}\times\frac{3}{4}\hat{k} + \frac{\mu_0 I}{4\pi R}\hat{k} \ = \frac{\mu_0 I}{4\pi R}\left[\frac{3\pi}{2}+1\right]\hat{k}$$

Sol 7: Magnetic Induction

$$\vec{B} = \frac{\mu_0 I}{2R}\hat{i} - \frac{\mu_0 I}{4\pi R}\hat{i} - \frac{\mu_0 I}{4\pi R}\hat{k}$$

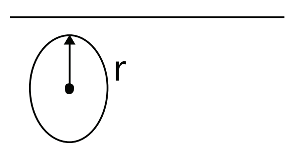
$$= \frac{\mu_0 I}{4\pi R}[2\pi - 1]\,\hat{i} - \frac{\mu_0 I}{4\pi R}\hat{k}$$

$$= \frac{\mu_0 I}{4\pi R}\sqrt{\left[4\pi^2+1-4\pi+1\right]}$$

$$= \frac{\mu_0 I}{4\pi R}\sqrt{2\left(2\pi^2-2\pi+1\right)}$$

Sol 8: We will find magnetic field B by ampere's law.

$$\oint \vec{B}.\vec{dl} = \mu_0 I_{IN}$$



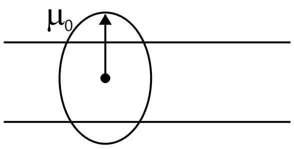
(a)For $r_1 < R$

$$B\times 2\pi r_1 = \mu_0\left(\int J dA\right)$$

$$=\mu_0\left(\int_0^{r_1} br 2\pi r\, dr\right)$$

$$B\times 2\pi r_1 = \mu_0\frac{2\pi b r_1^3}{3}$$

$$B = \frac{\mu_0 b r_1^2}{3}$$

(b) 

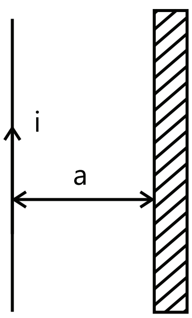
By ampere's law

$$B\times 2\pi r_2 = \mu_0\int (JdA) = \mu_0\left(\int_0^{r_0} br2\pi r\, dr\right)$$

$$B\times 2\pi r_2 = \mu_0 2\pi b\frac{r_0^3}{3}$$

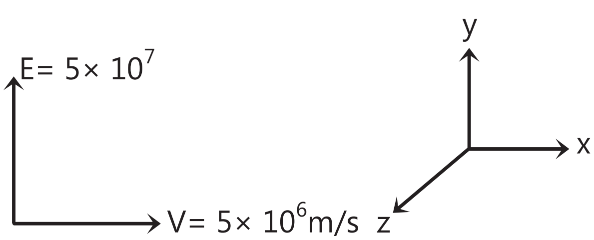
$$B = \frac{\mu_0 b r_0^3}{3r_2}$$

Sol 9:



Magnetic Force = qVB = $qV\left(\frac{\mu_0 I}{2\pi a}\right)$

Sol 10:



Magnetic force = qVB

Electric force = qE

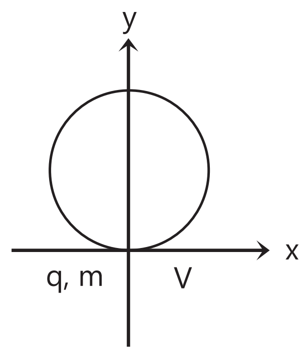
When both forces are equal in magnitude and opposite in direction then net force on charged particle is zero.

qVB = qE

$$B = \frac{E}{V} = \frac{5\times10^7}{5\times10^6} = 10\ T$$

and direction is in positive $\hat{k}$ direction

Sol 11:



y coordinate is equal to twice the radius of the circle

y = 2R

$$R = \frac{mV_0}{qB} \Rightarrow y = \frac{2mV_0}{qB}$$

Sol 12: We know that velocity of charged particle = v = $\frac{E}{B}$

Force = Change in momentum per sec= $\frac{mv}{t}$

$$I = \frac{e}{t} \Rightarrow F = \frac{mEI}{B\cdot e}$$

Sol 13: Force acting on a wire carrying current

$F = I\int d\vec{\ell}\times\vec{B}$

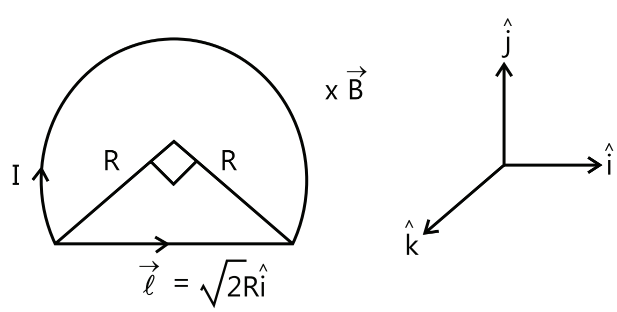
Since $\vec{B}$ is uniform so

$F = I\,(\int d\vec{\ell})\times\vec{B}$

For a loop $\int d\vec{\ell} = 0$

So F = 0

Sol 14:



Force = $I\int d\vec{\ell}\times\vec{B}$

Since $\vec{B}$ is constant so

$F = I\,(\int d\vec{\ell})\times\vec{B}$

$F = I\cdot\vec{\ell}\times\vec{B}$

$F = I\left(\sqrt{2}R\hat{i} \times (-B\hat{k})\right)$

$= I\sqrt{2}RB\hat{j} = \sqrt{2}IRB\hat{j}$

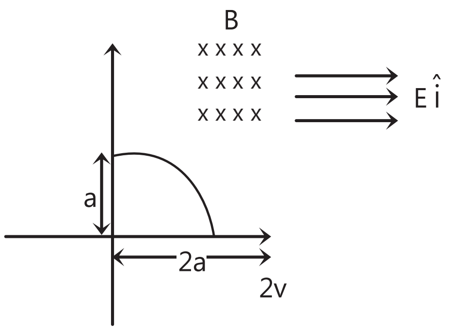
Sol 15: $F = F_1 + F_2 + F_3 + F_4$

$= i\int(d\ell_1 \times B_1) + i\int(d\ell_2 \times B_2) + i\int(d\ell_3 \times B_3) + i\int(d\ell_4 \times B_4)$

$$= \left(i\int_0^a dy\hat{j} \times (\alpha y)(-\hat{k})\right) + i(a\hat{i} \times \alpha y(-\hat{k})) + i\int_0^a dy\hat{j} \times (\alpha y)\hat{k} + I \times 0$$

$$F_1 = -i\alpha\frac{a^2}{2}\hat{i} + i\,\alpha a^2\hat{j} + i\frac{\alpha a^2}{2}\hat{i} = i\alpha a^2\hat{j}$$

Sol 16:



(a) Work done by Electric Field = Change in Kinetic Energy

$$\int F.dx = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2$$

$$qE \times 2a = \frac{3}{2}mv^2$$

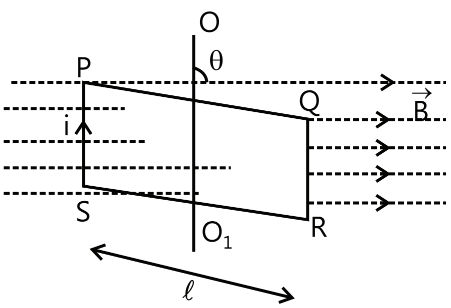
$$E = \frac{3mv^2}{4qa}$$

(b) Rate of work done = F.v = qE.v= $\frac{3}{4a}mv^3$

(c) Work done by magnetic field is always zero.

Work done by electric field = $\vec{F}.\hat{v} = qE\hat{i}.(-2v\hat{j}) = 0$

Sol 17:



Consider a loop PQRS placed in uniform magnetic field B in such a way that the normal to coil subtends an angle θ to the direction of B when a current I flows through the loop clockwise.

The sides PQ and RS are perpendicular to the field and equal and opposite forces of magnitude I and B act upwards and downwards respectively. Equal and opposite forces act on sides QR and PS towards right and left of coil.

The resultant force is zero but resultant torque is not zero. The forces on sides PQ and RS produce a torque due to a single turn which is given by

$\tau = I\ell^2 B\sin\theta$

for small θ, $\sin\theta \approx \theta$

$\tau = I\ell^2 B\theta$ ... (i)

$\tau = I\alpha$

$$= \left(\frac{m}{4}\frac{\ell^2}{12} \times 2 + \frac{m}{4}\frac{\ell^2}{4} \times 2\right)\alpha$$

$$= m\ell^2\left[\frac{1}{24} + \frac{1}{8}\right]\alpha = \frac{m\ell^2}{8}\left[\frac{4}{3}\right] = \frac{m\ell^2}{6} \quad \text{... (ii)}$$

By (i) and (ii)

$$I\ell^2 B\theta = \frac{m\ell^2}{6}\alpha$$

$$\alpha = \frac{6IB}{m}\theta$$

$$\omega^2 = \frac{6IB}{m}$$

$$\text{Time period} = 2\pi\sqrt{\frac{m}{6IB}} = 2\pi\sqrt{\frac{10^{-2}}{6 \times 2 \times 10^{-1}}}$$

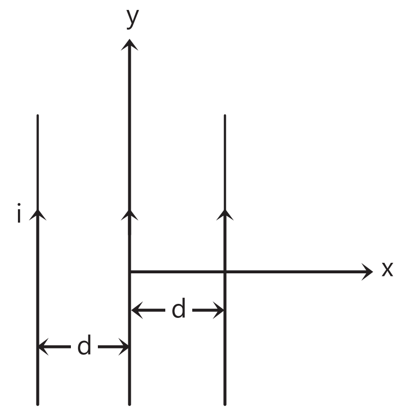
$$= 2\pi\sqrt{\frac{1}{120}} = 0.57 \text{ sec}$$

Sol 18: Net force acting on the loop = F

$$F = \frac{\mu_0 II'c}{2\pi a} - \frac{\mu_0 II'c}{2\pi b} = \frac{\mu_0 II'c}{2\pi}\left[\frac{1}{a} - \frac{1}{b}\right]$$

This loop will experience attractive forces.

Sol 19: (i)



Net Force at some point x, y is

$$F_{net} = \frac{\mu_0 I}{2\pi(x+d)} + \frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(x-d)} = 0$$

$$\Rightarrow \frac{1}{n+d} + \frac{1}{x} + \frac{1}{x-d} = 0$$

$$\frac{2x}{x^2-d^2} + \frac{1}{x} = 0$$

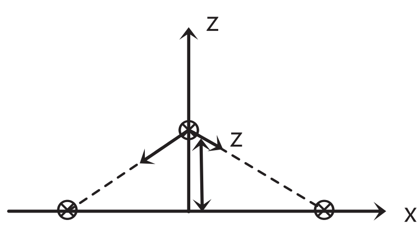
$$\frac{2x^2+x^2-d^2}{x(x^2-d^2)} = 0 \Rightarrow 3x^2 = d^2$$

$$x = \pm\sqrt{\frac{d}{3}}$$

Net force will be zero only in x–y plane

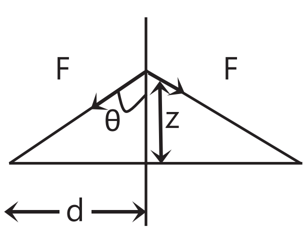
i.e. when z = 0 and $x = \pm\sqrt{\frac{d}{3}}$

(ii)



Let the middle wire is displaced by z distance in positive z-direction.

Attractive force acting on wire is F



$$\cos\theta = \frac{z}{\sqrt{d^2+z^2}}$$

$$F = \frac{\mu_0 i^2 \ell}{2\pi\sqrt{d^2+z^2}}$$

Resultant force is downward

$$F_{net} = -2\,F\cos\theta = \frac{-2\mu_0 i^2\ell}{2\pi\sqrt{d^2+z^2}}\cdot\frac{z}{\sqrt{d^2+z^2}}$$

$$F_{net} = \frac{-\mu_0 i^2 z\ell}{\pi(d^2+z^2)}$$

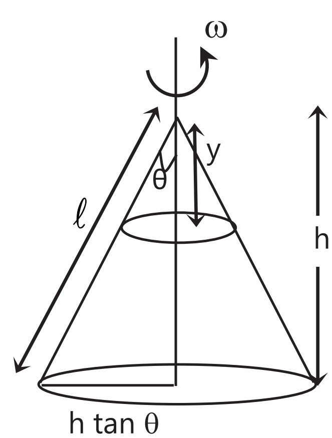
For small z

$$F_{net} = \frac{-\mu_0 i^2 z\ell}{\pi d^2} = \lambda a$$

$$\omega = \sqrt{\frac{\mu_0 i^2\ell}{\lambda\pi d^2}}$$

$$F = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\mu_0 i^2}{\lambda\pi d^2}} = \frac{i}{2\pi d}\sqrt{\frac{\mu_0}{\lambda\pi}}$$

Sol 20: $l\cos\theta = h$



Take a ring at distance y from the top point of the cone.

Magnetic moment M = IA

dM=

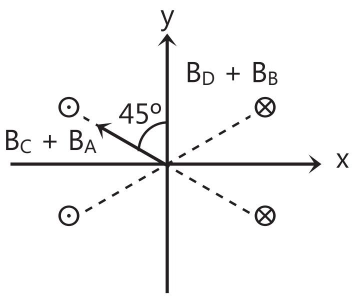
$$\left(\frac{Q}{\pi(h\tan\theta)\frac{h}{\cos\theta}}\cdot\frac{2\pi y\,dy\tan\theta}{\cos\theta}\right)\frac{\omega}{2\pi}\cdot\pi(y\tan\theta)^2$$

$$= \int_0^h \frac{Q\omega\tan^3\theta}{h^2\tan\theta}\cdot y^3dy = \frac{Q\omega\tan^2\theta}{h^2}\cdot\frac{h^4}{4}$$

$= \frac{1}{4} Q\omega \tan^2\theta\, h^2$

Sol 21: (i)$B_C = B_A = B_B = B_D = B$

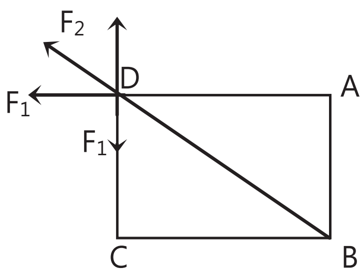
$B = \frac{\mu_0 I \times 2}{2\pi\sqrt{2}\,a}$



Net magnetic field is

$B_{net} = B\sqrt{2} = \frac{\mu_0 I \times 2}{2\pi a}$ along y-axis

(ii)



$F_1 = \frac{\mu_0 I^2 \ell}{2\pi(2a)}$

$F_2 = \frac{\mu_0 I^2 \ell}{2\pi(2\sqrt{2}a)}$

$F_x = F_1 + \frac{F_2}{\sqrt{2}} = \frac{\mu_0 I^2 \lambda}{4\pi a}\left[1 + \frac{1}{2}\right] = \frac{3\mu_0 I^2 \lambda}{8\pi a}$

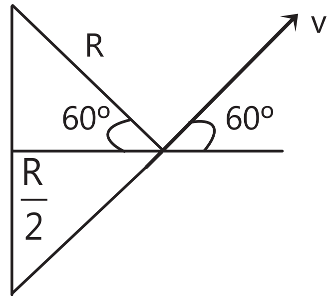
$F_y = F_1 - \frac{F_2}{\sqrt{2}} = \frac{\mu_0 I^2 \lambda}{4\pi a}\left[1 - \frac{1}{2}\right] = \frac{\mu_0 I^2 \lambda}{8\pi a}$

Net force $= \frac{\mu_0 I^2 \ell}{8\pi a}\sqrt{1+3^2}$; $\ell = 1$

$= \left(\frac{\mu_0}{4\pi}\right)\left(\frac{I^2}{2a}\right)\sqrt{10}$

Radius = R = a

Sol 22: (a)



$\vec{v} = v\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$

$B = \left(\frac{\mu_0 I}{2R}\left(\frac{\frac{2\pi}{3}}{2\pi}\right) - \frac{\mu_0 I}{\frac{4\pi R}{2}}\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)\right)\hat{k}$

$B = \left(\frac{\mu_0 I}{6R} - \frac{\sqrt{3}\mu_0 I}{2\pi R}\right)\hat{k} = \frac{\mu_0 I}{2R}\left[\frac{1}{3} - \frac{\sqrt{3}}{\pi}\right]\hat{k}$

Force $= qv \times B = \frac{qv}{2}\left[\left(\hat{i} + \sqrt{3}\,\hat{j}\right) \times \frac{\mu_0 I}{2R}\left(\frac{1}{3} - \frac{\sqrt{3}}{\pi}\right)\hat{k}\right]$

$= \frac{Qv}{m}\frac{\mu_0 I}{6a}\left[\frac{3\sqrt{3}}{\pi} - 1\right]$

(b)Net Torque $= \vec{M} \times \vec{B} = I\,AB\,\hat{j}$

$= I\left(\frac{\pi}{3}a^2 - \frac{\sqrt{3}a^2}{4}\right)B\,\hat{j}$

$= B\,I\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)a^2\,\hat{j}$

Sol 23: (a) Net Torque on the loop is

$\tau = -MB_x\hat{j} + MB_y\hat{i} = I\pi r^2\sqrt{B_x^2 + B_y^2}$ ...(i)

By Torque balance mgr = τ ...(ii)

By (i) and (ii)

$I = \frac{mg}{\pi r\sqrt{B_x^2 + B_y^2}}$

(b) Net Torque is $\tau = -MB_x\hat{j}$

$|\tau| = I\pi r^2 B_x$

By torque balance

$mgr = t \Rightarrow mgr = I\pi r^2 B_x$

$I = \frac{mg}{\pi r B_x}$

Sol 24: Magnetic field due to sheet of width d and infinite length at a distance h is given by

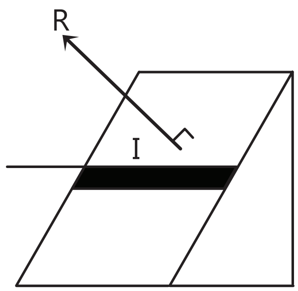
$$B = \frac{\mu_0 j_0}{\pi}\tan^{-1}\left(\frac{d/2}{h}\right)\hat{i}$$

$$\vec{\ell} = \hat{j}$$

$$F = i\,\vec{\ell}\times\vec{B}$$

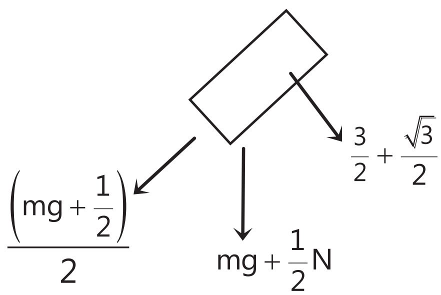
$$F = \frac{i\mu_0 j_0}{\pi}\tan^{-1}\left(\frac{d}{2n}\right)(-\hat{k})$$

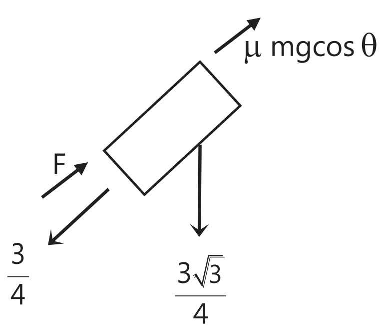
Sol 25:



Force= $I\,\vec{\ell}\times\vec{B}$ = 10 × 0.5 × 0.1

Force = $\frac{1}{2}$N upward on inclined plane





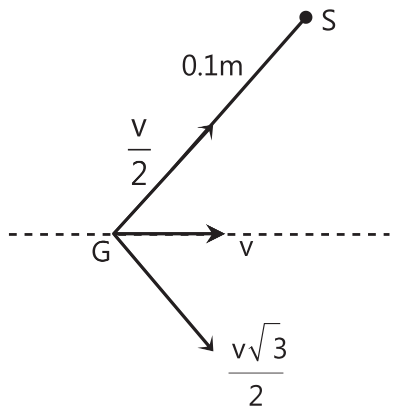
$$\mu.1 + F = \frac{3}{4} \Rightarrow F = \frac{3}{4} - \frac{3\sqrt{3}}{40}$$

$$F_{min} = \frac{3}{4}\left[1-\frac{\sqrt{3}}{10}\right] = 0.62\text{ N}$$

$$F = \frac{3}{4} + \frac{3\sqrt{3}}{40}$$

F_{max} = 0.88 N

Sol 26:



Electron will move in helical path with pitch = 0.1 m. For minimum value of B particle should reach at point S in a single revolution.

Time period $T = \frac{2\pi m}{qB}$

So $0.1 = \frac{v}{2}T$

$$0.1 = \frac{v\cdot 2\pi m}{2\cdot qB}$$

$$B' = \frac{20\pi mv}{2q}$$

$$= \frac{20\pi\sqrt{2\times 9.1\times 10^{-31}\times 2000\times 1.6\times 10^{-19}}}{2\times 1.6\times 10^{-19}}$$

$$= 10\pi\sqrt{\frac{2\times 9.1\times 10^{-31}\times 2000}{1.6\times 10^{-19}}}$$

$$B = 10\pi\sqrt{2.275}\times 10^{-4}$$

B = 4.7 × 10^{-3} T

Sol 27: To neutralize the magnetic field, current in vertical ring should be such that the magnitude of magnetic field is 3.49×10^{-5} T and current in horizontal ring should be such that the magnitude of magnetic field is $\frac{1}{\sqrt{3}}\times 3.49\times 10^{-5}$

For vertical ring

$$B = \frac{\mu_0 NI}{2r} = \frac{\mu_0\times 100\times I}{2\times 0.2}$$

$$3.49\times10^{-5} = \mu_0\times 250\,I$$

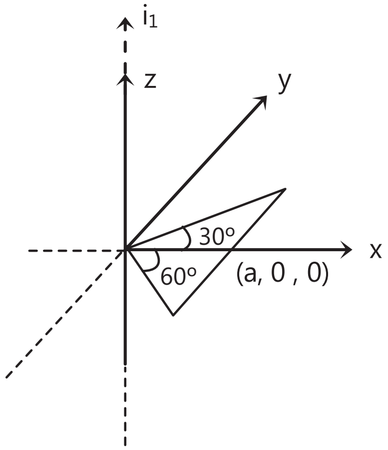
$$I = \frac{3.49\times10^{-5}}{\mu_0\times250} = \frac{3.49\times10^{-5}}{4\pi\times10^{-7}\times250} = 0.111\text{ A}$$

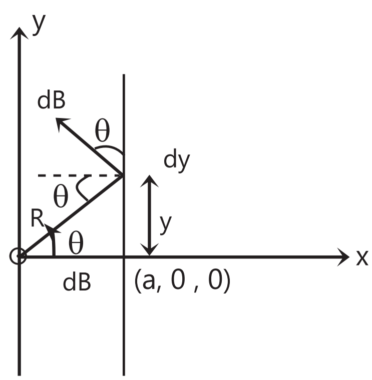
For horizontal ring

$$B = \frac{\mu_0 NI}{2r} \Rightarrow \frac{1}{\sqrt{3}}\times3.49\times10^{-5} = \frac{\mu_0\times100\,I}{2\times0.3}$$

$\Rightarrow$ I = 0.096 A

Sol 28:





Force on dy element in x direction is

$$\int dF = \int i_2 dy\, B\sin\theta$$

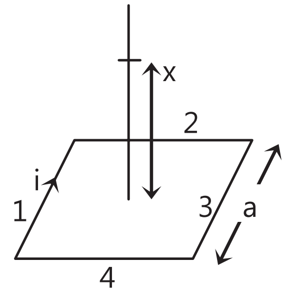
$$F = \int i_2 \frac{Rd\theta}{\cos\theta}\cdot\frac{\mu_0 i_1}{2\pi R}\sin\theta$$

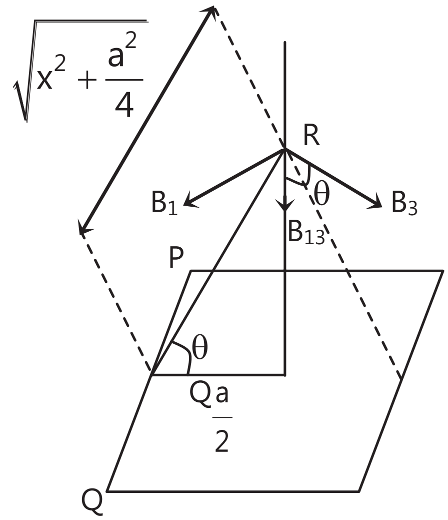
$$= \frac{\mu_0 i_1 i_2}{2\pi}\int_{-60}^{30}\tan\theta\, d\theta$$

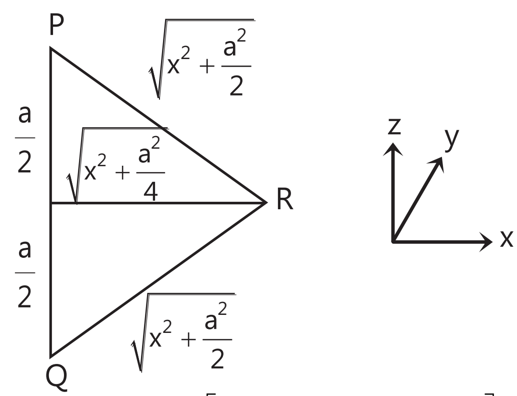
$$F = \frac{\mu i_1 i_2}{2\pi}[\log\cos]_{-60}^{30}$$

$$= \frac{\mu i_1 i_2}{2\pi}\log\sqrt{3} = \frac{\mu i_1 i_2}{4\pi}\log 3$$

Sol 29: (a) $|B_1| = |B_2| = |B_3| = |B_4|$







$$B_1 = \frac{\mu_0 I}{4\pi\sqrt{x^2+\frac{a^2}{4}}}\left[\frac{a}{2\sqrt{x^2+\frac{a^2}{2}}}+\frac{a}{2\sqrt{x^2+\frac{a^2}{2}}}\right]$$

$$B_1 = \frac{\mu_0 I}{4\pi\sqrt{x^2+\frac{a^2}{4}}}\cdot\frac{a}{\sqrt{x^2+\frac{a^2}{2}}}$$

Resultant of B_1 and B_3 is $B_{13} = 2B_1\cos\theta$

$$B_{13} = \frac{2\times\mu_0 I}{4\pi\sqrt{x^2+\frac{a^2}{4}}}\frac{a}{\sqrt{x^2+\frac{a^2}{2}}}\cdot\frac{\frac{a}{2}}{\sqrt{x^2+\frac{a^2}{4}}}$$

$$= \frac{\mu_0 I a^2}{4\pi\left(x^2+\frac{a^2}{4}\right)\left(x^2+\frac{a^2}{2}\right)^{\frac{1}{2}}}$$

$$B_{13} = \frac{\mu_0 I\, a^2}{\pi(4x^2+a^2)\left(x^2+\frac{a^2}{2}\right)^{\frac{1}{2}}}$$

Similarly $B_{24} = \dfrac{\mu_0 I a^2}{\pi(4x^2+a^2)\left(x^2+\frac{a^2}{2}\right)^{\frac{1}{2}}}$

Net resultant = B_{13} + B_{24}

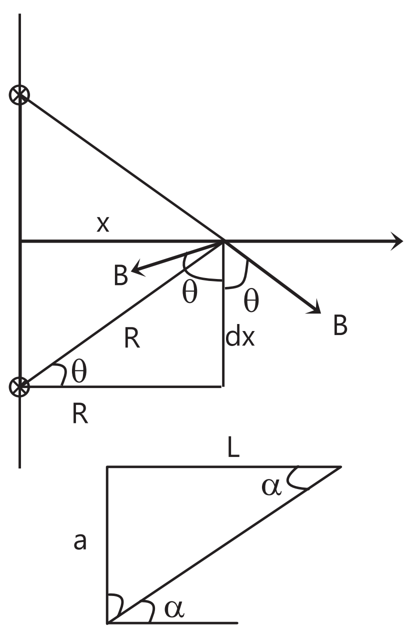
$$= \frac{2\mu_0 I a^2}{\pi(4x^2+a^2)\left(x^2+\frac{a^2}{2}\right)^{\frac{1}{2}}}$$

(b) Yes

Sol 30: $B_{res} = 2B\cos\theta$

$$F_{res} = I\int d\vec{\ell}\times B_{res}$$

$$= \int I\times\frac{R\,d\theta}{\sin\theta}\,\frac{2\,\mu_0 I}{2\pi R}\cos\theta;\ \sin\alpha = \frac{a}{\sqrt{L^2+a^2}}$$



$$= \frac{\mu_0 I^2}{\pi}\int_{\pi/2}^{\alpha}\cot\theta\, d\theta = \frac{\mu_0 I^2}{\pi}\ln(\sin\theta)_{90}^{\alpha}$$

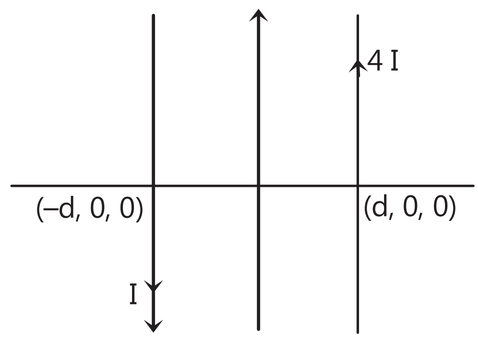
$$= \frac{\mu_0 I^2}{2\pi}\ln\left(\frac{a}{\sqrt{L^2+a^2}} - 1\right)$$

If direction of current in B is reversed then resultant magnetic field will become horizontal and so net force will be zero.

Exercise 2

Single Correct Choice Type

Sol 1: (C) Magnetic field at some x is given by

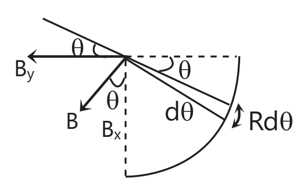


$$\frac{\mu_0(4I)}{2\pi(d-x)}\hat{k} + \frac{\mu_0 I}{2\pi(d+x)}\hat{k}$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{4}{d-x}+\frac{1}{d+x}\right] = \frac{\mu_0 I}{2\pi}\left[\frac{5d+3x}{d^2-x^2}\right]$$

It corresponds to graph (c)

Sol 2: (A) Magnetic field at the centre due to Rdθ component is



$$B_x = \int dB_x = \int\frac{\mu_0\left(\frac{I}{2\pi R}\cdot Rd\theta\right)}{2\pi R}\cos\theta = \frac{\mu_0 I}{4\pi^2 R}$$

$$B_y = \int dB_y = \int\frac{\mu_0\left(\frac{I}{2\pi R}\cdot Rd\theta\right)}{2\pi R}\sin\theta = \frac{\mu_0 I}{4\pi^2 R}$$

$$B = \frac{\mu_0 I}{4\pi^2 R}\sqrt{1+1}$$

Sol 3: (C) $V = \frac{E}{B}$ for no deflection to occur

$$V = \frac{3.2\times10^5}{2\times10^{-3}} = 1.6 \times 10^8 \text{ m/s}$$

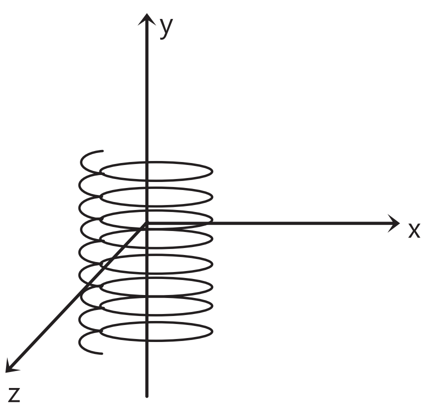
$$R = \frac{mv}{qB} = \frac{9.1\times10^{-31}\times1.6\times10^8}{1.6\times10^{-19}\times2\times10^{-3}} = 0.45 \text{ m}$$

Sol 4: (C) $\frac{q}{m} = \alpha$

Work done by electric field = $qE_0x_0 = \frac{1}{2}m(25–0)$

$$x_0 = \frac{25m}{2qE_0} = \frac{25}{2\alpha E_0}$$

Sol 5: (C) Particle is moving in helix along y-axis. So the time taken by particle to reach in x-z plane should be integral multiple of time taken to complete one revolution.



Helical motion of the particle

$$\Rightarrow \frac{2mv}{qE} = \left(\frac{2\pi m}{qB}\right)n$$

$$n = \frac{Bv}{\pi E}$$

So $\left[\frac{Bv}{\pi E}\right]$ should be an integer

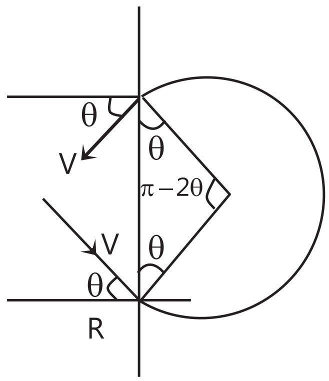
Sol 6: (D) Both particles will move in helix. They will meet for the first time when mass m will complete two revolutions and mass 2m will complete one revolution. Time taken to complete one rotation.

$$t_1 = \frac{2\times2\pi M}{QB};\ t_2 = \frac{2\pi 2M}{QB}$$

Distance from the point of projection = tv cosθ

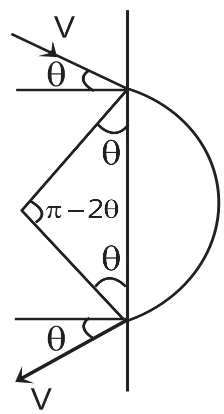
$$= v\cos\theta\frac{4\pi M}{QB} = \frac{4\pi Mv\cos\theta}{QB}$$

Sol 7: (C)



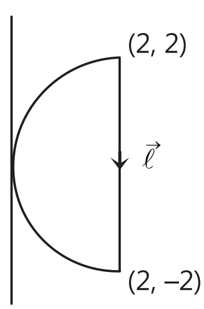
$$\text{Time taken} = \left(\frac{\pi+2\theta}{\omega}\right) = \left(\frac{\pi+2\theta}{2\pi}\right)T$$

Sol 8: (D)



$$\text{Time taken} = \left(\frac{\pi-2\theta}{\omega}\right) = \left(\frac{\pi-2\theta}{2\pi}\right)T$$

Sol 9: (B)



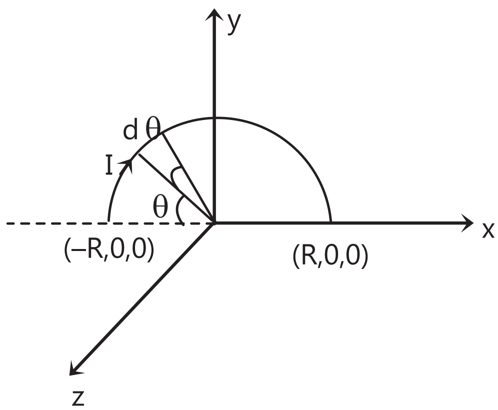
Magnetic force is given by

$dF_m = i\int d\vec{\ell}\times\vec{B} = i\int d\ell\,(-\hat{j}) \times (-4\hat{k}) = 4i\int d\ell\ \hat{i}$

since ℓ and B are perpendicular so

$df_m = 8\int d\ell\ \hat{i} = 8 \times 4\hat{i} = 32\hat{i}$

Sol 10: (A)



$$F = I\int Rd\theta(\sin\theta\ \hat{i} + \cos\theta\ \hat{j}) \times \frac{B_0}{2R}(-R\cos\theta)\hat{k}$$

$$= \frac{IB_0R}{2}\int_0^{\pi}(\sin\theta\,\hat{i} + \cos\theta\,\hat{j}) \times (-\cos\theta\ \hat{k})\ d\theta$$

$$= \int_0^{\pi}(\sin\theta\cos\theta\ \hat{j} - \cos^2\theta\ \hat{i})\ d\theta$$

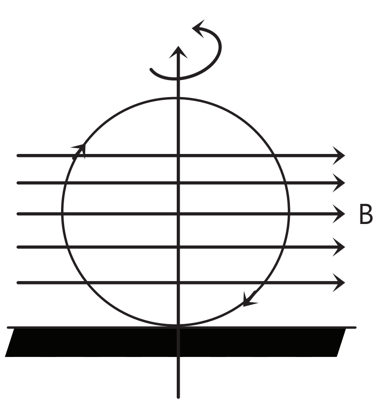
$$= \int_0^{\pi}\left(\frac{\sin 2\theta\,\hat{j}}{2} - \frac{(1+\cos 2\theta)\hat{i}}{2}\right)d\theta$$

$$= \left[\frac{-\cos 2\theta}{4}\right]_0^{\pi}\hat{j} - \left[\frac{\theta}{2}\right]_0^{\pi}\hat{i} - \left[\frac{\sin 2\theta}{4}\right]_0^{\pi}\hat{i}$$

$$= 0 - \frac{\pi}{2}\hat{i} - 0 = -\frac{\pi}{2}\hat{i}$$

Sol 11: (A) Refer Q.18 Exercise-I JEE Advanced.

Sol 12: (A) Torque on the ring due to magnetic field is



$\tau = MB\sin\theta$

$\tau = I \times \pi R^2 \times B = I\alpha$

$$I\pi R^2 \times B = \frac{MR^2\alpha}{2}$$

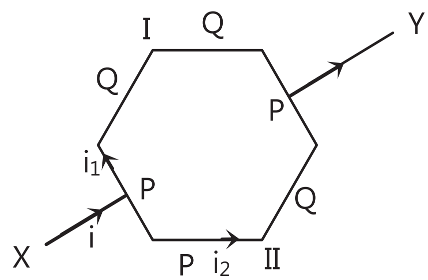
$$\alpha = \frac{2 \times 4 \times \pi \times 10}{2}$$

$= 40\pi$ rad/sec^2

Sol 13: (A) Let us assume that resistance of p material is ρ and that of Q is q.

$$i_1 = \frac{2\rho + q}{3(\rho + q)}i, \quad i_2 = \frac{2q + \rho}{3(\rho + q)}i$$

$$\frac{i_1}{i_2} = \frac{2\rho + q}{2q + \rho}$$



We know that $B \propto i$

SoB_1 = magnetic field due to I part

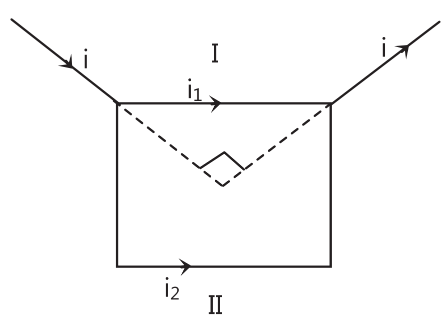
B_2 = magnetic field due to II part

For the magnetic field to be zero $B_1 = -B_2$ should hold.

$$\text{But } \frac{B_i}{B_i} \propto \frac{i_1}{i_2} = \frac{2\rho + q}{2q + \rho} \neq -1$$

So magnetic field will not be zero at centre.In (B), (C) and (D) $i_1 = i_2$ so magnetic field is zero at centre.

Sol 14: (C)

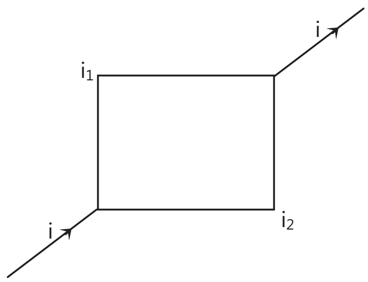


In (A)

$$i_1 = \frac{3}{4}i \quad ; \quad i_2 = \frac{i}{4}$$

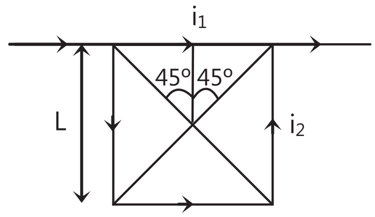
$$B = \frac{-\mu_0 i_1}{4\pi\frac{l}{2}}(\sin\theta_1 + \sin\theta_2) + \frac{3\mu_0 i_2}{4\pi\frac{\ell}{2}}(\sin\theta_1 + \sin\theta_2)$$

(B)



By symmetry $i_1 = i_2$ and magnetic field will be cancelled out by both the parts.

(C)



$i_1 = \frac{3}{4}i$; $i_2 = \frac{i}{4}$

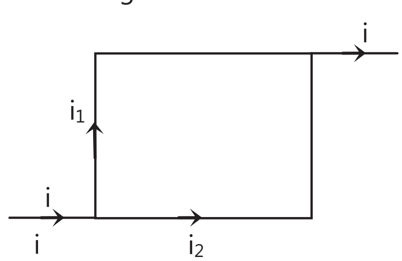
Let magnetic field due to sides of square be B_s

$$B_s = \frac{-\mu_0 \frac{3}{4} i_1}{4\pi\frac{L}{2}}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\hat{k} + \frac{3\mu_0 \frac{i}{4}}{4\pi\frac{L}{2}}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$B_s = 0$

But magnetic field due to 2 infinitely long wires is not zero so net magnetic field is zero.

(D)

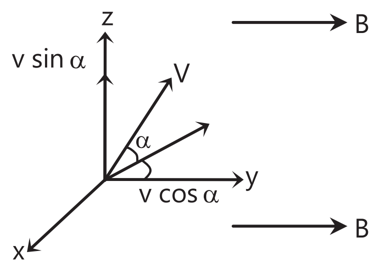


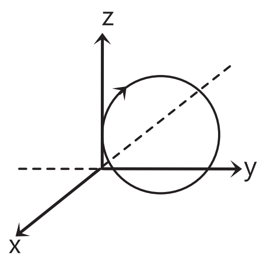
By symmetry $i_1 = i_2 = \frac{i}{2}$

So magnetic field due to four sides of square will cancel out. Magnetic field due to two infinitely long wires will also cancel out as they are equal in magnitude and opposite in direction.

So net magnetic field is zero.

Sol 15: (A) So its x coordinate cannot be positive.





Its x- and z- coordinate will be zero when particle will complete one revolution.

y - Coordinate = $v\cos\alpha\, t$

Multiple Correct Choice Type

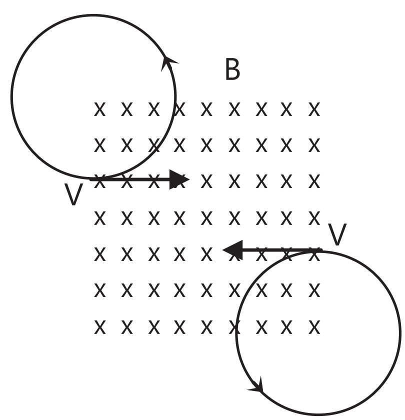
Sol 16: (B, C) (A)Motion is helical in nature

(B) They will follow circular path with radius

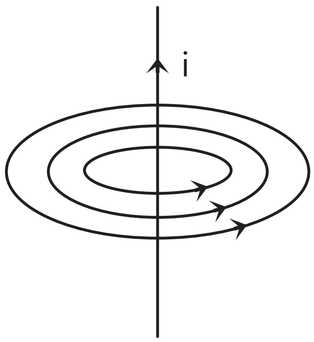
$$R = \frac{\sqrt{2mKE}}{qB}$$

(C) Work done by magnetic force is always zero.

(D)



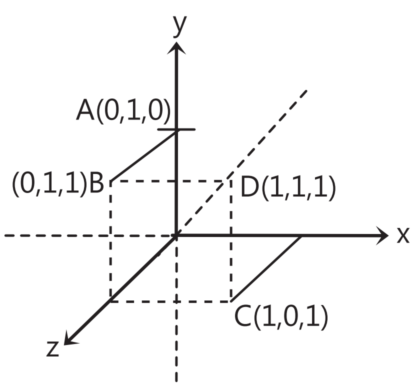
Sol 17: (A, B, C) $B = \frac{\mu_0 I}{2\pi r}(\sin\theta_1 + \sin\theta_2)$



By ampere's law magnetic field on a ring with centre as wire is same.

$B \not\propto \frac{1}{r}$ as θ_1 and θ_2 are also dependent on r.

Sol 18: (B, D) Magnetic field



at A = $B_A = \frac{\mu_0 I}{2\pi \times 1}; B_B = \frac{\mu_0 I}{2\pi\sqrt{2}}$

$B_C = \frac{\mu_0 I}{2\pi \times 1}; B_D = \frac{\mu_0 I}{2\pi \times \sqrt{2}}$

Sol 19: (A, B, C) $\frac{1}{\mu_0 \varepsilon_0} = c^2$

Sodimension of y is m/s

$v = \frac{E}{B}$ when E and B are both perpendicular and perpendicular to velocity

So dimension of x m/s

Dimension of RC = sec

So $Z = \frac{\ell}{CR}$ has dimension m/s

So x, y, z have same dimensions.

Sol 20: (A, B, C, D) On x-axis

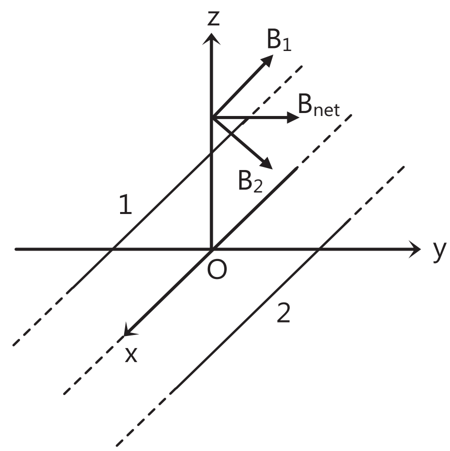
(A) $B = \frac{\mu_0 I}{2\pi a} - \frac{\mu_0 I}{2\pi a} = 0$

(B) On y-axis say at (y, 0, 0)

$$B = \frac{-\mu_0 I}{2\pi(a+y)}\hat{k} + \frac{\mu_0 I}{2\pi(a-y)}\hat{k}$$

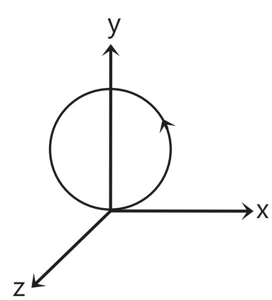
So except at origin, B has only z-components

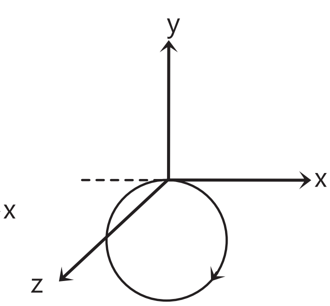
(C)



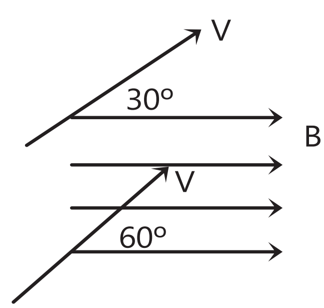
(D) B cannot has x-component as B is perpendicular to direction of I.

Sol 21: (A, B) This can be done by applying magnetic field in y-axis or z-axis.





Sol 22: (A, D)



Time period $T = \frac{2\pi m}{qB}$; $a = \frac{T_1}{T_2} = 1$

radii$R = \dfrac{mv\sin\theta}{qB}$

$$\frac{R_1}{R_2} = \frac{\sin(30^o)}{\sin(60^o)} = \frac{1}{\sqrt{3}}$$

pitch = $v\cos\alpha\, t$

$$\frac{P_1}{P_2} = \frac{v\cos(30^o)}{v\cos(60^o)} = \sqrt{3}$$

abc = 1; a = bc

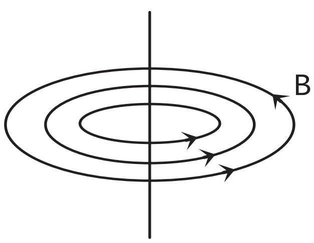
Sol 23: (C, D) If velocity is zero, then magnetic force is zero.Energy cannot increase in magnetic field as work done by magnetic force is zero.

$F = q\vec{v}\times\vec{B}$;So force is perpendicular to its velocity.

Assertion Reasoning Type

Sol 24: (D) If initially velocity of charged particle is in the direction of magnetic field then force acting on it is zero and particle will continue to move in the same direction. So statement 1 is false.

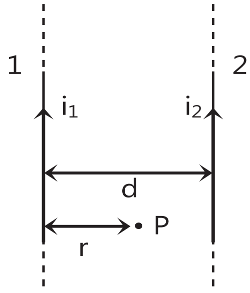
Sol 25: (B) Magnetic field at any point is in tangential direction. So it is not possible for a particle to move in tangential direction by the action of magnetic force.



Sol 26: (D) It's velocity vector must be perpendicular to both magnetic field and electric field.

Sol 27: (C) $F = I\int d\vec{\ell}\times\vec{B}$

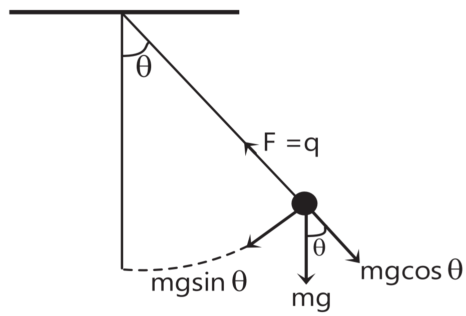
So force acting is attractive



Consider a point P in space between two wires at a distance r from one wire. The magnetic force due to wire 1 is in positive z-axis direction whereas due to wire 2 is in negative z-axis direction.

Sol 28: (D) Statement 1 is false as Ampere's circuital law holds good for a closed path of any size and shape around a current carrying conductor only if the relation is independent of distance.

Sol 29: (D) Since angular acceleration of the mass will not change so time period will also remain the same.



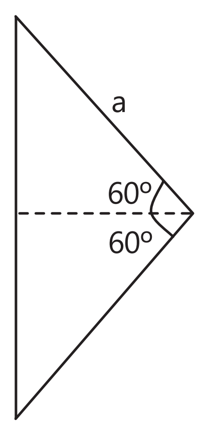
Comprehension Type

Paragraph 1

Sol 30: (A) Magnetic field due to curved part is

$$B = \frac{\mu_0 I}{4\pi a}\left(\frac{2\pi}{3}\right) = \frac{\mu_0 I}{6a}$$

Sol 31: (A)



$$B = \frac{\mu_0 I}{4\pi\frac{a}{2}}(\sin 60^o + \sin 60^o) = \frac{\sqrt{3}\mu_0 I}{2\pi a}$$

Sol 32: (D) Net magnetic field at C is

$$B = -\frac{\mu_0 I}{6a} + \frac{\sqrt{3}\mu_0 I}{2\pi a}$$

Paragraph 1

Sol 33: (D) I = 3A

r = 0.04m

N = 20

B = 0.5 T

Dipole moment M = INA= $3\times 20 \times \pi(0.01)^2$

$= 1.88 \times 10^{-2}$ Am^2

Sol 34: (B) PE $= -1.88 \times 10^{-2} \times \frac{1}{2}$

= –9.4mJ

Sol 35: (B) Torque, $\tau =$ in AB sin 90°

$$= 3\times 20\times \pi\times\left(\frac{1}{100}\right)^2\times 0.5\times 1$$

$$= 3\times 3.14\times 10^{-3}\,\text{Nm} \;= 9.4\times 10^{-3}\text{Nm}$$

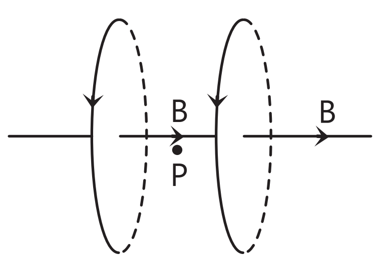
Match the Columns

Sol 36: A $\to$ q, r; B$\to$ p; C $\to$ q, r; D $\to$ q, or; A $\to$ q, r; B$\to$ p; C $\to$ q, r; D$\to$ q, s

τ = MB sin 90°= 9.4×10^{-3} Nm

(A) Magnetic field is in opposite direction. Since current is in same direction so they will attract each other. Magnetic field is equal in magnitude at P so magnetic field at P is zero.

(B)



Magnetic field at P is in the same direction.

Wires will attract as the current is in the same direction.

(C) Magnetic field at P is in opposite direction due to two wires and has same magnitude. So net magnetic field is zero at P. Wires will attract each other as current is in the same direction.

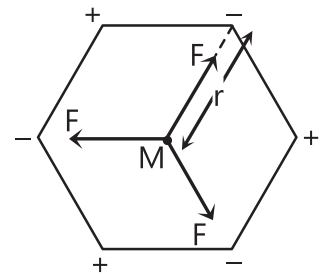
(D) Magnetic field will be in opposite direction and wires will repel each other as current is in opposite sense.

Sol 37: A $\to$ p, r, s; B$\to$ r, s; C $\to$ p, q; D$\to$ r, s

Electric field is zero at point M

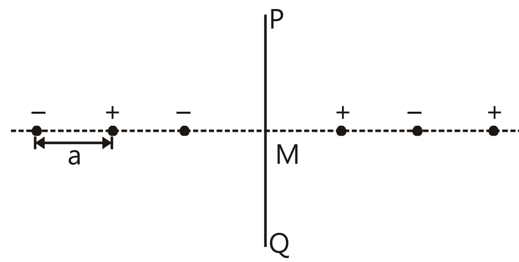
Electric potential = $\frac{3Kq}{r} - \frac{3Kq}{r} = 0$

Magnetic field is zero as current due to rotating charge is zero.



Magnetic moment= INA= 0 × NA = 0

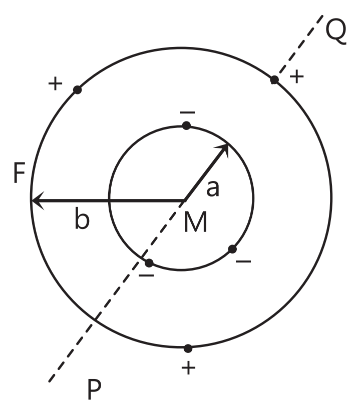
$$E = -\frac{Kq}{\left(\frac{5a}{2}\right)^2} - \frac{Kq}{\left(\frac{5a}{2}\right)^2} + \frac{Kq}{\left(\frac{3a}{2}\right)^2} + \frac{Kq}{\left(\frac{3a}{2}\right)^2} - \frac{Kq}{\left(\frac{a}{2}\right)^2} - \frac{Kq}{\left(\frac{a}{2}\right)^2} \neq 0$$



$$V = \frac{Kq}{\frac{5a}{2}} - \frac{Kq}{\frac{5a}{2}} + \frac{Kq}{\frac{3a}{2}} - \frac{Kq}{\frac{3a}{2}} + \frac{Kq}{\frac{a}{2}} - \frac{Kq}{\frac{a}{2}} = 0$$

B = 0 as current due to rotating charge is zero.

μ = 0 as current due to rotating charge is zero.



E = 0

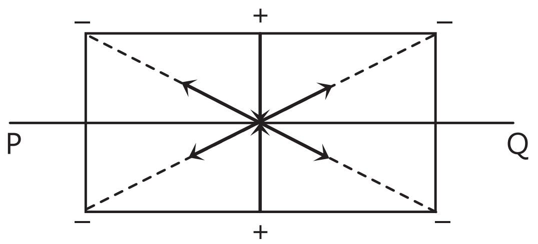
Electric field will cancel out due to symmetry

$$V = -\frac{Kq}{a}\times 3 + \frac{Kq}{b}\times 3 \neq 0$$

B is not zero as current due to rotating charge is non-zero.

μ = INA

as $I \neq 0 \Rightarrow \mu \neq 0$



Electric field is zero.By symmetry electric field will cancel out each other.

$$V = \frac{-Kq}{\left(\frac{\sqrt{5}\,a}{2}\right)} \times 4 + \frac{Kq}{\frac{a}{2}} \times 2 \neq 0$$

Let I be the current due to moving charge

$$\text{So } B = \frac{\mu_0 I}{2a} - \frac{2x\mu_0 I a^2}{2\left(2a^2\right)^{\frac{3}{2}}} \neq 0$$

μ = INA

$\mu = 2\times Ia^2 - Ia^2 = Ia^2$

Previous Years' Questions

Sol 1: (C) $c\phi$ = BINA

$$\therefore\ \phi = \left(\frac{BNA}{c}\right) I$$

Sol 2: (C) If $B_2 > B_1$, critical temperature, (at which resistance of semiconductors abruptly becomes zero) in case 2 will be less than compared to case1.

Using iron core, value of magnetic field increases. So, deflection increases for same current. Hence, sensitivity increases.

Soft iron can be easily magnetized or demagnetized.

Sol 3: (D) With increase in temperature, T_C is decreasing.

$T_C(0)$ = 100 K

T_C = 75 K at B = 7.5 T

Hence, at B = 5 T, T_C should lie between 75 K and 100 K.

Hence, the correct option should be (b).

Sol 4: (A, B, D) If both E and B are zero, then $\vec{F_e}$ and $\vec{F_m}$ both are zero. Hence, velocity may remain constant. Therefore, option (a) is correct.

If E = 0, B ≠ 0 but velocity is parallel or antiparallel to magnetic field, then also $\vec{F_e}$ and $\vec{F_m}$ both are zero. Hence, option (b) is also correct.

If E ≠ 0, B ≠ 0 but $\vec{F_e} + \vec{F_m} = 0$, then also velocity may remain constant or option (d) is also correct.

Sol 5: (A, B, D) Magnetic force does not do work. From work-energy theorem:

$$\overrightarrow{W}_{Fe} = \Delta KE \text{ or } (qE)(2a) = \frac{1}{2}m[4v^2 - v^2]$$

$$\text{or } E = \frac{3}{4}\left(\frac{mv^2}{qa}\right)$$

At P, rate of work done by electric field

$$= \vec{F}_e \cdot \vec{v} = (qE)(v)\cos 0°$$

$$= q\left(\frac{3}{4}\frac{mv^2}{qa}\right)v = \frac{3}{4}\left(\frac{mv^3}{a}\right)$$

Therefore, option (b) is also correct. Rate of work done at Q: of electric field = $\vec{F}_e \cdot \vec{v}$ = (qE)(2v)cos 90° = 0 and of magnetic field is always zero. Therefore, option (d) is also correct.

Note that $\vec{F}_e = qE\,\hat{i}$

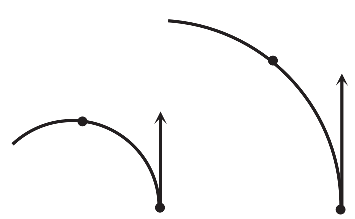
Sol 6: (A, C) $r = \frac{mv}{Bq} = \frac{P}{Bq} = \frac{\sqrt{2km}}{Bq}$

i.e., $r \propto \frac{\sqrt{m}}{q}$

If K and B are same.

i.e., $r_{H^+} : r_{He^+} : r_{O^{2+}} = \frac{\sqrt{1}}{1} : \frac{\sqrt{4}}{1} : \frac{\sqrt{16}}{2} = 1:2:3$

Therefore, He^+ and O^{2+} will be deflected equally but H^+ having the least radius will be deflected most.



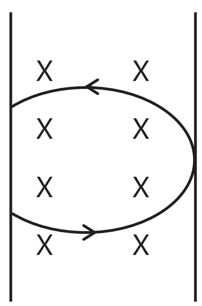
Sol 7: (A, C) $\vec{F}_{BA}$ = 0, because magnetic lines are parallel to this wire.

$\vec{F}_{CD}$ = 0, because magnetic lines are antiparallel to this wire.

$\vec{F}_{CB}$ is perpendicular to paper outwards and $\vec{F}_{AD}$ is perpendicular to paper inwards. These two forces (although calculated by integration)cancel each other but produce a torque which tend to rotate the loop in clockwise direction about an axis OO′.

Sol 8: (A, C, D) $v = \dfrac{BqI}{m}$

$\vec{v} \perp \vec{B}$ in region II. Therefore, path of particle is circle in region II.



Particle enters in region III if, radius of circular path, r >I

or $\dfrac{mv}{Bq} > I$

or $v > \dfrac{BqI}{m}$

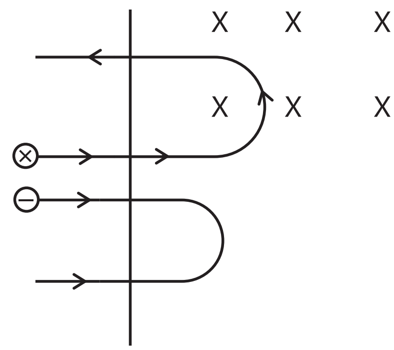
If $v = \dfrac{BqI}{m}$, $r = \dfrac{mv}{Bq}$ = I, particle will turn back and path length will be maximum. If particle returns to region I, time spent in region II will be:

$t = \dfrac{T}{2} = \dfrac{\pi m}{Bq}$, which is independent of v.

Sol 9: (B, D) $r = \dfrac{mv}{Bq}$ or $r \propto m$

$\therefore r_e < r_p$ as $m_e < m_p$

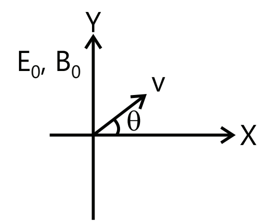
Further, $T = \dfrac{2\pi m}{Bq}$ or $T \propto m$



$\therefore T_e < T_p$, $t_e = \dfrac{T_e}{2}$ and $t_p = \dfrac{T_p}{2}$

or $t_e < t_p$

Sol 10: (C, D)

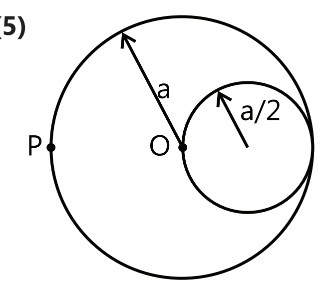


If $\theta = 0$ or 10^o

then particle moves in helical path with increasing pitch along Y-axis.

If $\theta = 90^o$ then magnetic force on the particle is zero and particle moves along Y-axis with constant acceleration.

Sol 11: (5)



$I = J \times \pi a^2$

$$B = \frac{\mu_0 J \times \pi a^2}{2\pi a} - \frac{\mu_0 J \times \pi}{2\pi \times \frac{3a}{2}} \times \frac{a^2}{4}$$

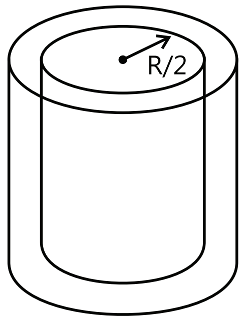
$$\Rightarrow B = \mu_0 Ja\left[\frac{1}{2} - \frac{1}{12}\right]$$

$$\Rightarrow B = \mu_0 Ja \times \frac{5}{12}$$

Sol 12: (B) $M = I \times \text{ Area of loop } \hat{k}$

$$= I \times \left[a^2 \times \frac{\pi a^2}{4 \times 2} \times 4 \right] \hat{k} = I \times a^2 \left[\frac{\pi}{2} + 1 \right] \hat{k}$$

Sol 13: (D)



$r < \frac{R}{2}$; $B = 0$

B at $r = \frac{R}{2}$

$$\Rightarrow B = \frac{\mu_0 JR}{2 \times 2} - \frac{\mu_0 JR}{2 \times 2} = 0$$

B at $r > \frac{R}{2}$

$$\Rightarrow B = \frac{\mu_0 JR}{2} - \frac{\mu_0 J \times \pi}{2\pi r} \times \frac{R^2}{4}$$

$$B = \frac{\mu_0 L}{2} \left[r - \frac{R^2}{4r} \right]$$

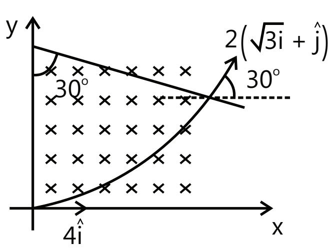
If we put $r = \frac{R}{2}$, B = 0

$\therefore$ B is continuous at r = R/2

Sol 14: (A, C) So magnetic field is along –ve, z-direction.

Time taken in the magnetic field $= 10 \times 10^{-1} = \frac{\pi M}{6QB}$

$$B = \frac{\pi M}{60 \times 10^{-3} Q} = \frac{1000\pi M}{60Q} = \frac{50\pi M}{3Q}$$



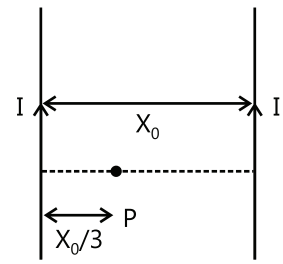
Sol 15: (B)

$$\frac{-2GMm}{L} + \frac{1}{2}mv^2 = 0 \Rightarrow v = 2\sqrt{\frac{GM}{L}}$$

Note: The energy of mass 'm' means its kinetic energy (KE) only and not the potential energy of interaction between m and the two bodies (of mass M each) – which is the potential energy of the system.

Sol 16: (3) Case – I

Case-I

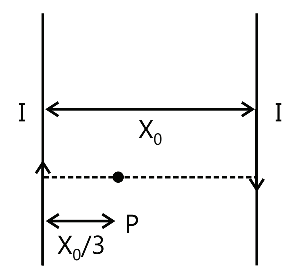


$$B_1 = \frac{1}{2}\left(\frac{\mu_0}{2\pi}\right)\left(\frac{3I}{x_0}\right)$$

$$R_1 = \frac{mv}{qB_1}$$

Case - II

Case-II



$$R_1 = \frac{mv}{qB_2}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{1/3}{1/9} = 3$$

Sol 17: (C) The net magnetic field at the given point will be zero if.

$$|\vec{B}_{wires}| = |\vec{B}_{loop}|$$

$$\Rightarrow 2\frac{\mu_0 I}{2\pi\sqrt{a^2 + h^2}} \times \frac{a}{\sqrt{a^2 + h^2}} = \frac{\mu_0 I a^2}{2(a^2 + h^2)^{3/2}}$$

$$\Rightarrow h \approx 1.2a$$

The direction of magnetic field at the given point due to the loop is normally out of the plane. Therefore, the net magnetic field due the both wires should be into the plane. For this current in wire I should be along PQ and that in wire RS should be along SR.

Sol 18: (B)

$$\tau = MB \sin\theta = I\pi a^2 \times 2 \times \frac{\mu_0 I}{2\pi d} \sin 30^\circ = \frac{\mu_0 I^2 a^2}{2d}$$

Sol 19: (A, B, C) $\vec{F} = 2I(L+R)[\hat{i} \times \hat{B}]$

2(L+R)

Sol 20: (A, D) $I_1 = I_2$

$\Rightarrow \; neA_1v_1 = neA_2v_2$

$\Rightarrow \; d_1w_1v_1 = d_2w_2v_2$

Now, potential difference developed across MK

V = Bvw

$$\Rightarrow \; \frac{V_1}{V_2} = \frac{v_1w_1}{v_2w_2} = \frac{d_2}{d_1}$$

Sol 21: (A, C) As $I_1 = I_2$

$n_1w_1d_1v_1 = n_2w_2d_2v_2$

Now $\dfrac{V_2}{V_1} = \dfrac{B_2v_2w_2}{B_2v_1w_1} = \left(\dfrac{B_2w_2}{B_1w_1}\right)\left(\dfrac{n_1w_1d_1}{n_2w_2d_2}\right) = \dfrac{B_2n_1}{B_1n_2}$