24. MODERN PHYSICS

1. INTRODUCTION

The developments in the post-classical/Newtonian physics, also known as modern physics, has given us revelatory insights into the structure and nature of fundamental forces/particles in the universe. The wave-particle duality/ paradox, which postulates that every elementary particle exhibits the properties of not only particles, but also waves is one such insight. For example, when electromagnetic radiation is absorbed by matter, it predominantly displays particle-like properties. It was de-Broglie who propounded the concept of matter waves, i.e. the particles exhibiting wave properties. We will be dealing here with the energy, wavelength, and frequency of electromagnetic waves and the relationship between these quantities. We will also be dealing with the photoelectric effect on which Einstein's based his photon theory of light. We will be discussing the Bohr atomic model, the hydrogen spectra, and the laws describing the characteristics of X-rays.

2. DUAL NATURE OF ELECTROMAGNETIC WAVES

Classical physics always defined motions in terms of particles or waves i.e., it treated particle and waves as distinct entities. An electron is considered as a particle because it possess mass (9.109×10^{-31} kilograms), electric charge (-1.602×10^{-19} coulomb) and they behave according to the laws of particle mechanics. However, we shall see that an electron in motion is as much a particle as it is a wave manifestation.

Electromagnetic radiation has properties in common with other forms of waves such as reflection, refraction, diffraction, and interference. It, however, also has particle-like properties in addition to those associated with wave motion (Photoelectric effect and Einstein's theory, black body radiation, Compton effect). Therefore, we can say that they a have dual nature.

Einstein's Equation: $E = hf = \dfrac{hc}{\lambda}$

2.1 Electromagnetic Spectrum

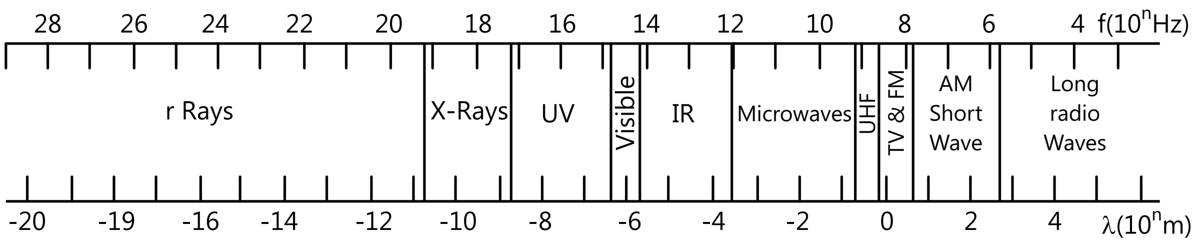


Figure 24.1 Electromagnetic spectrum

2.2 Electron Emission

Electrons are negatively charged particles. Therefore, though they move about arbitrarily in a conductor at room temperature, they cannot leave the surface of the conductor due to attraction of positively charged particles(protons). Therefore, some external energy has to be provided so that the electrons can be ejected from the atoms on the surface of the conductor. To eject the electrons which are just on the surface of the conductor, only minimal energy is required. This minimal energy or thermodynamic work that is needed to remove an electron from a conductor/ solid body to a point in the vacuum immediately outside the surface of the solid body/conductor is called the work function (denoted by W) of the conductor. Work function is the property of the metallic surface.

Heat, light, electric energy etc., can be employed to liberate an electron from a metal surface. Depending on the source of energy, the following methods are possible:

(a) **Thermionic emission:** In this method, the metal/conductor is heated to overcome binding potential of the conductor, and consequentially, free the electrons.

(b) **Field emission:** The emission of electrons induced by an electrostatic field is called field emission. In this process, the high electric field acting on the conductor exerts an electric force on the free electrons in the conductor in the opposite direction of field. This force overcomes the binding potential of the conductor and the electrons start coming out of the metal's surface

(c) **Secondary emission:** Ejection of electrons from a solid that is bombarded by a beam of charged particles (e.g., electrons or ions) is known as secondary emission.

(d) **Photoelectric emission:** The photoelectric effect refers to the emission/ejection of electrons from the surface of a metal in response to incident light (or electromagnetic wave). This happens when the incident light or electromagnetic wave has greater energy then the work function of the metal. The electrons emitted in this process are called **photoelectrons**.

3. PHOTOELECTRIC EFFECT

(a) The photoelectric effect was discovered by Wilhelm Ludwig Franz Hallwachs in 1888, the experimental verification which was done by Hertz.

(b) The photoelectric effect refers to the emission/ejection of electrons from the surface of a metal in response to incident light (or electromagnetic wave).

(c) The electron ejected due to photoelectric effect is is called a photoelectron and is denoted by e^-.

(d) Current produced as a result of the ejected electrons is called photoelectric current.

(e) Photoelectric effect proves quantum nature of light.

(f) Photoelectric effect can not be explained by the classical wave theory of light. The wave theory is incable of explaining the first 3 obserations of the photoelectric effect.

(g) Photoelectrons, generally, refer to the free electrons that are in the inter-molecular spaces in the metal.

(h) Explanation for Photoelectric effect was successfulyy explained given by Albert Einstein as being the result of light energy being carried in discrete quantized packets. For this excellent work he was honored with the Nobel prize in 1921.

(i) The law of conservation of energy forms the basis for photoelectric effect.

Threshold Frequency (v_0): The minimum frequency of the incident light or radiation that will produce a photoelectric effect i.e., ejection of photoelectrons from a metal surface is known as threshold frequency for that metal. Its value, though constant for a specific metal, may be different for different metals.

If v = frequency of incident photon & v_0 = Threshold Frequency

Then

(a) If $v < v_0$, there will be no ejection of photoelectron and, therefore, no photoelectric effect.

(b) If $v = v_0$, photoelectrons are just ejected from metal surface, in this case the kinetic energy of electron is zero.

(c) If $v > v_0$, then photoelectrons will come out of the surface along with kinetic energy.

Threshold Wavelength (λ_0): The greatest wavelength of the incident light or radiation for a specified surface for the emission of photoelectrons is known as threshold wavelength $\lambda_0 = \frac{c}{v_0}$ .For wavelengths above this threshold, there will be no photoelectron emission.

For λ = wavelength of incident photon, then

(a) If $\lambda < \lambda_0$, then photoelectric effect will take place and ejected electron will posses kinetic energy.

(b) If $\lambda = \lambda_0$, then just photoelectric effect will take place and kinetic energy of ejected photoelectron will be zero.

(c) If $\lambda > \lambda_0$, there will be no photoelectric effect.

3.1 Work Function or Threshold Energy (ϕ)

(a) The minimal energy or thermodynamic work that is needed to remove an electron from a conductor/solid body to a point in the vacuum immediately outside the surface of the solid body/conductor is called the work function or threshold energy for the conductor.

$$\phi = hv_0 = \frac{hc}{\lambda_0}$$

(b) Work function is the characteristic of given metal

(c) If E = energy of incident photon, then

- **(i)** If $E < \phi$, no photoelectric effect will take place.
- **(ii)** If $E = \phi$, just photoelectric effect will take place but the kinetic energy of ejected photoelectron will be zero.
- **(iii)** If $E > \phi$, photoelectric effect will take place along with possession of the kinetic energy by ejected electron.

3.2 Laws of Photoelectric Effect

Lenard postulated the following laws regarding photo emission on the basis of his experiments:

(a) For a given substance, there is a minimum value of frequency of incident light called threshold frequency below which no photoelectric emission is possible, howsoever, the intensity of incident light may be. It is given by $v_0 = \frac{c}{\lambda_0}$

(b) The number of photoelectrons emitted per second (i.e. photoelectric current) is directly proportional to the intensity of incident light provided the frequency is above the threshold frequency.

(c) The maximum kinetic energy of the photoelectrons is directly proportional to the frequency provided the frequency is above the threshold frequency. However, the relationship between the wavelength and kinetic energy is inversely proportional. With increasing frequency of incident light, the kinetic energy of photoelectron increases but with increasing wavelength it decreases. So $v \uparrow \lambda \downarrow$ K.E. of emitted electrons $\uparrow$ $v \downarrow \lambda \uparrow$ K.E. of emitted electrons $\downarrow$

(d) The maximum kinetic energy of the photoelectrons is independent of the intensity of the incident light.

(e) The process of photoelectric emission is instantaneous, i.e., as soon as the photon of suitable frequency falls on the substance, it emits photoelectrons.

(f) The photoelectric emission is one-to-one. i.e. for every photon of suitable frequency one electron is emitted.

(g) Value of threshold frequency or threshold wavelength depends upon photo sensitive nature of metal.

Illustration 1: The work function of silver is 5.26×10^{-19} J. Calculate its threshold wavelength. **(JEE MAIN)**

Sol: For any metal to eject photo electron the work function of surface is given as $\phi = \frac{hc}{\lambda_0}$

Threshold wavelength = $\lambda_0 = \frac{hc}{\phi}$; $\therefore \lambda = \frac{6.6\times10^{-34}\times3\times10^{8}}{5.26\times10^{-19}} = 3.764\times10^{-7}\,m$; $\lambda = 3764\,\mathring{A}$

Illustration 2: The work function of Na is 2.3 eV. What is the maximum wavelength of light that will cause photo electrons to be emitted from sodium? **(JEE MAIN)**

Sol: For any metal to eject photo electron the work function of surface is given as $\phi = \frac{hc}{\lambda_0}$

The threshold wavelength $\lambda_0 = \frac{hc}{\phi}$; $(\because \phi = hv_0 = \frac{hc}{\lambda_0})$;& hc = 1.24 × 10^{-6} (eV) m

$\lambda_0 = \frac{1.24\times10^{-6}}{2.3}$ m ; λ_0 = 0.539 × 10^{-6} m = 539 nm; $\lambda_o = 5930\,\mathring{A}$

3.3 Failure of Wave Theory to Explain Photoelectric Effect

Note - The assumptions of the classical wave theory could not explain some observations of the photoelectric effect. These aspects of the photoelectric effect were later explained by Albert Einstein's photon theory. The failures of the classical wave theory in explaining the photoelectric effect are enumerated below:

(a) The wave theory suggests that the intensity of the radiation should have a proportional relationship with the resulting maximum kinetic energy. However $K_{max} = eV_0$ suggests that it is independent of the intensity of light.

(b) According to the wave theory, the photoelectric effect should occur for any intense light, regardless of frequency or wavelength. However, the equation suggests that photo emission is possible only when frequency of incident light is either greater than or equal to the threshold frequency f_0.

(c) The wave theory states that there should be a delay on the order of seconds between the radiation's contact with the metal and the initial release of photoelectrons. It was assumed that between the impinging of the light on the surface and the ejection of the photoelectrons, the electron should be "soaking up" energy from the beam until it had accumulated enough energy to escape. However, no detectable time lag has ever been measured.

In reality, due to collision between atoms inside the metal, some energy is lost. Hence kinetic energy emitted by electrons is K< K_{max}. Hence the term K_{max} is used for the actual total kinetic energy.

3.4 Einstein's Photon Theory

Albert Einstein worked his way around the limitations of the classical wave theory by explaining that lights exists and travels as tiny packets/bundles called photons. The energy E of a single photon is given by E = hf

Applying the photon concept to the photoelectric effect, Einstein wrote:

$hf = W + K_{max}$ (Already discussed)

Discussed below is how Einstein's photon hypothesis overcomes the three objections raised against the wave theory interpretation of the photoelectric effect.

Objection 1: Intensity of the radiation should have a proportional relationship with the resulting maximum kinetic energy. This objection is overcome by Einstein's photon theory because, doubling the light intensity merely doubles the number of photons, thereby doubling the photoelectric current. It does not, however, change the energy of the individual photons.

Objection 2: Photoelectric effect should occur for any intense light, regardless of frequency or wavelength. The existence of a minimum frequency level (in Einstein's photon theory) follows from equation hf = W + K_{max}. If K_{max} equals zero, then $hf_0 = W$, which implies that the photon's energy will be barely adequate to eject the photoelectron and that there will be no residual energy to manifest as kinetic energy. The quantity W is the work function of the metal/substance. If the frequency f is reduced below f_0, the individual photons, irrespective of how numerous they are(in other words, no matter what the intensity of the incident light/radiation is), will not have enough energy to eject photoelectrons.

Objection 3: There should be a delay on the order of seconds between the radiation's contact with the metal and the initial release of photoelectrons. The absence of a time lag follows from the photon theory because the required energy is supplied in packets/bundles. Unlike in the wave theory, the energy is not spread uniformly over a large area.

Therefore, as far as photoelectricity goes, the photon/particle theory seems to be in total contradiction of the wave theory of light. Modern physicists have reconciled this apparent paradox by postulating the dual nature of light, i.e., light behaves as a wave under some circumstances and like a particle, or photon, under others.

3.5 Einstein's Equation of Photoelectric Effect

Einstein (1905) explained photoelectric effect on the basis of quantum theory.

According to Einstein, when photons fallon a metal surface, they transfer their energy to the electrons of metal. When the energy of photon is larger than the minimum energy required by the electrons to leave the metal surface, the emission of electrons take place instantaneously.

He proposed that after absorbing the photon, an electron either leaves the surface or dissipates its energy within the metal in such a short interval that it has almost no chance to absorb second photon

The energy supplied to the electrons is used in two ways:

(a) Removes the electron from the surface of metal

(b) Supplies some part of kinetic energy to the photoelectron. Therefore, Einstein's equation of photoelectric effect can be written as:

If v_{max} is the maximum velocity of emitted electrons then by law of conservation of energy

$hv = \phi + \frac{1}{2}mv^2$. If v_0 : Threshold frequency $\therefore$ $\phi_0 = hv_0$, So $\Rightarrow$ $hv = hv_0 + \frac{1}{2}mv_{max}^2$.

Einstein's equation explains the following concepts

(a) The frequency of the radiation/incident light is directly proportional to the kinetic energy of the electrons and the wavelength of radiation/incident light is inversely proportional to the kinetic energy of the electrons.

If v_0 is threshold frequency then maximum kinetic energy $E_{max} = hv - hv_0 \Rightarrow \frac{1}{2}mv_{max}^2 = h(v - v_0)$

So maximum velocity of photoelectrons: $\Rightarrow \quad v_{max} = \sqrt{\frac{2h(v - v_0)}{m}}$

m - mass of electron; v - frequency of incident light; v_0 - threshold frequency;

λ_0 - threshold wavelength λ - incident wavelength $\Rightarrow E_{max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \Rightarrow \frac{1}{2}mv_{max}^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$

(b) If $v = v_0$ or $\lambda = \lambda_0$ then v = 0

(c) $v < v_0$ or $\lambda > \lambda_0 \Rightarrow$ There will be no emission of photoelectrons.

(d) Intensity of the radiation or incident light refers to the number of photons in the light beam. More intensity means more number of photons and vice-versa. Intensity has no bearing on the energy of photons. Therefore, intensity of the radiation is increased, the rate of emission increases but there will be no change in kinetic energy of electrons. With increasing number of emitted electrons, value of photoelectric current increases.

Illustration 3: A light beam of wavelength 4000 Å is directed on a metal whose work function is 2 eV. Calculate the maximum possible kinetic energy of the photoelectrons. **(JEE MAIN)**

Sol: According to photoelectric equation the maximum kinetic energy of photoelectron after being ejected from metal is $E_K = h\nu - \phi$

Energy of the incident photon = $\frac{hc}{\lambda}$. Energy of the incident photon in eV = $\frac{19.8\times10^{-19}}{4\times1.6\times10^{-19}}$ = 3.09 eV ;

Kinetic energy of the emitted electron $E_K = h\nu - \phi$ = 3.09 – 2.00 = 1.09 eV

Illustration 4: Calculate the maximum kinetic energy of photoelectrons emitted from a metal with a threshold wavelength of 5800 Å, if the wavelength of the incident light is 4500 Å. **(JEE ADVANCED)**

Sol: The maximum kinetic energy of photoelectron with which it is ejected from metal is $E_K = h\nu - \phi$.

Therefore maximum velocity of photoelectron is $v_{max} = \sqrt{\frac{2E_{K_{max}}}{m_e}}$

$$E_{k_{max}} = \frac{hc[\lambda_0 - \lambda]}{\lambda_0\lambda} = 6.62 \times 10^{-34} \times 3 \times 10^8 \frac{[5800\times10^{-10} - 4500\times10^{-10}]}{5800\times4500\times10^{-20}} = 9.9 \times 10^{-20}\ J$$

$$E_{k_{max}} = \frac{9.9\times10^{-20}}{1.6\times10^{-19}} = 0.62\ eV ; \Rightarrow v_{max} = \sqrt{\frac{2hc(\lambda_0 - \lambda)}{m_e\lambda\lambda_0}} = \sqrt{\frac{2\times0.62\times1.6\times10^{-19}}{9.31\times10^{-31}}} = 4.67\times10^5\ m/s$$

3.6 Photoelectric Current

(a) When light/radiation is directed on a cathode, photoelectrons are emitted and these are attracted by an anode. The electric current, thus generated, flows in the circuit. This is called a photoelectric current.

(b) Value of photoelectric current depends upon following parameters:

(i) Potential difference between electrodes. **(ii)** Intensity of incident light.

3.6.1 Intensity of Light (I)

(a) It is quantity of light energy falling normally on a uniform surface area in unit time.

or $I = \frac{E}{A.t}$ where I = Intensity of light in $\frac{W}{m^2}$ E = total energy incident = nhv = $n\frac{hc}{\lambda}$

n = no. of photons; A = cross sectional area; T = time of exposure

(b) Intensity of light is proportional to saturation current

(c) For point source of light $I_r \propto \frac{I}{r^2}$

(d) For the Linear source of light $I_r \propto \frac{I}{r}$

Where r is the distance of the point from the light source.

3.7 Stopping Potential and Maximum Kinetic Energy

When the frequency f of the light/radiation is greater than the threshold frequency of the metal on which the light is directed, some photoelectrons are emitted from the metal with substantial initial speeds. Let us assume that E is the energy of light incident on a metal surface and W (<E) the work function of metal. In this case, as minimum energy is required to extract electrons from the surface, the emitted electrons will have the maximum kinetic energy which is E – W. $K_{max} = E - W$

As the potential V is increased, the electrons experience greater resistance/repulsion, and consequentially, less number of electrons reach the plate Q. This leads to a decrease in the flow of current in the circuit. At a certain value V_0, the electrons having maximum kinetic energy (K_{max}) also stop flowing and current in the circuit becomes zero. This is called the stopping potential.

(a) In photoelectric cell, when (+)ve voltage is applied on cathode and negative voltage is applied on anode applied, then the magnitude of photoelectric current decreases as the potential difference between the two points (cathode and anode) increases.

(b) The stopping potential is the negative potential (V_0) applied to the anode where the current gets reduced to zero or stops flowing in the circuit.

(c) When the magnitude of negative potential on anode is greater than or equal to magnitude of stopping potential the current in the circuit becomes zero.

(d) If emitted electrons do not reach from cathode to anode then stopping potential is given by

$$eV_0 = \frac{1}{2}mv_{max}^2 \text{ or } E_{max} = eV_0 \;;\; eV_0 = h(v - v_0) \;;\; V_0 = \frac{h(v - v_0)}{e}$$

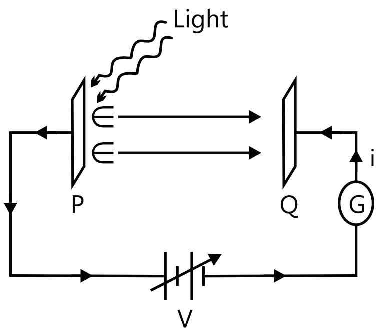


Figure 24.2: Photoelectric effect

(e) Value of stopping potential depends upon frequency of incident light.

(f) Stopping potential also depends upon nature of metal (or work function)

(g) Stopping potential does not depend upon intensity of light

(h) **Example**: Suppose stopping potential = -3 V, then $\frac{1}{2}mv_{max}^2 = 3eV$

If we apply – 5 V, then also there will be zero current in the circuit but $\frac{1}{2}mv_{max}^2 \neq 5eV$

Because stopping potential is not equal to 5V which cannot be used in Einstein's equation.

3.7.1 Graphs

(a) Kinetic energy V/s frequency: At $v = v_0$, $E_{max} = 0$

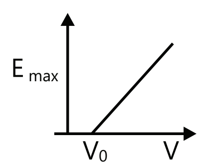


Figure 24.3

(b) V_{max} V/s v : At $v = v_0$ $V_{max} = 0$

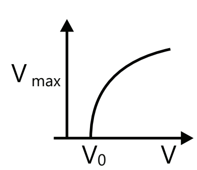


Figure 24.4

(c) Saturated Current V/s Intensity:

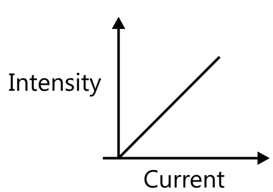


Figure 24.5

(d) Stopping potential V/s frequency:

$\because eV_0 = hv - hv_0$

$\tan\theta$ = slope

$= \frac{h}{e}$ (constant for all type of metals)

Intercept on x-axis = v_0

Intercept on y-axis = v

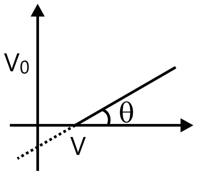


Figure 24.6

(e) Potential V/s current: (v : constant)

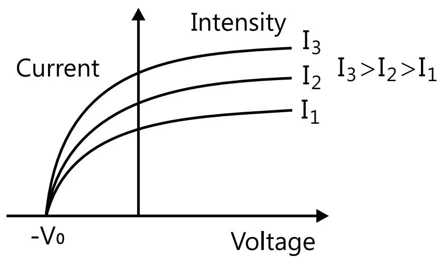


Figure 24.7

$\Rightarrow$ Stopping potential does not depend upon intensity of light.

(f) Photoelectric current V/s Retarding potential:

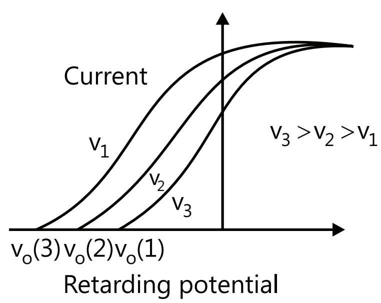


Figure 24.8

Illustration 5: Calculate the value of the stopping potential if one photon has 25 eV energy and the work function of material is 7 eV. **(JEE MAIN)**

Sol: The stopping potential required to stop the photoelectrons to reach cathode is $V_0 = \frac{E - \phi_0}{e}$

Stopping potential is $V_0 = \frac{E - \phi_0}{e} = \frac{25 - 7}{e} = \frac{18\text{ eV}}{e} \Rightarrow V_0 = 18\text{ V}$

3.8 Derivation of de-Broglie Wavelength

De broglie equation is given by: $\lambda = \frac{h}{p}$

Derivation: Let us start with the energy of a photon in terms of its frequency v, E = hv

Albert Einstein's special theory of relativity gives a new expression with reference to the velocity of light. This expression is $E = mc^2$, where m refers to the relativistic mass of light which is non-zero as it is travelling with velocity c. If it were at rest, it's mass would be zero.

Now, by equating both the energy equations we get $E = h\nu = mc^2$. Also, as seen earlier $\nu = \frac{c}{\lambda}$

Wavelength of a photon. $\therefore \frac{h}{\lambda} = mc$ and $\lambda = \frac{h}{mc}$

Analogously, de Broglie argued that a particle with non-zero rest mass m and velocity v would have a wavelength given $\lambda = \frac{h}{mv}$

Also, mv = p, where p is the particle's momentum. Substituting p for mv we get $\lambda = \frac{h}{p}$

3.8.1 Criterion for Type of Behaviour

Like electromagnetic waves, moving bodies also exhibit the wave-particle duality and the wave and particle aspects of moving bodies cannot be observed simultaneously. A moving body will exhibt particle behavior if the wavelength of the body is negligible in comparison to its dimensionwhereas it will exhibit a wave nature if its wavelength is in order of the dimension of body.

Illustration 6: Determine Broglie wavelengths of (a) a 46g golf ball with a velocity of 30 m/s and (b) an electron with a velocity of 10^7 m/s. **(JEE ADVANCED)**

Sol: The de-Broglie wavelength of the particle of mass m and moving with velocity v is given by $\lambda = \frac{h}{mv}$, where h is Planck's constant

(a) Since v<<c, we can let (effective mass = Rest mass) m =m_0 .

Hence $\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}\,J.s}{(0.04\,kg)(30\,m/s)} = 4.8 \times 10^{-34}$ m.

Thus, we see that the wavelength of the golf ball is so negligible compared with its dimensions that we would not be able to observe expect to find any wave aspects in its behavior.

(b) Again v << c, so with m = m_0 = 9.1 × 10^{-31} kg,

we have $\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}\,J.s}{9.1 \times 10^{-31} \times 10^7\,(kg\,m/s)} = 7.3 \times 10^{-11}\,m = 0.73\,\mathring{A}$

The dimensions of atoms are comparable with the radius of the hydrogen atom which in reality is 5.93 × 10^{-11} m. So, it it is clear that an electron with a wavelength of 7.3 × 10^{-11} m would demonstrate a wave behavior. Also, we can see that the wave character of moving electrons is facilitates the understanding atomic structure and behavior.

4. ENERGY, MOMENTUM, AND WAVELENGTH OF PHOTONS

(a) The quantum theory states that the light photons are undivided energy packets.

(b) Energy of photons is denoted by E = hv, where h is Planck constant, v is the frequency of photons, and E is the energy of photons.

(c) The velocity of photons and the velocity of light are equal (c). Therefore, c = $v\lambda$

Here, λ is the wavelength of wave connected to photon. $\therefore$ $E = hv = \frac{hc}{\lambda}$

(d) The mass of photons at rest is zero but it will be non-zero if the photons are moving.. Assuming m to be the effective mass of photons, energy of photon according to Einstein:

$E = mc^2 \Rightarrow E = hv = \frac{hc}{\lambda} = mc^2$

(e) Momentum of moving photon $p = mc = \frac{mc^2}{c} = \frac{E}{c} = \frac{1}{c}\left(\frac{hc}{\lambda}\right) = \frac{h}{\lambda} \Rightarrow p = \frac{h}{\lambda}$

(f) Effective mass $m = \frac{E}{c^2} = \frac{hv}{c^2} = \frac{h}{c\lambda} = \frac{p}{c}$

(g) Wavelength connected to moving photons $\lambda = \frac{h}{p} = \frac{h}{mc} = \frac{hc}{E}$

(h) **From Point (e) and (f):-** Momentum of photon $p \propto m$ $p \propto E$ Energy of photons $E \propto m$

Wavelength of wave connected to photons $\lambda \propto \frac{1}{p}$; $\lambda \propto \frac{1}{m}$; $\lambda \propto \frac{1}{E}$

(i) **Graphs**

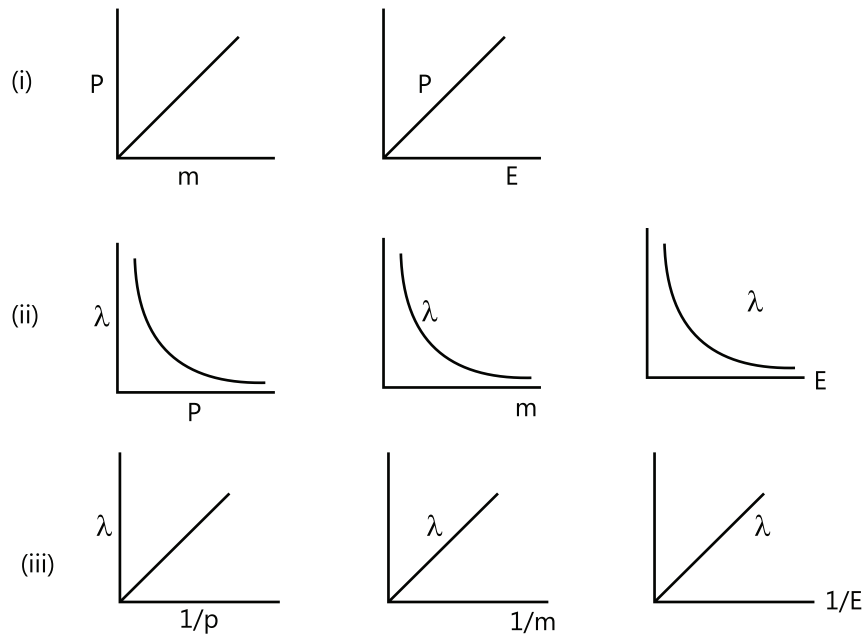


Figure 24.9

(j) There is no charge on photons

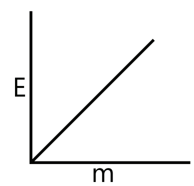


Figure 24.10

Illustration 7: A Determine the velocity of a light wave, given that frequency of the photon is ν, energy is hν, and momentum is $p = \frac{h}{\lambda}$ **(JEE MAIN)**

Sol: For light of frequency ν, the energy is E = hν and the frequency of light wave is $\nu = \frac{c}{\lambda}$. Hence speed of light is easily determined.

As E = hν and $P = \frac{h}{\lambda}$,

$E = \frac{hc}{\lambda} = Pc \Rightarrow c = \frac{E}{P}$

Illustration 8: Determine the mass of a photon witha wavelength of 0.01 $\overset{\circ}{A}$. **(JEE MAIN)**

Sol: Using equation of equivalent mass of photon, $m = \frac{h}{c\lambda}$, we can find the mass of proton.

$m = \frac{E}{c^2} = \frac{h}{c\lambda} = \frac{6.62 \times 10^{-34}}{3 \times 10^8 \times 10^{-12}}$; $m = 2.21 \times 10^{-30}$ kg

Illustration 9: Determine the momentum of a photon with a of frequency 10^9 Hz. **(JEE MAIN)**

Sol: The momentum of photon is $p=\frac{h\nu}{c}$

$$p=\frac{h}{\lambda}=\frac{h\nu}{c}=\frac{6.62\times10^{-34}\times10^{9}}{3\times10^{8}}\ ;\ p=2.2\times10^{-33}\ \text{kg m/s}$$

Illustration 10: Determine the energy and momentum of a γ-ray photon with a wavelength of 0.01 Å. **(JEE MAIN)**

Sol: For wave of wavelength λ the energy and momentum is given by $E=\frac{hc}{\lambda}$ and $p=\frac{h}{\lambda}=\frac{E}{c}$

$$E=\frac{hc}{\lambda}=\frac{1240(\text{eV})\times1\times10^{-9}}{0.01\text{Å}}\ ;$$

$$E=\frac{1240\times10^{-9}}{10^{-2}\times10^{-10}}(\text{eV})=1.24\times10^{6}\ \text{eV}\ ;\ E=1.24\ \text{MeV}\ ;\ \text{The momentum is } P=\frac{E}{c}=1.24\ \frac{\text{MeV}}{c}$$

$$P=\frac{1.24\times10^{6}\times1.6\times10^{-19}}{3\times10^{8}}=6.62\times10^{-22}\ \text{kg m/s}$$

5. ENERGY, MOMENTUM, AND WAVELENGTH OF A MOVING PARTICLE

Suppose the mass of a particle at rest is m and it is moving with velocity v.

(a) Mass at rest = m

(b) Effective mass (or relativistic mass) $=\sqrt{\frac{m}{1-v^2/c^2}}$

(c) Momentum or $\vec{p}=m\vec{v}$ or p = mv $\therefore$ $p=mv=\sqrt{2mE}$ Here E : Kinetic energy

(d) Kinetic energy $E=\frac{1}{2}mv^2=\frac{p^2}{2m}$

(e) If λ is the wavelength of connected wave to the moving particle, then $\lambda=\frac{h}{p}=\frac{h}{mv}=\frac{h}{\sqrt{2mE}}$

Illustration 11: A body of 10 gm is moving with velocity 2×10^{3} m/s. Determine the value of its associated de-Broglie wavelength. **(JEE ADVANCED)**

Sol: The de-Broglie wavelength associated with particle moving with speed v is calculated as $\lambda=\frac{h}{mv}$.

de-Broglie wavelength $\lambda=\frac{h}{mv}=\frac{6.62\times10^{-34}}{10\times10^{-3}\times2\times10^{3}}$;

$\lambda=3.3\times10^{-35}$ m

So $\lambda\propto\frac{1}{p}$; $\lambda\propto\frac{1}{\sqrt{E}}$

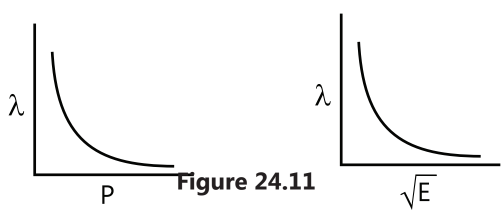


Figure 24.11

$\lambda\propto\frac{1}{v}$ $\lambda\propto\frac{1}{\sqrt{E}}$

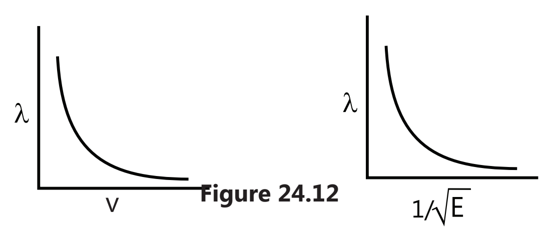
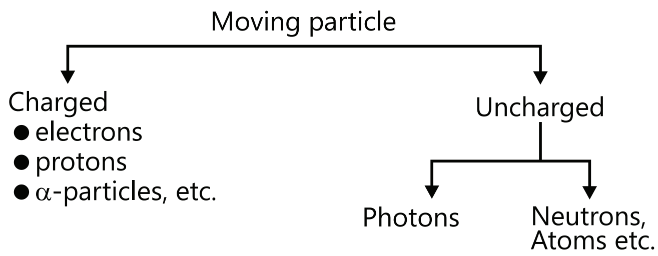


Figure 24.12



5.1 Energy, Momentum, and Wavelength of Charged Particle Accelerated by V-volt

(a) Potential difference or electric field can be used to accelerate a charged.

(b) The kinetic energy of a charged particle having charge q, mass m, accelerated by V volt, and a velocity v is denoted by $E = \frac{1}{2}mv^2 = qV$

(c) Velocity V = $\sqrt{\frac{2qV}{m}} = \sqrt{\frac{2E}{m}}$

(d) Momentum p = $\sqrt{2mE} = \sqrt{2mqV}$

(e) Wavelength $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$

(Here it is assumed that initial potential given to electron is zero)

If the particle is given some initial potential V_i and if final potential is V_f then, $\lambda = \frac{h}{\sqrt{2mq(V_f - V_i)}}$

From above Relation

$\lambda \propto \frac{1}{\sqrt{V}}$

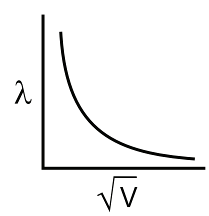


Figure 24.13

$\lambda^2 \propto \frac{1}{V}$

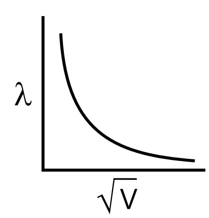


Figure 24.14

$\lambda \propto \frac{1}{\sqrt{V}}$

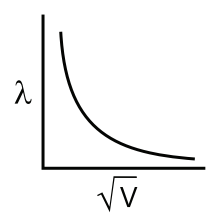


Figure 24.15

Cases:

(a) If the moving charged particle is an electron, then

(i) $v_e = \sqrt{\frac{2eV}{m_e}}$ **(ii)** $p_e = \sqrt{2\, m_e eV}$ **(iii)** $\lambda_e = \frac{h}{\sqrt{2m_e eV}} = \frac{12.27}{\sqrt{V}}$ Å

(b) If the moving charged particle is a proton, then

(i) $v_P = \sqrt{\frac{2eV}{m_p}}$ **(ii)** $p_P = \sqrt{2m_e pV}$ **(iii)** $\lambda_P = \frac{h}{\sqrt{2m_e pV}} = \frac{0.286}{\sqrt{V}}$ Å

(c) If the charged particle is an α-particle, then

(i) $v_\alpha = \sqrt{\frac{2(2e)V}{m_\alpha}} = \frac{eV}{m_P} = \frac{1}{\sqrt{2}} v_P$ **(ii)** $P_\alpha = \sqrt{2m_\alpha e_\alpha V} = \sqrt{2\times 8m_p \times eV} = 2\sqrt{2}\, p_p$

(iii) $\lambda_\alpha = \frac{h}{\sqrt{16m_p eV}} = \frac{0.101}{\sqrt{V}}$ Å

Illustration 12: Determine the potential to be applied to accelerate an electron such that its de-Broglie wavelength becomes 0.4 Å. **(JEE MAIN)**

Sol: The de-Broglie wavelength of an electron in terms of accelerating potential difference is $\lambda_e = \frac{12.27}{\sqrt{V_0}}$ Å

Where V is the applied potential on electron to accelerate it. $\lambda = \frac{12.27}{\sqrt{V_0}}$ Å ; $0.4 = \frac{12.27}{\sqrt{V_0}}$

Squaring on both the sides we get $0.16 = \frac{(12.27)^2}{V_0} \Rightarrow V_0 = \frac{12.27\times 12.27}{16\times 10^{-2}}$;$\Rightarrow V_0 = 941.0$ V

5.2 Wavelength of Wave Connected to Uncharged Particle

(Like neutron atoms, molecules etc.)

(a) If m is the mass and v is the velocity of particle, then kinetic energy $E = \frac{1}{2}mv^2$, momentum p = mv

(b) Wavelength $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$

(c) If λ is the wavelength of wave connected to matter particle then particle energy E will be

$$E = \frac{h^2}{2m\lambda^2}\ J = \frac{h^2}{(2m\lambda^2)e} eV$$

(d) Energy E of particle (e.g., electron, neutron, or atom) at equilibrium temperature TE = (3/2) KT

Here K = Boltzman constant

(e) $\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{3mKT}}$ here m : mass of a single atom.

Illustration 13: Determine the associated de-Broglie wavelength if the energy of a thermal neutron is 0.02 eV, **(JEE MAIN)**

Sol: For neutron having kinetic energy K, the associated de-Broglie wavelength is found to be $\lambda = \frac{h}{\sqrt{2mK}}$

de-Broglie Wavelength $\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.6\times10^{-34}}{\sqrt{2\times1.6\times10^{-27}\times0.02\times1.6\times10^{-19}}}$; $\lambda = 2\times10^{-10}\,m = 2Å$

6. EXPERIMENTAL VERIFICATION OF MATTER WAVES

The Davisson–Germer experiment conducted by American physicists Clinton Davisson and Lester Germer confirmed the De Broglie hypothesis which says that the particles of matter such as electrons have wave-like properties (see Fig. 24.16).

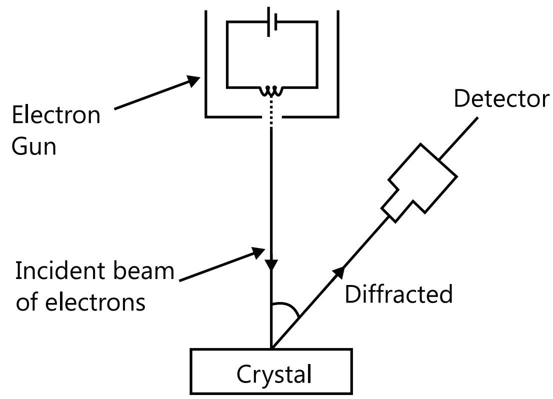


Figure 24.16: Diffraction of matter waves

(a) Davisson–Germer's experiment

(i) Experimental confirmation of De Broglie waves was done by scientist Davisson and Germer by firing slow-moving electrons at a crystalline nickel target. The diffraction pattern of electron beam through the nickel crystal was same as those predicted by Bragg for X-rays.

(ii) Since diffraction is the property of waves, the diffraction of electronic beam confirmed that a wave is connected to moving electron beam.

(iii) The electron gun was used to obtain electrons with different energies. This was done by accelerating the e^- by V volt in the electron gun.

(iv) When these accelerated electrons fall on a crystal, they are diffracted in various directions.

(v) Electrons were collected by a detector of Faraday cup which was connected to an electrometer.

(vi) Electron beams with different energies produced different intensities of diffracted electrons.

(vii) Results of the Davisson–Germer's experiment

- Intensity at any angle is proportional to the distance of the curve at that angle from the point of scattering.

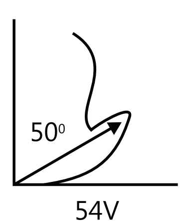


Figure 24.17

- Intensity is maximum at 54 V potential difference and 50° diffraction angle.

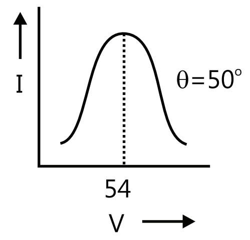


Figure 24.18

(viii) From Bragg's Law :- $D\sin\theta = n\lambda$ (constructive interference)

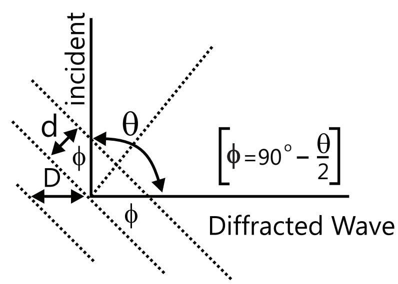


Figure 24.19

θ : Angle between incident ray and nth maxima.

n: Diffraction order

D: Distance between atoms or 2d $\sin\phi = n\lambda$

D: Distance between lattice planes

ϕ : Angle between diffraction plane and incident ray.

(ix) The critical value of wavelength of an accelerated electron at 54 V = 1.67Å

Experimental value = 1.65 Å

(x) Any wave or particle is diffracted by crystal plane only when wavelength is in order of distance between lattice planes of an atom.

Illustration 14: In a Davisson-Germer experiment, a, electron beam of wavelength 1.5 Å is normally incident on a crystal, having 3Å distance between atoms. Determine the angle at which first maximum occurs. **(JEE MAIN)**

Sol: According to Davison–Germer's experiment, when electrons accelerating at some potential difference V are incident on a crystal, they diffract. The angle at which the first maxima of diffraction pattern occurs can be found by Bragg's law i.e., $D\sin\theta = n\lambda$

$D\sin\theta = n\lambda \;\therefore\; \sin\theta = \frac{n\lambda}{D} = \frac{1\times1.5}{3} = \frac{1}{2},\ \theta = 30^\circ$

7. ATOMIC MODELS

Model : A model is simply a testable idea or hypothesis based on logical and scientific facts.

Theory : A model becomes a theory when it is verified by rigorous scientific analysis and experiments. . Otherwise, the model is simply not accepted.

7.1 Dalton's Atomic Model

(a) All matter is made of tiny particles called atoms. Atoms are indivisible and indestructible.

(b) All atoms of a given element are identical in mass and properties, while atoms of different elements differ in mass and properties.

(c) All matter is made up of hydrogen atoms. The mass and radius of heaviest atom is about 250X and 10X of the than that of the hydrogen atom, respectively.

(d) Atoms are stable and electrically neutral.

Reason of Failure of model: The discovery of electron by J.J. Thomson (1897) proved that atoms are not indivisible. Hence, the model is no longer valid.

7.2 Thomson's Atomic Model (or Plum-Pudding Model)

In this model, the atom is composed of electrons (which Thomson still called "corpuscles") surrounded by a soup of positive charge to balance the electrons' negative charges, like negatively charged "raisins" surrounded by positively charged "pudding".

Achievements of model: Explained successfully the phenomenon of thermionic emission, photoelectric emission, and ionization.

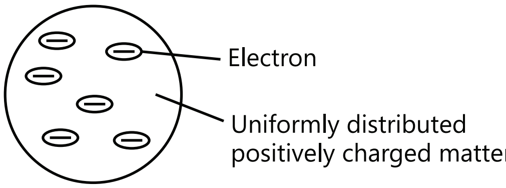


Figure 24.20: Thomson's Atomic Model

Failure of the model:

(a) It could not explain the line spectrum of H-atom.

(b) It could not explain the Rutherford's α – particle scattering (Rutherford gold foil) experiment.

7.3 Rutherford's Experiment and Atomic Model

α -Scattering Experiment:

Results of Experiment:

(a) It was seen that in the experiment that when the α - particles were fired at the gold foil, some of the particles (<1 in 8000) bounced off the metal foil in all directions, some right back at the source. This should have been impossible according to Thomson's model; the alpha particles should have all gone straight through. Obviously, those particles had encountered an electrostatic force far greater than Thomson's model suggested they would, which in turn implied that the atom's positive charge was concentrated in a much tinier volume than Thomson imagined. This was possible only in the case when there exists a solid positive mass confining in a very narrow space.

(b) However, most of the α - particles just flew straight through the foil. This suggested that those tiny spheres of intense positive charge were separated by vast gulfs of empty space.

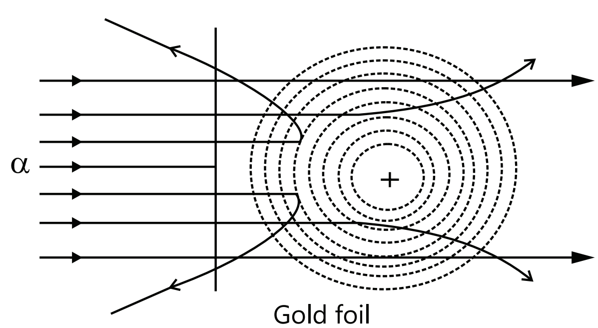


Figure 24.21: Scattering of alpha particles by gold nucleus

(c) $N \propto \dfrac{1}{\sin^4\left(\dfrac{\theta}{2}\right)} \Rightarrow$ If $\theta\uparrow$ then $N\downarrow$, N = No. of particles scattered per unit time

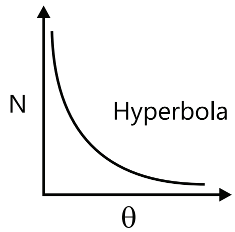


Figure 24.22

Equation indicates that at larger deflection (scattering) angle, number of particles deflected are very-very less.

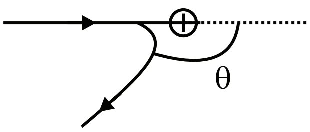


Figure 24.23

Graph for N & θ show that coulomb's law holds for atomic distances also.

(d) $N \propto$ (Nuclear charge)^2

Illustration 15: In an α - particle scattering experiment using gold foil, the number of particles scattered at 60° is 1000 per minute. What will be the number of particles per minute scattered at 90° angle? **(JEE ADVANCED)**

Sol: In Rutherford's experiment, the number of particles deflected at an angle θ by the gold atoms per minute are best represented by relation $N \propto \dfrac{1}{\sin^4\left(\dfrac{\theta}{2}\right)}$

Let N = No. of α - particles scattered per minute at an angle 90°.

$$\therefore \quad N \propto \frac{1}{\sin^4\left(\frac{90}{2}\right)} \qquad \text{... (i)}$$

Given that $1000 \propto \dfrac{1}{\sin^4\left(\dfrac{60}{2}\right)}$ ... (ii)

Taking ratio of (i) to (ii) we get $N = 1000 \times \dfrac{\sin^4\left(\dfrac{60}{2}\right)}{\sin^4\left(\dfrac{90}{2}\right)} = 250$ /min

Rutherford's Atomic Model

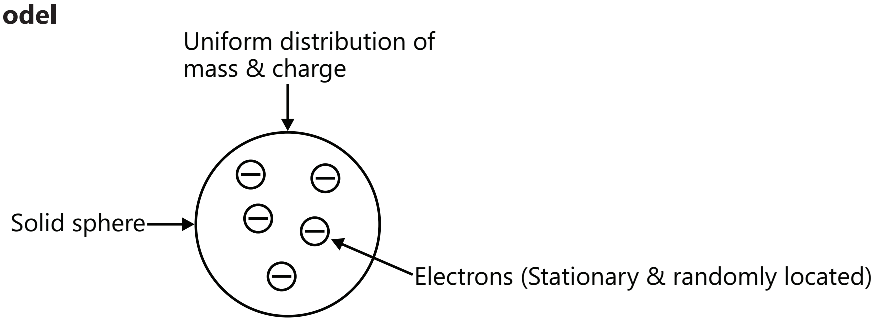


Figure 24.24: Thomson's Atomic Model

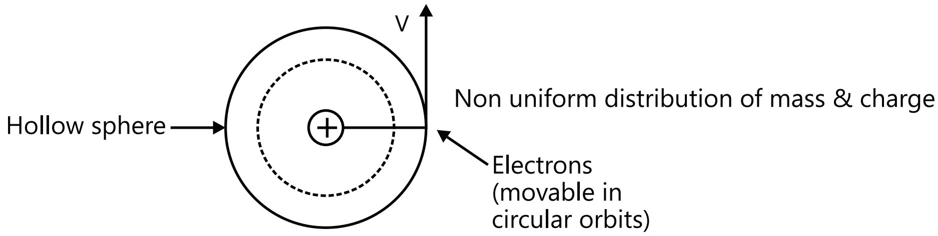


Figure 24.25: Ruthorford's Atomic Model

(a) The whole positive charge and almost whole mass of an atom (leaving aside the mass of revolving e^- in various circular orbits) remains concentrated in nucleus of radius of the order of 10^{-15} m.

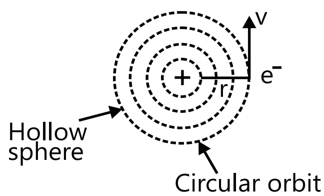


Figure 24.26: Motion of electron in atom

(b) $\Sigma q(+)ve$ on proton in a nucleus = $\Sigma q(-)ve$ on e^- in various circular orbits & hence, the atom is electrically neutral.

(c) The necessary centripetal force for revolving round the nucleus in circular orbit is provided by coulomb's electrostatic force of attraction $\frac{mv^2}{r} = \frac{k(ze)(e)}{r^2}$

Reason of failure of model

(a) It could not explain the line spectrum of H-atom.

Justification: Asper Maxwell's electromagnetic theory, every accelerated moving charged particle emits energy in the form of electromagnetic waves and, therefore, the frequency of an Modern Physics - Solution (1) while moving in a circular orbit around the nucleas will steadily decline, resulting in the continuous emission of lines thereby mandating that the spectrum of an atom be continuous, but in reality, one obtains line spectrum for atoms.

(b) It could not explain the stability of atoms.

Justification : Since revolving electron continuously radiates energy, the radii of circular path will continuously decrease and in a time of about 10^{-8} s the revolving electron must fall down in a nucleus by adopting a spiral path.

Application of Rutherford's model

Determintion of distance of closest approach: When a positively charged particle approaches a stationary nucleus (which is the positively charged core of the atom), then due to repulsion between the two (like charges repel), the kinetic energy of positively charged particle gradually decreases, reaching a stage where its kinetic energy becomes zero and from where it again starts retracing its original path.

Definition: The distance of closest approach is the minimum distance of a stationary nucleus from a point where the kinetic energy of a positively charged particle approaching the nucleas for a head-on collision becomes zero. Suppose a positively charged particle A of charge q_1 ($=z_1e$) approaches from infinity towards a stationary nucleus of charge z_2e then,

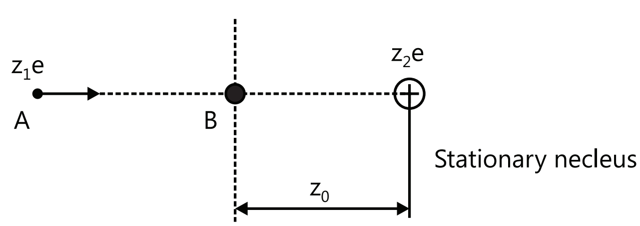


Figure 24.27: Distance of closest approach

Let at point B, kinetic energy of particle A becomes zero then by the law of conservation of energy at point A & B.

$TE_A = TE_B$; $KE_A + PE_A = KE_B + PE_B$; $E + 0 = 0 + \dfrac{k(z_1e)(z_2e)}{r_0}$ (in joule) $\therefore$ $r_0 = \dfrac{k(z_1e)(z_2e)}{E}$ m

Illustration 16: Calculate the distance of closest approach where an α-particle with kinetic energy 10 MeV is heading towards a stationary point-nucleus of atomic number 50. **(JEE MAIN)**

Sol: The nucleus of tin (atomic number 50) being more massive than the alpha particle, remains stationary. So the kinetic energy of the alpha particle is converted into electric potential energy at the distance of closest approach.

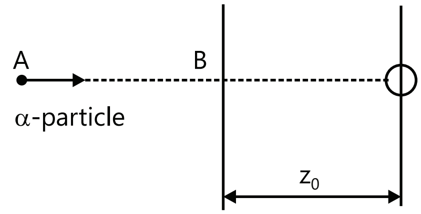


Figure 24.28

The electric potential energy of alpha particle is $TE_\alpha = \dfrac{K \times (Z_1e) \times (Z_2e)}{r_0}$

where $K = \dfrac{1}{4\pi\varepsilon_0}$ and r_0 is the distance of closest approach of alpha particle from nucleus of tin.

$TE_A = TE_B$; $\therefore$ $10 \times 10^6 \text{ eV} = \dfrac{K \times (2e)(50e)}{r_0}$

$r_0 = 1.44 \times 10^{-14}$ m ; $r_0 = 1.44 \times 10^{-4}$ Å

Illustration 17: Find the distance of closest approach for a proton moving with a speed of 7.45×10^5 m/s towards a free proton originally at rest. **(JEE MAIN)**

Sol: As the moving proton approaches the free proton originally at rest, it exerts an electric force of repulsion on the proton at rest. At the distance of closest approach, both the protons move with same velocity along the line of impact. The initial kinetic energy of moving proton is equal to the final kinetic energy of both the protons plus the electric potential energy at the distance of closest approach, given by $\dfrac{Ke^2}{r_0}$. Here r_0 is the distance of closest approach.

$\rightarrow$ V = 7.45×10^5 m/s u = 0

O Proton O Free proton

Originally

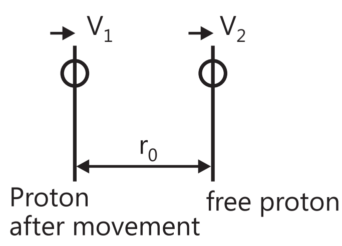


Figure 24.29

At the time of distance of closest approach

By the law conservation of energy

$$\frac{1}{2}mv^2 + 0 = \frac{ke^2}{r_0} + \frac{1}{2}mv_1^2 + \frac{1}{2}mv_1^2 \qquad \text{... (i)}$$

By the conservation of momentum mv + 0 = $mv_1 + mv_1$ $\therefore$ $v_1 = \dfrac{v}{2}$

From equation (i) $\dfrac{1}{2}mv^2 = \dfrac{ke^2}{r_0} + m\left(\dfrac{v}{2}\right)^2$; $r_0 = \dfrac{4}{mv^2} \times ke^2 = \dfrac{4 \times (9 \times 10^9)(1.6 \times 10^{-19})^2}{(1.66 \times 10^{-27})(7.45 \times 10^5)^2}$ $r_0 = 1.0 \times 10^{-12}$ m

7.4 Bohr's Model

Bohr combined the concepts of classical physics with quantum mechanics to propose his model for H or H-like atoms. This model is based on law of conservation of angular momentum.

(a) According to de Broglie, in a stationary orbit the circumference of Bohr's orbit must be an integral multiple of the wavelength associated with the moving particle

or $2\pi r = n\lambda$ (Constructive interference)

or $2\pi r = \dfrac{nh}{mv}$ or $mvr = \dfrac{nh}{2\pi}$ which is Bohr's quantum condition.

(b) In an orbit, waves are always formed in whole numbers..

7.4.1 Concept of Stable, Stationary, Quantized, Fixed Allowed Radii Orbit, or Maxwell's Licensed Orbits

According to Bohr, if an electron revolves in these orbits the electron neither radiates nor absorbs any energy.

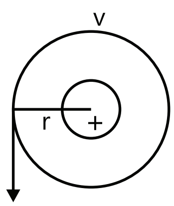


Figure 24.30: Bohr radius

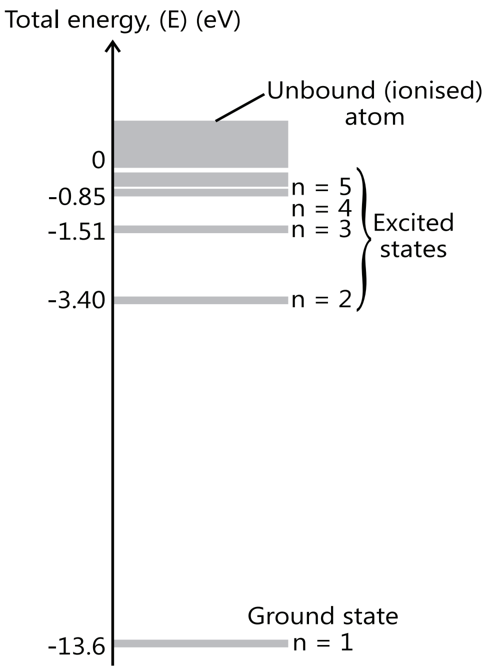


Figure 24.31: Energy level diagram

(b) **Emission of energy**

Where n = principle quantum no.

E_n = energy of e^- in nth orbit

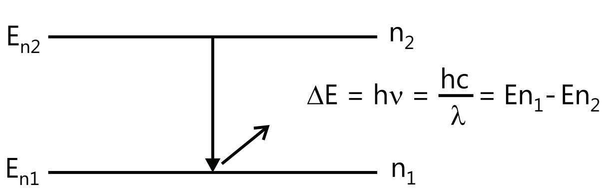


Figure 24.32: Emission of energy by electron

(c) Absorption of energy

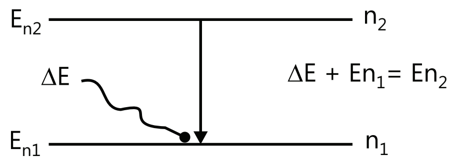


Figure 24.33: Absorption of energy by electron

Electron revolves only in those orbits in which its angular momentum is integer multiple of $\frac{h}{2\pi}$

$$mvr = I\omega = n\frac{h}{2\pi}$$

$$\frac{mv^2}{r} = \frac{kZe^2}{r^2}$$

7.4.2 Determination of Radius, Velocity & Energy of e⁻ in Bohr's Orbit

(a) Determination of radius of circular path (orbit)

$$\because\ mvr = \frac{nh}{2\pi} \quad \text{... (i)}$$

$$\therefore\ v = \frac{nh}{2\pi mr} \quad \text{... (ii)}$$

and $\frac{mv^2}{r} = \frac{kZe^2}{r^2}$; $\quad \therefore\ m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{kZe^2}{r}$; $\quad r_n = v\left(\frac{n^2h^2}{4\pi^2 mkZe^2}\right)$; $\quad r_n = \frac{n^2}{Z} \times \frac{h^2}{4\pi^2 mke^2}$

$$r_n = \frac{n^2}{Z} \times 0.529\ \mathring{A}$$

Results:

(i) $\because\ r_1 = \frac{(1)^2}{z} \times 0.529\,\mathring{A}$; $\therefore\ r_n = n^2 r_1$

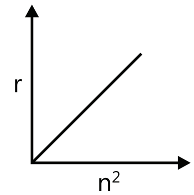


Figure 24.34

(ii) $\therefore\ r \propto n^2$

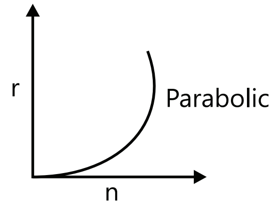


Figure 24.35

Illustration 18: The radius of the shortest orbit of a single-electron system is 18 pm. This system can be represented as **(JEE MAIN)**

Sol: According to Bohr's model, the radius of orbit of electron is directly proportional to square of principle quantum number i.e., $r_n \propto n^2$. When the electron is in ground state (i.e., for principle quantum number =1) $r_1 = \frac{0.529}{Z}\mathring{A}$

For shortest orbit n = 1; $r_n = n^2 r_1$; $\frac{(1)^2}{Z}$ × 0.529 Å = 18 × 10^{-2} Å

$\Rightarrow Z = 3$ system is Li^{2+} since only single e is present.

Illustration 19: What will be the ratio of the area of circular orbits in doubly ionized lithium atom in 2nd & 3rd Bohr orbit? **(JEE MAIN)**

Sol: According to Bohr's theory as $r_n \propto n^2$, but $A \propto r^2$. Therefore $A \propto n^4$.

$$\therefore \frac{A_2}{A_3} = \frac{(2)^4}{(3)^4} = \frac{16}{81}$$

7.4.3 Determination of Velocity of Electron in Circular Orbit

$$\therefore mvr = \frac{nh}{2\pi} \qquad ...(i)$$

$$r = \frac{nh}{2\pi mv}; \Rightarrow \frac{mv^2}{r} = \frac{kZe^2}{r^2} \Rightarrow v = \frac{2\pi kZe^2}{nh}; \Rightarrow \quad v = \frac{Z}{n} \times \frac{2\pi ke^2}{h} \Rightarrow v = 2.18 \times 10^6 \frac{Z}{n} \text{ m/s}$$

$v = \frac{c}{137}\frac{Z}{n}$ m/s; where c = velocity of light in vacuum = 3×10^8 m/s

Results:

(i) $v \propto \frac{1}{n}$ (Z = constant)

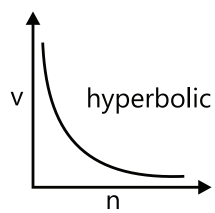


Figure 24.36

Illustration 20: What will be the ratio of speed of electrons in hydrogen atom in its 3rd & 4th orbit? **(JEE MAIN)**

Sol: According to the Bohr's theory $v \propto \frac{Z}{n}$ where v is the speed of electron in its orbit, n is the principle quantum number and Z is the atomic number of the element.

$$\because \; v \propto \frac{Z}{n} \quad \therefore \frac{v_3}{v_4} = \frac{4}{3}$$

Illustration 21: What will be the the ratio of speed of electron in 3rd orbit of He⁺ to 4th orbit of Li⁺⁺ atom? **(JEE MAIN)**

Sol: According to the Bohr's theory $v \propto \frac{Z}{n}$, where v is the speed of electron in its orbit n is the principle quantum number and Z is the atomic number of the element.

Here the element in consideration differs in atomic number, i.e., Z(He) = 2 and Z (Li) =3

$$\therefore \frac{\left(v_3\right)_{He^+}}{\left(v_4\right)_{Li^{2+}}} = \frac{\frac{2}{3}}{\frac{3}{4}} = \frac{8}{9}.$$

7.4.4 Determination of Energy of Electron in Bohr's Circular Orbit

(a) Kinetic energy of electron $KE = \frac{1}{2}mv^2$ **;** $KE = \frac{kZe^2}{2r}$

Results:

(i) KE of an e^- = positive quantity

(ii) $r\uparrow$, $KE\downarrow$

(iii) when, $r = \infty$, KE = 0

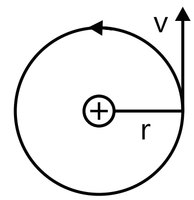


Figure 24.37: Bohr's Orbit

(b) Potential energy of an electron $PE = \frac{K(+Ze)(-e)}{r}$; $PE = -\frac{KZe^2}{r}$

Results:

(i) Potential energy (PE) of an e^- = negative quantity **(ii)** $r\uparrow$, $PE\uparrow$ (c) If $r = \infty$, PE = 0

(c) Total energy of electron: The total energy of an electron in any orbit equals the sum of its kinetic and potential energy in that orbit. $TE = KE + PE = \frac{KZe^2}{2r} - \frac{KZe^2}{r}$; $TE = -\frac{KZe^2}{2r}$

Results:

(i) TE of an electron in atom = (-)ve quantity. (-)ve sign indicates that electron is in bound state.

(ii) If $r\uparrow$, $TE\uparrow$

(iii) if $r = \infty$, TE = 0

(iv) TE = -KE = $\frac{PE}{2}$ in any H-like atom

Total energy of terms of n

$$TE = -\frac{kZe^2}{2\times\left(\frac{n^2h^2}{4\pi^2 mkZe^2}\right)}$$

$$TE = -\frac{2\pi^2 mk^2Z^2e^4}{n^2h^2} \Rightarrow TE = -R\,ch\,\frac{Z^2}{n^2} \Rightarrow TE = -13.6\frac{Z^2}{n^2}ev$$

where R = Rydberg constant = $\frac{2\pi^2 mk^2e^4}{ch^3} = \frac{me^4}{8\epsilon_0^2 ch^3} = 1.097\times10^7\ m^{-1}$

Note: Rydberg constant is not a universal constant. In Bohr calculation, it is determined by assuming the nucleus to be stationary

For Bohr Rydberg constant, $R_\infty = 1.097\times10^7 m^{-1}$, if nucleus is not assumed stationary then

$$R = \frac{R}{1+\left(\frac{m_e}{m_N}\right)}$$, m_N = mass of nucleus

7.4.5 Results Based on Total Energy Equation

(a) With the increase in principal quantum number n (relative overall energy of each orbital), both total energy and potential energy of an electron increases, whereas the kinetic energy decreases.

(b) With the increase in principal quantum number, the difference between any two consecutive energy level decreases.

(c) Total energy of an electron in any orbit in H-like atom = (Total energy of an electron in that orbit in H-atom $\times Z^2$)

(d) PE of an electron in any orbit in H-like atom = (PE of an electron in that orbit in H-atom) $\times Z^2$ (v) KE of an electron in any orbit in H-like atom= (KE of an electron in that orbit in H-atom) $\times Z^2$ (vi) $\Delta E_{n_1 n_2}$ in any H-like atom= ($\Delta E_{n_1 n_2}$ in H-atom) $\times Z^2$

7.4.6 Success of Bohr's Theory

(a) Bohr successfully combined Rutherford's model with the Planck hypothesis on the quantified energy states at atomic level

(b) Bohr's theory explained the atomic emission and absorption spectra

(c) It explained the general characteristics of the periodic table

(d) Bohr's theory offered the first "working" model for the atom

7.4.7 Short Coming of Bohr's Model

(a) Bohr's model holds true only for atoms with one electron. E.g, H, He^+, Li^{+2}, Na^{+1}

(b) Bohr's model posits circular orbits whereas according to Somerfield these are elliptical.

(c) The model could not explain the intensity of spectral lines.

(d) It assume the nucleus to be stationary, but it also rotates on its own axis.

(e) It failed to account for the minute structure in spectrum line.

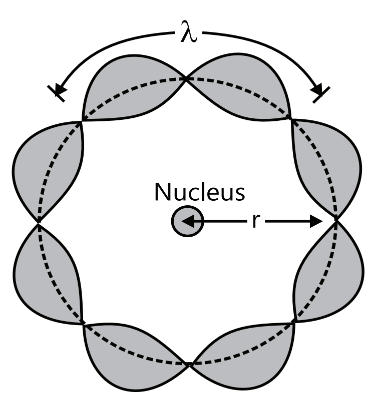
(f) The model offered no explanation for the Zeeman effect (splitting up of spectral lines in magnetic field) and Stark effect (splitting up of spectral lines in electric field)

(g) Doublets observed in the spectrum of some of the atoms like sodium (5890 Å & 5896 Å) could not be explained by Bohr's model.

7.4.8 de Broglie's Explanation of Bohr's Second Postulate of Quantization

In Bohr's model of the atom, it is stated that the angular momentum of the electron orbiting around the nucleus is quantized (that is, $L_n = \frac{nh}{2\pi}$; n = 1, 2, 3,.....). Why is it that the values of angular momentum are only integral multiples of $\frac{h}{2\pi}$

De Broglie, speculated that nature did not single out light as being the only matter which exhibits a wave-particle duality. He proposed that ordinary ``particles" such as electrons, protons, or bowling balls could also exhibit wave characteristics in certain circumstances.C.J. Davisson and L.H. German later experimentally verified the wave nature of electron in 1927. It was De Broglie's contention (like Bohr) that an electron in motion around the nucleas must be seen as a particle wave. Analogous to waves travelling on a string, particle waves too can lead to standing waves under resonant conditions.



A standing wave is shown in a circular orbit where four de Broglie wavelength fit into the circumference of the orbit.

Figure 24.38: De broglie model

We know that when a string is peturbed, it generates a number of wavelengths along the length of the string. Of these, only those wavelengths that have nodes at either ends and form standing waves survive, while other wavelengths get reflected upon themselves resulting in their amplitudes quickly dropping to zero. Therefore, standing waves are formed when a wave travels the along the enrire length of the string and back in one, two, or any integral number of wavelengths. For an electron moving in nth circular orbit of radius r_n, the total distance is the circumference of the orbit, $2\pi r_n$.

Thus, $2\pi r_n = n\lambda$, n = 1, 2, 3 .... We have, $\lambda = \frac{h}{p}$, where p is the magnitude of the electron's momentum.

If the speed of the electron is much less than the speed of light, the momentum is mv_n.

Thus, $\lambda = \frac{h}{mv_n}$.; $2\pi r_n = \frac{nh}{mv_n}$ or $mv_n r_n = \frac{nh}{2\pi}$

This is the quantum condition proposed by Bohr for the angular momentum of the electron. Thus de Broglie hypothesis provided an explanation for Bohr's second postulate for the quantization of angular momentum of the orbiting electron by postulating the wave nature of matter particles like electrons. The quantized electron orbits and energy states are due to the wave nature of the electron and only resonant standing waves can persist.

7.4.9 Limitations

(a) The Bohr model is applicable to hydrogenic atoms with a single electron. All attempts to use Bohr's Model to analyze atoms with more than one electron failed as Bohr's model deals only with interaction between the electron and the positively charged nucleus but does not account for the interaction of an electron with other electrons as would be the case with multi-electron atoms.(ii) While the Bohr's model correctly predicts the frequencies of the light emitted by hydrogenic atoms, it cannot predict the relative intensities of spectral lines. Some frequencies in the hydrogen emission spectrum, for example, have weak intensity while others have strong intensity. Bohr's model is unable to account for the intensity variations.

7.4.10 Some Important Definitions and their Meaning

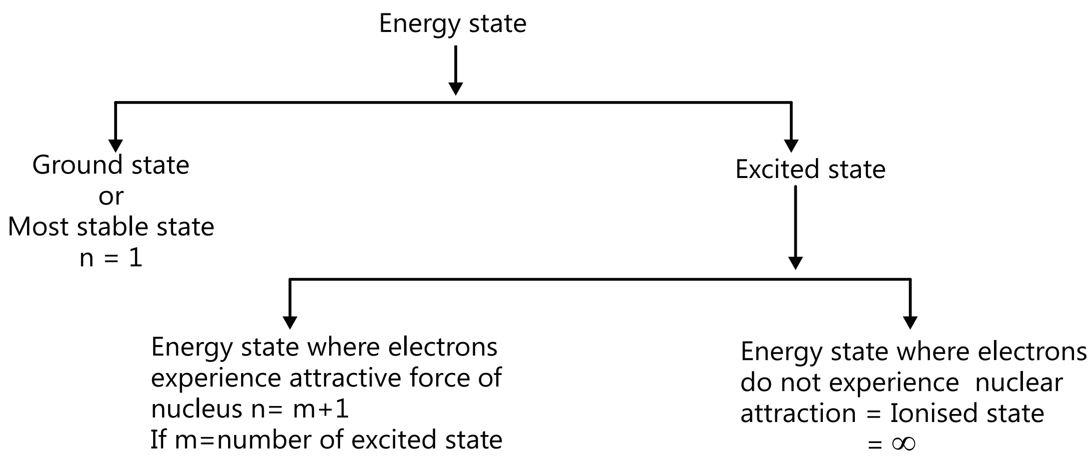


Figure 24.39: Energy level classification

(a) **Ionization energy and ionization potential:** The ionization energy is the energy necessary to remove an electron from the neutral atom. It is a minimum for the alkali metals which have a single electron outside a closed shell. The ionization potential is the potential through which an electron is accelerated for removal an electron from the neutral atom is called ionization potential.

I.E. = $E_\infty - E_1 = -E_1$ = Binding energy of e^- (e_∞ assumed to be zero)

(b) **Excitation energy and excitation potential:** The minimum energy required to excite an atom i.e., alteration from the condition of lowest energy (ground state) to one of higher energy (excited state) is called excitation energy of the particular excited state and corresponding potential is called excitation potential.

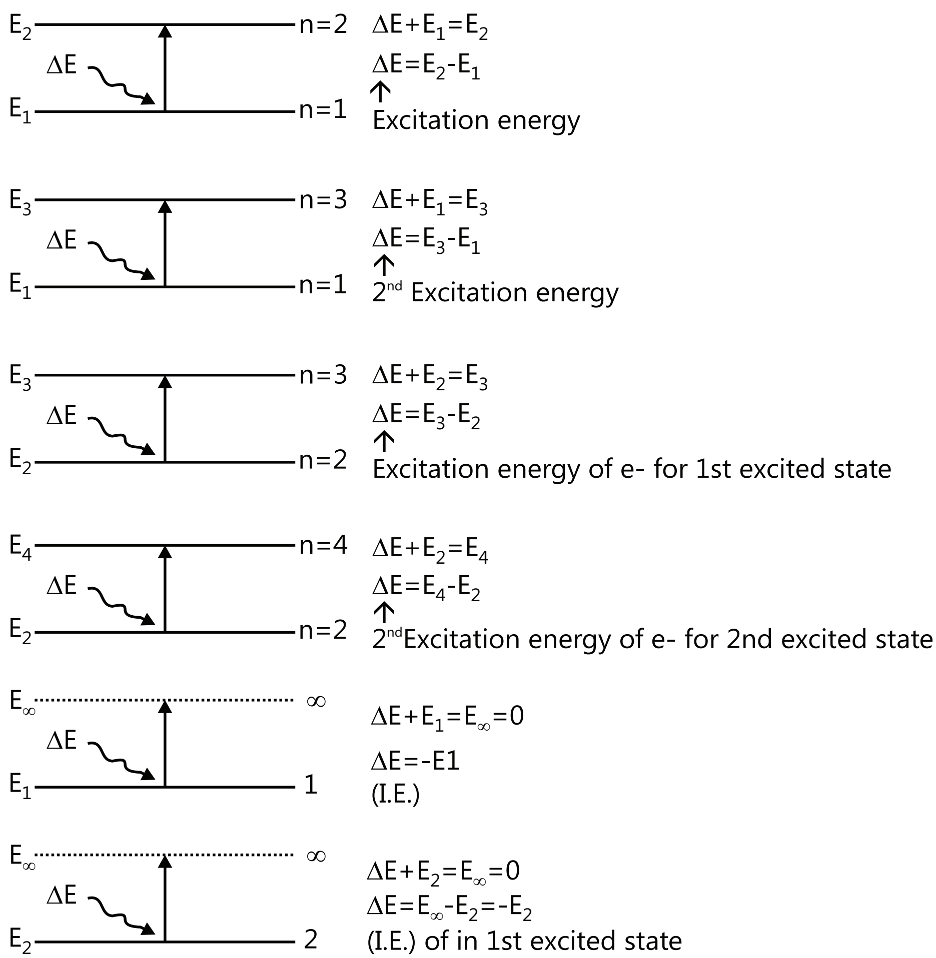


Figure 24.40

If excitation energy and ionization energy are represented in eV, then corresponding value in volt is termed as excitation potential and ionization potential, respectively.

For Example: Excitation energy and ionization energy for H-atom are 10.2 eV and 13.6 eV, respectively and, therefore, 10.2V and 13.6V are excitation and ionization potential, respectively.

PLANCESS CONCEPTS

Reduced mass: Both the proton and electron revolve in circular orbits about their common centre of mass. However, we can account for the motion of the nucleus simply by replacing the mass of electron m by the reduced mass μ of the electron and the nucleus.

Here $\mu = \frac{Mm}{M+m}$ ....(i)

Where M = mass of nucleus. The reduced mass can also be written as, $\mu = \frac{m}{1+\frac{m}{M}}$

Note: If motion of the nucleus is also considered, then m is replaced by μ, where μ = reduced mass of electron – nucleus system = $\frac{mM}{m+M}$. In this case, $E_n = (-13.6\text{eV})\frac{Z^2}{n^2}.\frac{\mu}{m_e}$

Vaibhav Krishnan (JEE 2009, AIR 22)

8. SPECTRUM

8.1 Types of Line Spectrum

Emission line spectrum: When an electric current passes through a gas which is at less than atmospheric pressure, it gives energy to the gas. This energy is then given out as light of several definite wavelengths (colours). This is called a emission line spectrum. These are caused when an electron hops from excited states to lower states. Different The wavelength of emission lines of different elements have emissions of different wavelengths. For one element the emission spectrum are unique for each element.

Absorption line spectrum: It is the electromagnetic spectrum, broken by a specific pattern of dark lines or bands, observed when radiation traverses a particular absorbing medium and through a spectroscope. The absorption pattern of an element is unique and can be used to identify the substance of the medium. When white light is passed through a gas, the gas is absorbs light of certain wavelength. The bright background on the photographic plate is then crossed by dark lines that corresponds to those wavelengths which are absorbed by the gas atoms, resulting in transition of an atom from lower energy states to higher energy states.

(The emission spectrum consists of bright lines on dark background.)

The spectrum of sunlight has dark lines called **Fraunhoffer lines**. These lines are produced when the light emanating from the core of the sun passes through the layer of cooler gas. This layer absorbs light of certain wavelengths corresponding to the elements present in the cooler gas. This results in dark lines (absorption of certain wavelengths) on a brighter background. Fraunhoffer lines reveal the composition of the star.

8.2 Time Period and Frequency of Electron's Motion

(a) **Time period** of revolution of an electron in the nth Bohr orbit is $T_n = \frac{2\pi r_n}{v_n} = \frac{n^3}{Z^2}\frac{h^3}{4\pi^2 mk^2e^4} = 1.5\times10^{-16}\frac{n^3}{Z^2}$ sec

For H-atom, Z = 1 ; then for n = 1, $T_1 = 1.5\times10^{-16}$ sec , $T_1 : T_2 : T_3$ = 1 : 8 : 27

(b) **Frequency of revolution** $v_n = \frac{1}{T_n}$ $\quad v_n \propto \frac{Z^2}{n^3}$

For H-atom $v_1 = 6.6\times10^{15}$Hz, $v_1 : v_2 : v_3 = 1 : \frac{1}{8} : \frac{1}{27}$

(c) Current and Magnetic field Due to Electron's Motion: The motion of an electron in a circular orbit, gives rise to some equivalent current in the orbit. It is equal to (in the nth orbit) M = current × area; $M_n = I_n.\pi r_n^2$;

$M_n = \frac{nhe}{4\pi m}$; $M_n = \frac{eL}{2m}$

Where L = $\frac{nh}{2\pi}$, angular momentum of the electron in its orbit.

What you must memorise is their dependence on Z and n and order of magnitudes in first Bohr orbit.

$T_n \propto \frac{n^3}{Z^2}$; $\qquad T_1 \approx 1.5\times10^{-16}$ s

$v_n \propto \frac{Z^2}{n^3}$; $\qquad v_1 \approx 6.6\times10^{15}$ Hz

$\omega_n = 2\pi v_n$; $\qquad \omega_n \propto \frac{Z^2}{n^3}$

$L_n = \frac{nh}{2\pi}$; $\qquad L_n \propto n$

PLANCESS CONCEPTS

Total energy of an electron in an atom = $\frac{1}{2} \times$ potential energy of electron = – kinetic energy of electron

Nivvedan (JEE 2009, AIR 113)

8.3 Determination of Number of Spectral Lines (Theoretical) in Emission and in Absorption Transitions

8.3.1 Number of Emission Spectra Lines

When an electron is in an excited state with principal quantum number n, then the electron may go to $(n-1)^{th}$ state, ………., 2^{nd} state or 1^{st} state from the n^{th}state. Therefore, there could be (n – 1) possible transitions starting from the n^{th} state. The electron reaching $(n-1)^{th}$ state may make (n – 2) different transitions. Similarly for other lower states, the total number of possible transitions is

$$(n-1) + (n-2) + (n-3) + \ldots\ldots\ldots 2 + 1 = \frac{n\,(n-1)}{2}$$

8.3.2 Number of Absorption Spectral Line

At ordinary temperatures almost all the atoms remain in their lowest energy level (n = 1) and, therefore. absorption transition can start only from the lowest energy level i.e., n = 1 level (not from n = 2, 3, 4, …… levels). Hence, only Lyman series is found in the absorption spectrum of hydrogen atom (which as in the emission spectrum, all the series are found)

Number of absorpton spectral lines = (n – 1)

Remember: The absorpton spectrum of sun has Balmer series also besides the Lyman series. Many H-atoms remain in n = 2 also due to very high temperature.

8.4 Explanation of H-Spectrum and Spectral Line Formula

In a single-electron atom, the transition of an electron from any higher energy state n_2 to any lower energy state n_1 causes a photon of frequency v or wavelength λ to be emitted.

Then $\Delta E = h\nu = \frac{hc}{\lambda} = E_{n_2} - E_{n_1}$;∵ $E = -Rch\frac{Z^2}{n^2}J = -13.6\frac{Z^2}{n^2}$ eV

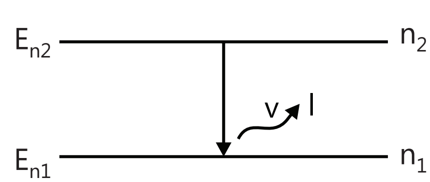


Figure 24.41

$$\therefore \Delta E = -\frac{RchZ^2}{n_2^2} - \left(\frac{RchZ^2}{n_1^2}\right) \Rightarrow \Delta E = Rch\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

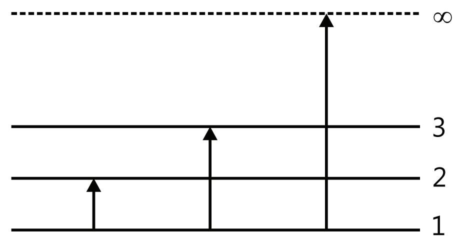
$$\Rightarrow \quad h\nu = \frac{hc}{\lambda} = RchZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \nu = \frac{1}{\lambda} = R\,Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$\bar{\nu}$ = wave number = number of wave in unit length $\nu = c\bar{\nu}$

For H-atom, Z= 1 & there for, $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

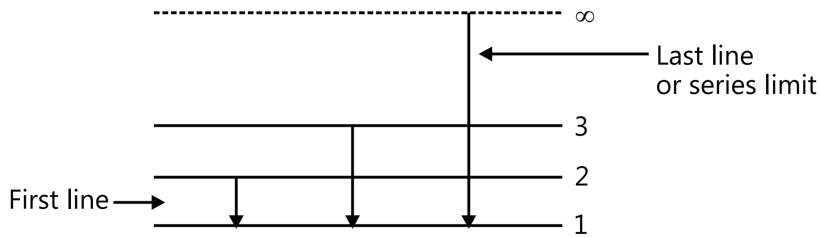
8.5 Hydrogen Spectral Series

(a) Lyman series: $n_1 = 1$, $n_2 = 2$, 3, 4, …..∞



Absorption line

Figure 24.42



Emission line

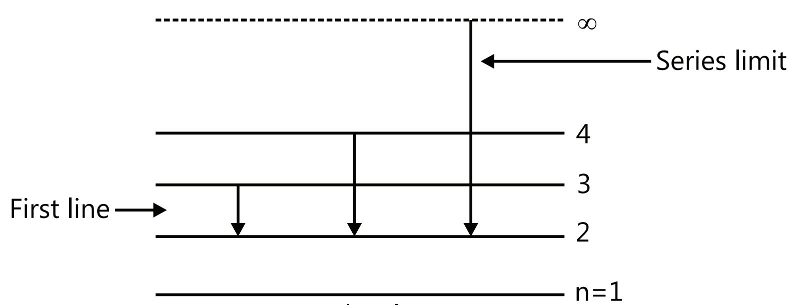
Figure 24.43

For 1st line or series beginning $n_1 = 1,\ n_2 = 2;\ \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right];\ \lambda_{max} = \frac{4}{3R} = 1216\,\text{Å}$

For series limit or last line $n_1 = 1,\ n_2 = \infty;\ \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right];\ \lambda_{min} = \frac{1}{R} = 912.68\,\text{Å}$

***** Remember – Lyman series is found in UV region of electromagnetic spectrum

(b) Balmer series:



Energy level

Figure 24.44

$n_1 = 2,\ n_2 = 3, 4, 5, 6, \ldots\ldots\infty$ Wavelength of first line

i.e. maximum wavelength $\frac{1}{\lambda_{max}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right];\ \therefore\ \lambda_{max} = 6563\,\text{Å}$

Wavelength of last line or series limit i.e. minimum wavelength

$\lambda_{min} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right];\ \lambda_{min} = \frac{4}{R} = 3646\ \text{Å}$

***** Balmer series is found only in emission spectrum.

***** Balmer series lies in the visible region of electromagnetic spectrum. Only the first four lines of Balmer series lies in visible region. Rest of them lie in the infrared region of EM spectrum.

(c) Paschen series: $n_1 = 3, n_2 = 4, 5, 6 \ldots \infty$

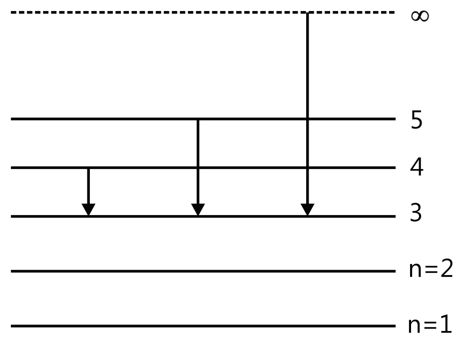


Figure 24.45

For first line $n_1 = 3, n_2 = 4$, then $\frac{1}{\lambda_{max}} = R \times \left[\frac{1}{3^2} - \frac{1}{4^2}\right]$

$\lambda_{max} = 18751$ Å For last line or series limit

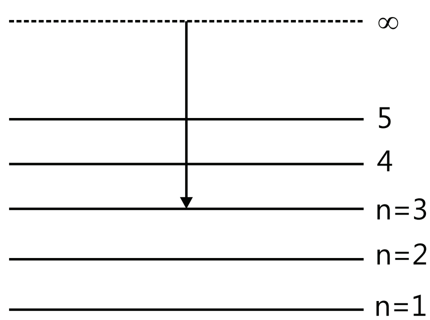


Figure 24.46

$n_1 = 3,\ n_2 = \infty;\ \frac{1}{\lambda_{min}} = R\left[\frac{1}{3^2} - \frac{1}{\infty^2}\right];\ \lambda_{min} = \frac{9}{R} = 8107$ Å

***** Paschen series is also found only in emission spectrum.

***** Paschen series is obtained in infrared region of electromagnetic spectrum.

(d) Brackett series – $n_1 = 4, n_2 = 5, 6, 7 \ldots \infty$

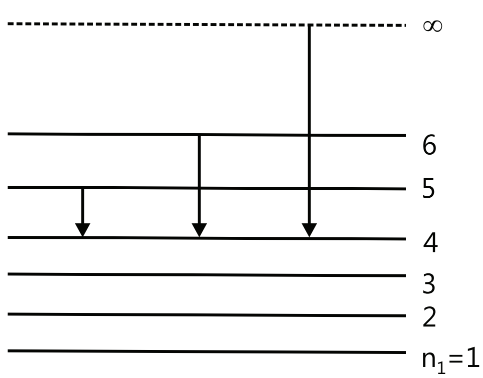


Figure 24.47

For first list $\frac{1}{\lambda_{max}} = R\left[\frac{1}{4^2} - \frac{1}{5^2}\right];\ \lambda_{max} = 40477$ Å

For last line or series limit $\frac{1}{\lambda_{min}} = R\left[\frac{1}{4^2} - \frac{1}{\infty^2}\right]$; $\lambda_{min} = \frac{16}{R} = 14572$ Å

* Brackett series is also found only in emission spectrum.

* Brackett series is also obtained in infrared region of electromagnetic spectrum.

(e) Pfund series-

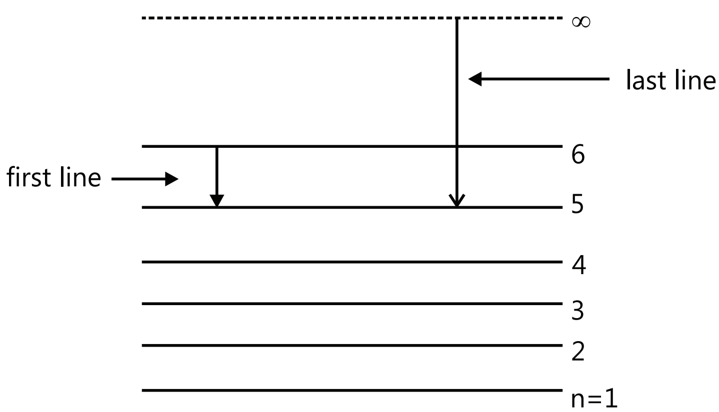


Figure 24.48

$n_1 = 5$, $n_2 = 6, 7, 8, \ldots\ldots \infty$

For first line $\frac{1}{\lambda_{max}} = R\left[\frac{1}{5^2} - \frac{1}{6^2}\right]$; $\lambda_{max} = 74515$ Å For last line or series limit

$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{5} - \frac{1}{\infty^2}\right] \Rightarrow \lambda_{min} = \frac{25}{R} = 22768 \text{ Å}$$

* Pfund series is also obtained only in emission spectrum.

* Pfund series is situated in the infrared region of electromagnetic spectrum.

PLANCESS CONCEPTS

The minimum wavelength of a series (Lyman, Balmer, Paschen, Brackett etc.) correlates with the ionization potential of the electron from that shell.

Chinmay S Purandare (JEE 2012, AIR 698)

General Point for Spectral Lines in Every Spectral Series

(a) Wavelength of first line is maximum and last line is minimum.

(b) As the order of spectral series increases, wavelength also usually increases

$\lambda_{PF} > \lambda_{BR} > \lambda_P > \lambda_B > \lambda_L$

(c) Frequency of energy emission in Lyman transitions are highest among all other series.

PLANCESS CONCEPTS

Total energy of an electron in an atom = $\frac{1}{2}$* Potential energy of electron

= – Kinetic energy of electron

***** If an electron jumps from then $\Delta E = E_{high} - E_{low}$

Where E_{low} is the low-energy state from where the jump begins and E_{high} is the high-energy state where the jump ends.

Nitin Chandrol (JEE 2012, AIR 134)

Illustration 22: What will be the two highest wavelengths of the radiation emitted when hydrogen atoms make transitions from higher states to n = 2 states? **(JEE ADVANCED)**

Sol: For electronic transition from energy state E_n > E_2 (where n = 3, 4, 5...) to E_2, the spectral series corresponds to Balmer series. Therefore the wavelength of this transition is $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$ where $n = 3, 4, 5....\infty$ and R is Rydberg's constant.

The highest wavelength corresponds to the lowest energy of transition. This will be the case for the transition n = 3 to n = 2. The second highest wavelength corresponds to the transition n = 4 to n = 2.

The energy of the state n is $E_n = \frac{E_1}{n^2}$

Thus, $E_2 = -\frac{13.6\text{eV}}{4} = -3.4\,\text{eV}$; $E_3 = -\frac{13.6\text{eV}}{9} = -1.5\,\text{eV}$; and $E_4 = -\frac{13.6\text{eV}}{16} = -0.85$ eV

The highest wavelength is $\lambda_1 = \frac{hc}{\Delta E} = \frac{1242\,\text{eV} \times 1\text{ nm}}{(3.4\text{eV} - 1.5\text{eV})}$ = 654 nm

The second highest wavelength is $\lambda_2 = \frac{1242\,\text{eV} \times 1\text{ nm}}{(3.4\,\text{eV} - 0.85\,\text{eV})}$ = 487 nm.

Illustration 23: The particle μ-meson has a charge equal to that of an electron and a mass that is 208 times that of the electron. It moves in a circular orbit around a nucleus of charge +3e. Assume that the mass of the nucleus is infinite. Supposing that Bohr's model is applicable to this system, (a) derive an equation for the radius of the n^{th} Bohr orbit, (b) find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for a hydrogen atom (c) find the wavelength of the radiation emitted when the μ-meson jumps from the third orbit to the first orbit. **(JEE ADVANCED)**

Sol: According to Bohr's theory, the radius of n^{th} Bohr's orbit is $r_n = \frac{n^2h^2\varepsilon_0}{\pi m e^2 Z}$ and energy of μ-meson in n^{th} orbit is $E_n = -\frac{mZ^2e^4}{8\varepsilon_0^2 n^2 h^2}$. If μ-meson jumps from a higher energy orbit to a lower energy orbit, the energy emitted is $\Delta E = Z^2 \times 13.6 \times \left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]\text{eV}$. To derive the expression for the n^{th} orbit we have to keep in mind that the electrostatic force of attraction between μ-meson and the nucleus provides the required centripetal force for circular orbit. According to Bohr's postulate, the magnitude of angular momentum of μ-meson must be integral multiple of $\frac{h}{2\pi}$.

(a) We have, $\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$ or $v^2 r = \frac{Ze^2}{4\pi\varepsilon_0 m}$ ... (i)

The quantization rule is $vr = \frac{nh}{2\pi m}$

The radius is $r = \frac{(vr)^2}{v^2 r} = \frac{n^2h^2}{4\pi^2 m^2}\ \frac{4\pi\varepsilon_0 m}{Ze^2} = \frac{n^2h^2\varepsilon_0}{Z\pi me^2}$ ... (ii)

For the given system, Z = 3 and m = 208 m_e.;Thus $r_\mu = \frac{n^2h^2\varepsilon_0}{624\pi m_e e^2}$.

(b) From (ii), the radius of the first Bohr orbit for the hydrogen atom is $r_h = \frac{h^2\varepsilon_0}{\pi m_e e^2}$.

For $r_\mu = r_h$, $\frac{n^2h^2\varepsilon_0}{624\pi m_e e^2} = \frac{h^2\varepsilon_0}{\pi m_e e^2}$ or, n^2 = 624 or, n = 25

(c) From (i), the kinetic energy of the atom is $\frac{mv^2}{2} = \frac{Ze^2}{8\pi\varepsilon_0 r}$ and the potential energy is $-\frac{Ze^2}{4\pi\varepsilon_0 r}$.

The total energy is $E_n = -\frac{Ze^2}{8\pi\varepsilon_0 r}$ Using (ii),

$$E_n = -\frac{Z^2\pi me^4}{8\pi\varepsilon_0^2 n^2h^2} = -\frac{9\times 208 m_e e^4}{8\varepsilon_0^2 n^2h^2} = \frac{1872}{n^2}\left(-\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right) \quad \text{... (iii)}$$

But $\left(-\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right)$ is the ground state energy of hydrogen atom and hence is equal to $-13.6\,\text{eV}$.

From (iii), $E_n = -\frac{1872}{n^2}$ × 13.6 eV = $\frac{-25459.2\,\text{eV}}{n^2}$

Thus, E_1 = - 25459.2 eV and $E_3 = \frac{E_1}{9}$ = -2828.8 eV, The energy difference is $E_3 - E_1$ = 22630.4 eV.

$$E_3 - E_1 = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{hc}{E_3 - E_1} = \frac{12375\,\text{eV} - \overset{\circ}{A}}{22630.4\,\text{eV}} = 0.5468\ \overset{\circ}{A}$$

Illustration 24: A neutron moving with speed v makes a head-on collision with a stationary hydrogen atom in ground state. Determine the minimum kinetic energy of the neutron for which inelastic (completely or partially) collision may take place. The mass of neutron $\approx$ mass of hydrogen = $1.67\times 10^{-27}\,\text{kg}$. **(JEE ADVANCED)**

Sol: It is important to remember the hydrogen atom will absorb the kinetic energy lost in an inelastic collision, causing the atom to reach one of its excited states. The quantum of energy thus absorbed by hydrogen atom will be equal to what is required to reach a possible excited state, and not more. Since the hydrogen atom is initially in ground state (n = 1), the minimum energy it can absorb will be equal to that required to reach the first excited state (n = 2). If the colliding neutron's kinetic energy is less than this minimum energy, no energy will be absorbed, i.e., inelastic collision may not take place.

Let us assume that the neutron and the hydrogen atom move at speeds v_1 and v_2 after the collision. The collision will be inelastic if a part of the kinetic energy is used to excite the atom. Suppose an energy ΔE is used in this way.

Considering collision to be inelastic, using conservation of linear momentum and energy,

$$mv = mv_1 + mv_2 \quad \text{...(i)}$$

And $\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E$ ...(ii)

From (i), $v^2 = v_1^2 + v_2^2 + 2v_1v_2$; From (ii), $v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}$ Thus, $2v_1v_2 = \frac{2\Delta E}{m}$

Hence, $(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = v^2 - \frac{4\Delta E}{m}$; As $v_1 - v_2$ must be real, ; $v^2 - \frac{4\Delta E}{m} \geq 0$;

or $\frac{1}{2}mv^2 > 2\Delta E$.

The minimum energy that can be absorbed by the hydrogen atom in ground state to go in an excited state is 10.2 eV. Thus, the minimum kinetic energy of the neutron needed for an inelastic collision is $\frac{1}{2}mv_{min}^2 = 2 \times 10.2\,eV = 20.4\,eV$

Illustration 25: The potential energy U of a small moving particle of mass m is $\frac{1}{2}m\omega^2r^2$, where ω is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, show that radius of the n^{th} allowed orbit is proportional to $\sqrt{n}$. **(JEE ADVANCED)**

Sol: The force acting on the particle in the radial direction $F_r = -\frac{dU}{dr}$ provides the necessary centripetal acceleration for the particle to move in a circular orbit.

The force at a distance r is $F_r = -\frac{dU}{dr} = -m\omega^2 r$ ... (i)

Suppose the particle moves along a circle of radius r. The net force on it should be $\frac{mv^2}{r}$ along the radius.

Comparing with (i), $\frac{mv^2}{r} = m\omega^2 r \Rightarrow v = r\omega$ ... (ii)

The quantization of angular momentum gives $mvr = \frac{nh}{2\pi}$ or, $v = \frac{nh}{2\pi mr}$ ... (iii)

From (ii) and (iii), $r = \left(\frac{nh}{2\pi m\omega}\right)^{1/2}$.

Thus, the radius of the n^{th} orbit is proportional to $\sqrt{n}$.

9. BINDING ENERGY

Binding energy,is amount of energy required to separate a particle from a system of particles or to disperse all the particles of the system. Conversely it also defined as the energy released when particles are brought together to form a system of particles. For example, if an electron and a proton are initially at rest and brought from large distances to form a hydrogen atom, 13.6 eV energy will be released. The binding energy of a hydrogen atom is, therefore, 13.6 eV, same as its ionization energy.

10. CONCEPT OF RECOILING OF AN ATOM DETERMINATION OF MOMENTUM & ENERGY FOR RECOIL ATOMS

When a nuclear particle is emitted or ejected at high velocity from an atom the remainder of the atom recoils with a velocity inversely proportional to its mass. This happens when an electron makes transition from any higher energy state to any lower energy state. The atom is recoiled by sharing some energy from the energy evolved during electronic transition.

If m = mass of recoiled atom, V = velocity of recoiled atom

Then $\frac{1}{2}mv^2 + \frac{hc}{\lambda} = E_{n_2} - E_{n_1} = \Delta E$

Recoil momentum of atom = $\frac{h}{\lambda}$ = momentum of photon

Recoil energy of atom = $\frac{p^2}{2m}$

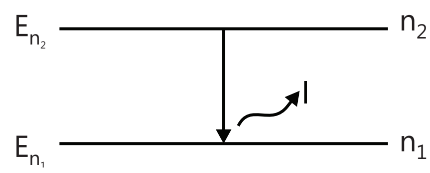


Figure 24.49

Illustration 26: Given that the excitation energy of a hydrogen-like ion in its first excited state is 40.8 eV, determine the energy needed to remove the electron from the ion. **(JEE MAIN)**

Sol: The excitation energy for hydrogen like ion for (n-1)th excited state (nth orbit) is $E = hc \times R \times Z^2\left(1 - \frac{1}{n^2}\right)$ where n = 2, 3, 4, ...etc. The energy needed to remove the electron from the ion is $E = hc \times R \times Z^2$.

The excitation energy in the first excited state (n=2) is

$E = RhcZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$ = (13.6 eV) × Z^2 × $\frac{3}{4}$.

Equating this to 40.8 eV, we get Z = 2. So, the ion in question is He^+.

The energy of the ion in the ground state is E = $-\frac{RhcZ^2}{1^2}$ = $-4 \times (13.6\,eV)$ = –54.4 eV

Thus 54.4 eV is required to remove the electron from the ion.

PLANCESS CONCEPTS

The energy of a photon and its wavelength are inversely proportional.

B Rajiv Reddy JEE 2012, AIR 11

11. THE WAVE FUNCTION OF AN ELECTRON

Quantum mechanics has enabled physicists to develop a mathematically and logically rigorous theory which describes the spectra in a much better way. The following is a very brief introduction to this theory.

We have already seen that to understand the behavior of light, we understand it as both a wave (the electric field $\vec{E}$ (as well as a particle (the photon). The energy of a particular 'photon' is related to the 'wavelength' of the $\vec{E}$ wave. Light going in x direction is represented by the wave function. $E(x,t) = E_0 \sin(kx - \omega t)$ In general, if light can go in any direction, the wave function is $\vec{E}(\vec{r}, t) = \vec{E}_0 \sin(\vec{k}.\vec{r} - \omega t)$ ... (i)

Where $\vec{r}$ is the position vector; $\vec{k}$ is the wave vector.

11.1 Quantum Mechanics of the Hydrogen Atom

The wave function $\Psi(\vec{r}, t)$ of the electron and the possible energies E of a hydrogen atom or a hydrogen-like ion are obtained from the Schrodinger's equation.

$$\frac{-h^2}{8\pi^2 m}\left[\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}\right] - \frac{Ze^2\Psi}{4\pi\varepsilon_0 r} = E\Psi \quad \text{... (ii)}$$

Here (x, y, z) refers to a point with the nucleus as the origin and r is the distance of this point from the nucleus. E refers to energy. The constant Z is the number of protons in the nucleus. For hydrogen, we have to put Z = 1. There are infinite number of functions $\Psi(\vec{r})$ which satisfy equation (ii). These functions, which are solutions of equation (ii), may be characterized in terms of three parameters n, l and m_l With each solution Ψ_{nlm_l}, there is associated a unique value of the energy E of the atom or the ion. The energy E corresponding to the wave function Ψ_{nlm_l} depends only on n and may be written as $E_n = -\dfrac{mZ^2e^4}{8\varepsilon_0^2h^2n^2}$

12. LASER

12.1 Basic Process of Laser

The basic strategy to get light amplification by stimulated emission is as follows:

A system is chosen which has a metastable state at having an energy E_2 (See Fig. 24.50). There is another allowed energy E_1 which is less than E_2. The system could be any of the following: a gas or a liquid in a cylindrical tube or a solid in the shape of a cylindrical rod. Let us assume that the number of atoms in the metastable state E_2 is increased to more than that in E_1. Let us also assume that a photon of light of energy E_2–E_1 is incident on one of the atoms in the metastable state E_2. Then this atom drops to the state E_1 i.e., emitting a photon in the same phase, energy, and direction as the first one. Then these two photons interact with two more atoms in the state E_2 and so on. Therefore, the number of photons keeps on increasing. All these photons will have the same phase, the same energy, and the same direction. Thus, the amplification of light is achieved.

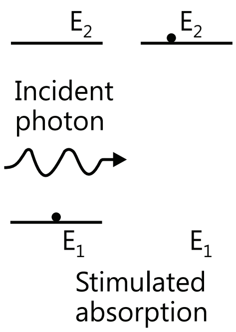


Figure 24.50: Laser

12.2 Working

When power is suppliedy and the electric field is established, some of the atoms of the mixture get ionized. These ionized atoms release some electrons which are accelerated by the high electric field. Consequentially, these electrons collide with helium atoms to take them to the metastable state at energy E_3. These atoms collide with a neon atom and transfer the extra energy to it. As a result, the helium atom returns to its ground state and the neon atom is excited to the state at energy E_2. This process keeps looping so that the neon atoms are continuously pumped to the state at energy E_2, keeping the population (of atoms) of this state large.

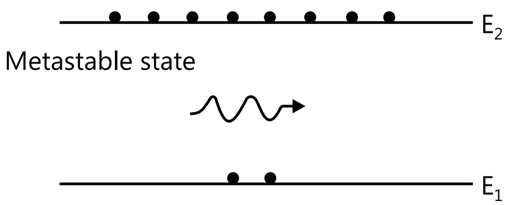


Figure 24.51: metastable state of electron

12.3 Uses of Laser

(a) Spectroscopy: Most lasers being inherently pure source of light, emit near monochromatic light with a very clear range of wavelengths. This makes the laser ideal for spectroscopy.

(b) Heat Treatment: In laser heat treating, energy is transmitted to the material's surface in order to create a hardened layer by metallurgical transformation. The use of lasers results in little or no distortion of the component and, as such, eliminates much of part reworking that is currently done. Therefore, the laser heat treatment system is cost-effective.

(c) Lunar laser ranging: The Apollo astronauts planted retroreflector arrays on te moon to make possible the Lunar Laser Ranging Experiment. In this experiment laser beams are focused, through large telescopes on Earth, on the arrays, and the time taken for the beam to be reflected back to Earth is measured to determine the distance between the Earth and Moon with high accuracy.

(d) Photochemistry: Extremely brief pulses of light – as short as picoseconds or femtoseconds (10^{-12} to 10^{-15} s) – produced by some laser systems are used to initiate and analyze chemical reactions. This technique is known as photochemistry.

(e) Laser Cooling: This technique involves atom trapping, wherein a number of atoms are enclosed in a specially shaped arrangement of electric and magnetic fields.

(f) Nuclear Fusion: Powerful and complex arrangements of lasers and optical amplifiers are used to produce extremely high-intensity pulses of light of extremely short duration. These pulses are arranged to impact pellets of tritium-deuterium, simultaneously, from all directions, hoping that the compression effect of the impacts will induce atomic fusion in the pellets.

13. X-RAYS

X-radiation is a form of electromagnetic radiation. Most X-rays have a wavelength ranging from 0.01 to 10 nanometers, corresponding to frequencies in the range 30 petahertz to 30 exahertz (3×10^{16} Hz to 3×10^{19} Hz) and energies in the range 100 eV to 100 keV. X-radiation is also referred to as Röntgen radiation, after Wilhelm Röntgen, who is usually credited as its discoverer, and who had named it X-radiation to signify an unknown type of radiation produced when electron collided with the walls of the tube.

The wave nature of X-rays, was established by Laue who demonstrated that they are diffracted by crystals.

13.1 Production

The modern X-ray tube, called Coolidge tube, is shown in the Fig. 24.52. A heated element emits electronswhich are accelerated towards a cooled copper anode under a high potential difference. A target metal of high atomic number and high melting point is lodged on the anode.

The intensity of the X-ray beam is controlled by The filament current controls the intensity of the X-ray beam by regulating the number of electrons striking the target per unit time. The potential difference between the cathode and anode controls the penetrating power of the beam.

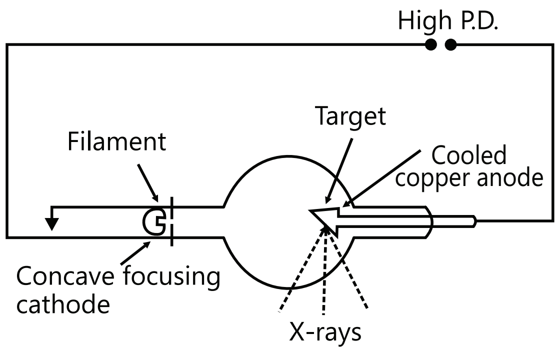


Figure 24.52: X- Ray tube

13.2 X-Ray Spectra

A typical X-ray spectrum given by a target is shown in the Fig.24.53. The spectrum is basically continuous range of wavelengths starting from a minimum value. A line spectrum having sharp wavelengths is superimposed on this.

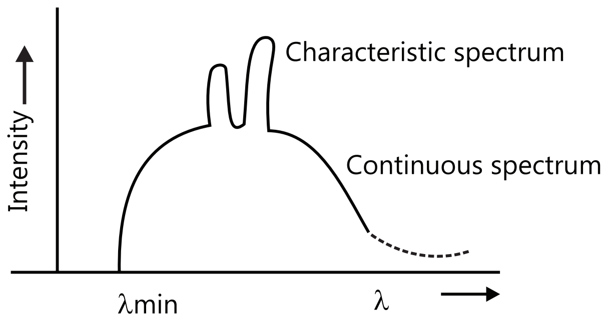


Figure 24.53: X-Ray spectra

13.3 Origin Of Characteristic Spectrum

If an incoming electron knocks out an electron in one of the inner shells, the exiting electron creates a vacancy in that shell. This vacancy gets fille by another electron from a higher shell that makes a transition to this shell, creating another vacancy in the higher shell. This hopping of electrons from higher to lower shells continues till the inner shells are filled up. This process produces a series of radiations, some of which pertain to the X-ray region. These radiations are typical of the target element. The X-ray spectrum of a substance is classified into K-series, L-series, M-series etc.

13.4 Moseley's Experiment and the Concept of Atomic Number

Moseley's experiment involved the analysis of the X-ray spectra of 38 different elements, ranging from aluminum to gold. He measured the frequency of principal lines of a particular series (the α-lines in the K-series) of the spectra and was able to show that the frequencies of certain characteristic X-rays emitted from chemical elements are proportional to the square of a number which was close to the element's atomic number (Z). He presented the following relationship: $\sqrt{\nu} = a\,(Z - b)$

where v = frequency of X-rays, Z = atomic number, a and b are constants. On plotting the values of square root of the frequency against atomic numbers of the elements producing X-rays on a graph, a straight line was obtained.

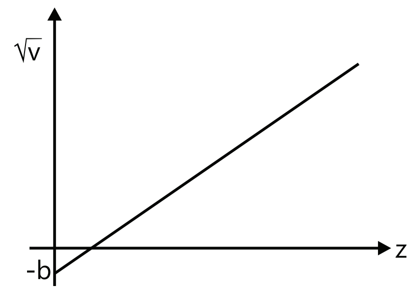


Figure 24.54

When electron is knocked out from n_1 energy state and it is filled with electron from n_2 energy state wavelength of X-ray emitted is $\frac{1}{\lambda} = R\,(Z - b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For K-series b = 1, $n_1 = 1$ and $n_2 = 2, 3, \ldots\ldots$ For L-series : b = 7.4, $n_1 = 2$, $n_2 = 3, 4, \ldots\ldots$

For K_α -line, the electron jumps from L-level to a vacancy in the K-kevel. So for the L electron, there are Z protons in the nucleus and an electron in the K-shell which screens off the positive charge. So the net charge the L electron faces can be taken as (Z – 1)e.

Now, E = Rhc $(Z-1)^2$ $= \Rightarrow h\nu = Rhc(Z-1)^2 \times \left[\frac{3}{4}\right]$

$$\sqrt{v} = \sqrt{\frac{3Rc}{4}}\,(Z-1)$$

13.5 Origin of Continuous Spectrum

When an incident electron comes very close to a target nucleus, it is suddenly accelerated due to the electrostatic field around the nucleus (Coulomb field) and emits electromagnetic radiation. This radiation is referred to as breaking radiation and is continuous. The minimum wavelength (and maximum frequency) correlates with an electron losing all its energy in a single collision with a target atom.

If V is the acceleration p.d., then $h\nu_{max} = \frac{hc}{\lambda_{min}} = eV$ or $\lambda_{min} = \frac{hc}{eV} = \frac{12420}{V}$ Å

13.6 Properties of X-rays

(a) X-rays ionize the material which they penetrate.

(b) They produce the same effect on photographic plates as visible light.

(c) They cause fluorescence when they act on certain chemical compounds like zinc sulphide.

(d) X-rays penetrate matter and get absorbed as they pass through it. If I_0 is the intensity of incident relation and I is the intensity after travelling through a distance x, then $I = I_0 e^{-\mu x}$ where μ is called the absorption coefficient of the material. The atomic number of the material and its absorption coefficient are directly proportional. This is the basis of radiography.

(e) They cause photoelectric emission.

(f) Electric and magnetic fields have no effect on X-rays as they contain no charged particles.

Note: X-Rays are not affected by electric or magnetic fields. Intensity of X-rays depends on number of electrons in the incident beam.

PROBLEM-SOLVING TACTICS

(a) This section of Physics is more fact-based. The key to answering questions of these sections is establish a lonk between the known and asked quantities

(b) One has to be very conversant with the formulae and standard scientific constants.

(c) In this section, graphical questions seeking relationship between various fundamental quantities are usually asked. Assign the dependent variable as y and the independent variable as x and then look for a relation between them.

(d) One must not get confused about approaching the questions from a wave nature or particle nature or try to combine both. Just solve questions on the basis of the known and asked quantities and the relationship between the two.

(e) It is important to learn the scientific constants in various units to avoid unnecessary unit conversion. (e.g., if energy of a photon is in eV units and wavelength asked in angstrom, one can directly use the relation = 12400/E, here 12400 is the product of Planck's constant and speed of light.)

(f) Analytical questions pertaining to H-atom can be solved easily if one knows proportionality relation between quantities. They need not be learnt by heart. They can be derived without bothering about constants appearing in these relations. (e.g., radius of nth shell is directly proportional to n^2, keeping Z constant.)

FORMULAE SHEET

Speed of E.M.W. in vacuum c = $3\times10^8\ m/s = \nu\lambda$

Each photon having a frequency ν and energy $E = h\nu = \frac{hc}{\lambda}$ where h = 6.63×10^{-34} Js is Planck's Constant

Einstein's Photo Electric Equation:

Photon energy = K.E. of electron + work function.

$$h\nu = \frac{1}{2}mv^2 + \phi$$

ϕ = Work function = energy needed by the electron in freeing itself from the atoms of the metal.

$$\phi = hv_0$$

The minimum value of the retarding potential to prevent electron emission is:

$$eV_{cut\ off} = (KE)_{max}$$

De Broglie wave length given by $\lambda = \frac{h}{p}$ (wave length of a particle)

The electron in a stable orbit does not radiate energy i.e. $\frac{mv^2}{r} = \frac{kZe^2}{r^2}$

A stable orbit is that in which the angular momentum of the electron about nucleus is an integral (n) multiple of $\frac{h}{2\pi}$ i.e. mvr = $n\frac{h}{2\pi}$; n = 1, 2, 3, ... $(n \neq 0)$.

For Hydrogen atom : (Z = atomic number = 1)

(i) L_n = angular momentum in the nth orbit = $n\frac{h}{2\pi}$

(ii) r_n = radius of nth circular orbit = $r_n = \dfrac{n^2h^2\varepsilon_0}{\pi me^2}$ (0.529 Å) n^2 ; (1 Å = 10^{-10} m) ; $r_n \propto n^2$

(iii) E_n energy of the electron in the nth orbit = $\dfrac{-13.6\text{eV}}{n^2}$ i.e. $E_n \propto \dfrac{1}{n^2}$

(iv) n^{th} orbital speed $v_n = \dfrac{e^2}{2\varepsilon_0 nh}$

Note: Total energy of the electron in an atom is negative, indicating that it is bound.

Binding Energy $(BE)_n = -E_n = \dfrac{13.6\text{eV}}{n^2}$

(iv) $E_{n_2} - E_{n_1}$ = Energy emitted when an electron jumps from n_2 orbit to n_1 orbit $(n_2 > n_1)$.

$$\Delta E = (13.6 \text{ eV}) \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$\Delta E = h\nu$; ν= frequency of spectral line emitted.

Wave number= $\bar{\nu} = \dfrac{1}{\lambda}$ = [no. of waves in unit length (1m)] = $R\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

Where R = Rydberg's constant for hydrogen = $1.097 \times 10^7 m^{-1}$

(v) For hydrogen like atoms of atomic number Z:

$$r_{nz} = \frac{\text{Bohr radius}}{Z} \times n^2 = (0.529\text{Å})\frac{n^2}{Z} \;;$$

$$E_{nZ} = (-13.6)\frac{Z^2}{n^2}\text{ev}$$

Note: If motion of the nucleus is also considered, then m is replaced by μ.

Where μ = reduced mass of electron – nucleus system = $\dfrac{mM}{m+M}$

In this case, $E_n = (-13.6\text{eV})\ \dfrac{Z^2}{n^2} \cdot \dfrac{\mu}{m_e}$

Excitation potential for quantum jump from $n_1 \to n_2$ = $\dfrac{E_{n_2} - E_{n_1}}{\text{electron charge}}$

From Mosley's Law $\sqrt{\nu} = a(z-b)$ where b (shielding factor) is different for different series.

For x-rays $\dfrac{1}{\lambda} = R \times (Z-b)^2 \times \left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$R = R_0 A^{1/3}$. Where R_0 = empirical constant = 1.1×10^{-15}m ; A = Mass number of the atom.

Solved Examples

JEE Main/Boards

Example 1: Calculate the energy of α-particle in the event of its head-on collision with gold nucleus if the closest distance of approach is 41.3 Fermi.

Sol: The kinetic energy of α-particle is converted into the electric potential energy at the distance of closest approach in the event of a head-on collision. The kinetic energy of alpha particle is thus $E = \frac{q_1q_2}{4\pi\varepsilon_0 r}$

Given r_0 = 41.3 × 10^{-15} m, Z = 70, q_1= Ze= 79e and q_2 = 2e,

$$\text{As } E = \frac{Ze(2e)}{4\pi\varepsilon_0 r} = \frac{9\times10^9\times79\times2(1.6\times10^{-19})^2}{41.3\times10^{-15}}$$

$$= \frac{9\times79\times2\times1.6\times1.6\times10^{-14}}{41.3}\ \text{J}$$

$$= 8.814\times10^{-13}\ \text{J} = \frac{8.814\times10^{-13}}{1.6\times10^{-19}}\ \text{eV} = 5.51\ \text{MeV}$$

Example 2: If the wavelength of the incident light is 5000 Å and the photoelectric work function of the metallic plate is 1.90 eV, find

(a) Energy of the photon in eV

(b) Kinetic energy of the photoelectrons emitted

(c) Stopping potential

Sol: The energy of photon is $E = h\nu = \frac{hc}{\lambda}$, where λ is the wavelength of the light. This photon knocks out photoelectron from the surface of metal with the maximum kinetic energy $E_{max} = h\nu - \phi_0 = eV$ where f_0 is the work function of metal and V is the stopping potential.

(a) Energy of the incident photon,

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{5000\times10^{-10}}$$

= 3.96 × 10^{-19} joule = 2.47 eV

(b) Kinetic energy of the photo-electrons emitted KE_{max}

$$= \frac{1}{2}mv^2 = h\nu - \phi_0 = (2.47 - 1.90)\ \text{eV} = 0.57\ \text{eV}$$

(c) e V = KE_{max} Where V is stopping potential

$$V = \frac{KE_{max}}{e} = \frac{0.57\times1.6\times10^{-19}}{1.6\times10^{-19}} = 0.57\ \text{V.}$$

Example 3: Determine the de Broglie wavelength of an electron having kinetic energy of 500 eV?

Sol: The de-Broglie wavelength of electron moving with Kinetic energy K is given as $\lambda = \frac{h}{\sqrt{2mK}}$

Using $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$ we get

$$\lambda = \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times500\times1.6\times10^{-19}}}$$

$$\lambda = 0.5467\times10^{-10}\ \text{m}.$$

Example 4: If an X-ray tube produces a continuous spectrum of radiation with its short wavelength end 0.65Å, what is the maximum energy of a photon in the radiation?

Sol: The energy of radiation having wavelength λ is

$$E = \frac{hc}{\lambda}$$

Given λ_{min} = 0.65Å = 0.65×10^{-10} m,

h = 6.63×10^{-34} Js, c = $3\times10^8\,\text{ms}^{-1}$

We know, maximum energy of X-ray photon is

$$E_{max} = h\nu_{max} = \frac{hc}{\lambda_{min}} = \frac{6.63\times10^{-34}\times3\times10^8}{0.65\times10^{-10}\times1.6\times10^{-19}}$$

$$= 19.13\times10^3\,\text{eV} = 19.13\ \text{keV}$$

Example 5: If ultra-violet light of $\lambda = 2600\,\text{Å}$ is incident on a silver surface with a threshold wavelength for photoelectric emission of $\lambda = 3800\,\text{Å}$, calculate:

(i) Work function

(ii) Maximum kinetic energy of the emitted photoelectrons.

(iii) Maximum velocity of the photoelectrons.

Sol: The work function of metal is $\phi = h\nu_{th} = \frac{hc}{\lambda_{th}}$. The kinetic energy with which the photoelectron is ejected from the metal surface is $E = h\nu - \phi = \frac{1}{2}mv^2$

(i) $\phi = h\nu_{th} = \frac{hc}{\lambda_{th}} = \frac{6.63\times10^{-34}\times3\times10^{8}}{3800\times10^{-10}}$ J

$= 5.23\times10^{-19}$ J $= \frac{5.23\times10^{-19}}{1.6\times10^{-19}}$ eV = 3.27 eV

(ii) Incident wavelength $\lambda = 2600$ Å

Then KE_{max} of emitted photoelectrons $= h\nu - \phi$;

here $h\nu = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{2600\times10^{-10}}$

$= 7.65\times10^{-19}$ J $= \frac{7.65\times10^{-19}}{1.6\times10^{-19}} = 4.78$ eV

KE_{max} = (4.78 – 3.27) eV; = 1.51 eV

(iii) $V_{max} = \sqrt{\frac{KE_{max}\times2}{m}} = \sqrt{\frac{1.51\times1.6\times10^{-10}\times2}{9.1\times10^{-31}}}$ m/s.

$= 7.29\times10^{5}$ m/s

Example 6: The photocurrent generated when a surface is irradiated with light of wavelength 4950 Å, vanishes if a stopping potential greater than 0.6 V is applied across the photo tube. When a different source of light is used, it is found that the stopping potential has changed to 1.1 V. Determine the work function of the emitting surface and the wavelength of second source.

Sol: The maximum kinetic energy of emitted photoelectron is the product of stopping potential and electron charge, given by $KE_{max} = eV = h\nu - \phi$, where φ is the work function of the metal. For two different stopping potentials we have two different wavelengths of light used.

Let λ_1 = 4950Å, V_1 = 0.6 V

$KE_{max} = \frac{hc}{\lambda_1} - \phi$; $\because KE_{max} = eV_1 \Rightarrow \phi = \frac{hc}{\lambda_1} - eV_1 =$

$\frac{(6.6\times10^{-34})\times(3\times10^{8})}{4950\times10^{-10}\times1.6\times10^{-19}} - 0.6 = 1.9$ eV

(b) $\frac{hc}{\lambda_2} = \phi + eV_2$;

$\frac{hc}{\lambda_2} = 3.04\times10^{-19} + \left(1.6\times10^{-19}\times1.1\right) = 4.8\times10^{-19}$ J

$\therefore \lambda_2 = \frac{hc}{4.8\times10^{-19}} = \frac{(6.6\times10^{-34})\times(3\times10^{8})}{4.8\times10^{-19}}$

$\lambda_2 = 4.125\times10^{-7}$ m = 4125 Å

Example 7: A hydrogen-like atom (atomic number Z) in a higher excited state of quantum number n can make a transition to the first excited state by successively emitting two photons of energies 10.20 eV and 17.00 eV, respectively. On the other hand, the atom from the same excited state can make a transition to the second excited state by successively emitting two photon of energies 4.25 eV and 5.95 eV, respectively. What are the values of n and Z. (Ionization energy of hydrogen atom = 13.6 eV)?

Sol: For any hydrogen-like atom, the energy released in transition from a higher excited state to a lower excited state is $\Delta E = Z^2\times13.6\times\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$eV where n_f and n_i are principle quantum numbers of final (lower) and initial (higher) energy states respectively.

In first case, the excited atom makes a transition from n^{th} state to n = 2 state and two photons of energies 10.2 eV and 17.0 eV are emitted. Hence, if Z is the atomic number of H-like atom, then using

$\Delta E = Z^2\times13.6\times\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$eV;

$$(10.2 + 17.0)\text{eV} = Z^2\times13.6\times\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \quad \text{...(i)}$$

In second case, the excited atom makes a transition from nth state to n = 3 state and two photons of energies 4.25 eV and 5.95 eV are emitted.

$$(4.25 + 5.95)\ \text{eV} = Z^2\times13.6\times\left[\frac{1}{3^2} - \frac{1}{n^2}\right] \quad \text{...(ii)}$$

Dividing equation (i) and (ii), we get

$\frac{27.2}{10.2} = \frac{9(n^2-4)}{4(n^2-9)}$ or $\frac{n^2-4}{n^2-9} = 1.185$ or $\frac{2n^2-13}{5} = \frac{2.185}{0.185}$

or $n^2 = 36$ or n = 6

Putting in equation (i), we get

$27.2 = Z^2\times13.6\left(\frac{1}{4} - \frac{1}{36}\right)$ or $Z^2 = \frac{27.2}{13.6}\times\frac{9}{2} = 9$

or, Z = 3.

Example 8: In a hydrogen sample, if the atoms are excited to states with principal quantum number n, how many different wavelengths may be observed in the spectrum?

Sol: The hydrogen atom excited to the principal quantum number n will emit radiations as the electron hop back to lower energy states. Each transition to a lower energy state emits radiation of different wavelength. Thus, we get a radiation spectrum.

From the nth state, the atom may go to $(n - 1)^{th}$ state, ……, n = 2 state or n = 1 state. So there are (n – 1) possible transitions starting from the n^{th} state. The atoms reaching $(n - 1)^{th}$ state may make (n – 2) different transitions to reach n = 1 state. In the same way, for other lower states, the total number of possible transitions is (n – 1) + (n – 2) + (n – 3) + …. 2+1

$= \frac{n(n-1)}{2}$.

Example 9: For a hydrogen-like, doubly ionized lithium atom with atomic number Z=3, determine the wavelength of the radiation required to excite the electron in Li^{2+} from the first to the third Bohr orbit. The ionization energy of hydrogen atom is 13.6 eV.

Sol: The energy required by the hydrogen-like atom for transition from ground state (n=1) to any of the excited states (n^{th} orbit) is $\Delta E = 13.6 Z^2 (1 - \frac{1}{n^2})$.

Wavelength of radiation having energy E is, $\lambda = \frac{hc}{E}$.

The energy of nth orbit of a hydrogen like atom is given as $E_n = -\frac{13.6}{n^2}$

Thus for Li^{2+} atom, as Z = 3, the electron energies for the first and third Bohr orbits are:

For n = 1, $E_1 = -\frac{13.6 \times (3)^2}{1^2}$ eV = –122.4 eV

For n = 3, $E_3 = -\frac{13.6 \times (3)^2}{(3)^2}$ eV = – 13.6 eV

Thus the energy required to transfer an electron from E_1 level to E_3 level is,

$E = E_1 - E_3$ = – 13.6 – (– 122.4) = 108.8eV

Therefore, the radiation needed to cause this transition should have photons of this energy.

hν = 108.8 eV. The wavelength of this radiation is $\frac{hc}{\lambda}$ = 108.8 eV or $\lambda = \frac{hc}{108.8\,eV}$

$= \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8)}{108.8 \times 1.6 \times 10^{-19}} m = 113.74$ Å.

Example 10: For a hypothetical hydrogen-like atom, the wavelength in Å for the spectral lines for transitions from n = p to n = 1 are given by $\lambda = \frac{1500p^2}{p^2 - 1}$, where p = 2, 3, 4, ….

(i) Find the wavelength of the least energetic and the most energetic photons in this series.

(ii) Construct an energy level diagram for this element representing at least three energy levels.

(iii) Determine the ionization potential of this element?

Sol: If wavelength of spectral lines for transitions from n = p to n = 1 are given, then the energy of radiation for each transition is given as $E = \frac{hc}{\lambda} = \frac{hc}{1500}(1 - \frac{1}{p^2})$. The least energy is obtained from transition from p = 2 to p = 1 and maximum energy is obtained from transition from p=∞ to p=1. The ionization corresponds to the maximum energy in the spectrum.

Given $\lambda = \frac{1500p^2}{p^2 - 1}$ and energy is $E = \frac{hc}{\lambda}$

Substituting for λ we get $E = \frac{hc}{1500}\left(1 - \frac{1}{p^2}\right) \times 10^{10}$ J

$= \frac{hc}{(1500)(1.6 \times 10^{-19})} \left(1 - \frac{1}{p^2}\right) \times 10^{10}$ eV

$= 8.28 \left(1 - \frac{1}{p^2}\right)$ eV.

Hence energy of n^{th} state is given b $E_n = \frac{8.28}{n^2}$ eV

(i) Maximum energy is released for transition from p = ∞ to p = 1; hence wavelength of most energetic photon is 1500 Å.

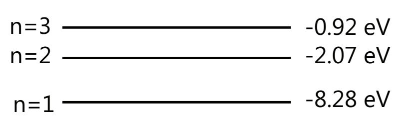
Least energy is released for transition from n = 2 to n = 1 transition. For p = 2 l = 2000Å

(ii) The energy level diagram is shown in the Fig. 24.60.

(iii) The ionization potential corresponds to energy required to liberate an electron from its ground state.

i.e., ionization energy = 8.28 eV

Hence, ionization potential = 8.28 V



Example 11: A single electron orbiting a stationary nucleus of charge +Ze, where Z is a constant and e is the magnitude of the electronic charge, requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit. Find

(i) The value of Z

(ii) Energy required to excite the electron from the third to the fourth Bohr orbit.

(iii) Wavelength of the electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.

(iv) Kinetic energy, potential energy, and angular momentum of the electron in the first Bohr orbit.

(v) The radius of the first Bohr orbit.

(The ionization energy of hydrogen atom = 13.6 eV. Bohr radius = 5.3×10^{11} m)

Sol: For a hydrogen-like atom, the total energy of electron in n^th^ orbit is $E=-\frac{13.6\times Z^2}{n^2}$ eV and radius of n^th^ orbit is

$$r_n=\frac{5.3\times10^{-11}n^2}{Z}$$

The kinetic energy in n^{th} orbit is equal to the magnitude of total energy in n^{th} orbit. The potential energy in n^{th} orbit is equal to twice the total energy in n^{th} orbit.

The energy required to excite the atom from n_1 state to n_2 state is $E=13.6Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$ eV. To remove the electron from n = 1 state to infinity, $E=\frac{hc}{\lambda}=13.6\times Z^2$.

So $\lambda=\frac{hc}{13.6\times Z^2}$

This atom is hydrogen like

Z = atomic number of the nucleus

E_n = Energy of the electron in the nth orbit.

= $(Z)^2$ (energy of the electron in the nth orbit of the hydrogen atom) = $-(Z)^2\,\frac{13.6}{n^2}$ eV = $\frac{E_1}{n^2}$

Where E_1 = Energy of the electron in the 1^st^ Bohr orbit of the given atom.

(i) Given (Z^2) (13.6) $\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$ =47.2 eV

$\Rightarrow$ Z = 5.

(ii) $E_1 = -(25)\,\frac{13.6}{1} = -340$ eV

$$E_4-E_3 = 13.6\ Z^2\left[\frac{1}{4^2}-\frac{1}{3^2}\right] = E_1\left[\frac{1}{3^2}-\frac{1}{4^2}\right]$$

$$= 340\left[\frac{1}{3^2}-\frac{1}{4}\right] \text{ eV} = 16.53 \text{ eV}$$

(iii) Minimum energy required to remove electron from first orbit = 340 eV.

$$\Rightarrow \frac{hc}{\lambda} = 340\times1.6\times10^{-19}$$

$$\text{or } \lambda = \left[\frac{6.6\times10^{-34}\times3\times10^8}{340\times1.6\times10^{-19}}\right]\text{m} = 36.40\,\text{Å}$$

(iv) KE of the electron in the 1^st^ orbit

$$KE_1 = -E_1 = 340 \text{ eV}\ ;\ PE_1 = 2E_1 = -680 \text{ eV}$$

Angular momentum of the electron in the 1^st^ Bohr orbit

$$=\frac{h}{2\pi} = \frac{6.63\times10^{-34}}{2\pi} = 1.055\times10^{-34}\text{ kg m / s}$$

(v) Radius of the 1^st^ Bohr orbit for the given atom

$$=\frac{\text{Bohr radius}}{Z} = \frac{5.3\times10^{-11}}{5} = 1.06\times10^{-11}\text{ m}$$

JEE Advanced/Boards

Example 1: If a hydrogen atom in its ground state is excited by means of a monochromatic radiation of wavelength 975 Å

(a) How many different lines are possible in the resulting spectrum?

(b) Calculate the longest wavelength amongst them.

The ionization energy for hydrogen atom is 13.6 eV.

Sol: First calculate the energy of the incident photon of given wavelength. From the formula $E=13.6\left(1-\frac{1}{n^2}\right)$ eV, find the value of n, i.e., the maximum excited state the hydrogen atom will reach after absorbing the photon of given wavelength. Longest wavelength in the resulting spectrum will correspond to transition from n^{th} orbit to $(n-1)^{th}$ orbit.

Energy of the ground state

(n = 1) = – (ionization energy) = –13.6 eV

The wavelength of the incident radiation

= λ = 975Å ∴ Energy of the incident photon

$$=\frac{hc}{\lambda}=\frac{6.63\times10^{-34}\times3\times10^8}{975\times10^{-10}\times1.6\times10^{-19}} = 12.75\text{ eV}$$

Let electron be exerted to nth orbit

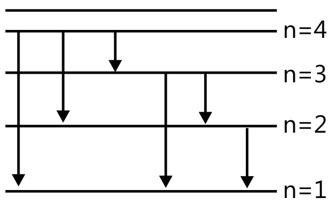
$$\Rightarrow 12.75 = 13.6\left(\frac{1}{1^2}-\frac{1}{n^2}\right) \Rightarrow n = 4$$

The quantum transitions to the less excited states give six possible lines as follows:

n = 4 : (4 $\rightarrow$ 3), (4 $\rightarrow$ 2), (4 $\rightarrow$ 1)

n = 3 : (3 $\rightarrow$ 2), (3 $\rightarrow$ 1); n = 2 : (2 $\rightarrow$ 1)

The longest wavelength emitted is for the transition (4 $\rightarrow$ 3) where energy difference is minimum



$$E_{min}\ (E_4 - E_3) = 13.6\left(\frac{1}{3^2}-\frac{1}{4^2}\right)$$

$$= 0.661\text{eV; Thus } \lambda_{max} = \frac{hc}{E_{min}}$$

$$= \frac{6.63\times10^{-34}\times3\times10^8}{0.661\times1.6\times10^{-19}} \approx 18807\text{Å}$$

Example 2: An X-ray tube operating at a potential difference of 40 kV produces heat at the rate of 720 W. Assuming 0.5% of the energy of the incident electrons is converted into X-rays, calculate

(a) The number of electrons per second striking the target.

(b) The velocity of the incident electrons.

Sol: When X-Rays are produced in an X-Ray tube, the power consumed is denoted by P = IV. Some of this power is wasted as heat and the rest is converted to X-Rays. The electron incident per second on target is n = I/e

As 0.5% of energy is converted into X-ray, therefore heat produced per second at the target is P = 0.995 VI

where, I is current inside tube

$$\Rightarrow I = \frac{P}{0.995V} = \frac{720}{0.995\times40\times10^3} = 0.018\text{Å}$$

Number of electrons per second incident of the target

$$n = \frac{I}{e} = \frac{0.018}{1.6\times10^{-19}} = 1.1\times10^{17}\ \text{electrons/s.}$$

(b) Kinetic energy of incident electron $\frac{1}{2}mv^2 = eV$

$$\text{Or } v = \sqrt{\frac{2eV}{m}}$$

$$= \sqrt{\frac{2\times1.6\times10^{-19}\times40\times10^3}{9.1\times10^{-31}}} = 1.19\times10^8\ m/s$$

Example 3: If one milliwatt of light of wavelength 4560 Å is incident on a cesium surface, calculate the photoelectric current liberated assuming a quantum efficiency of 0.5%.

Planck's constant h = 6.62×10^{-34} Js and velocity of light 3×10^8 m/s.

Sol: The energy of one photon of light is $E=\frac{hc}{\lambda}$. The number of photons incident on the surface per second can be determined by dividing power by energy of one photon. The number of photons multiplied by quantum efficiency gives the number of photoelectrons emitted per second.

The energy of each photon of incident light

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34})(3\times10^8)}{4560\times10^{-10}} = 4.35\times10^{-19}\ J$$

Number of photons in one milliwatt source

$$= \frac{\text{Power of light}}{\text{Energy of one photon}}$$

$$= \frac{10^{-3}}{4.35\times10^{-19}} = 2.29\times10^{15}/\text{s.}$$

Number of electrons released = $2.29\times10^{15}\times\frac{0.5}{100}$

$= 1.14\times10^{13}$ /s

$\therefore$ Photoelectric current

= Photo charge flowing per second

= Total electrons emitted per sec. × charge of one electron

$$= (1.14\times10^{13})\times(1.6\times10^{-19}) = 1.824\ \mu A$$

Example 4: Consider the following data: Incident beam: wavelength 3650 Å; intensity 10^{-8} W/m^2. Surface: Absorption coefficient 0.8; work function 1.6 eV.

Determine the time rate of number of electrons emitted per m^2, power absorbed per m^2, and the maximum kinetic energy of emitted photoelectrons.

Sol: The energy of one photon of light is $E=\frac{hc}{\lambda}$.

Number of photons incident on the surface per m^2 per second is the intensity divided by energy of one photon. Number of photons × absorption coefficient = the number of photons absorbed by the surface. The remaining number of photons is equal to the photoelectrons emitted per m^2 per second.

If N is the number of photons crossing per unit area per unit time,

Number of photons falling per second on unit area

$$=\frac{\text{Intensity}}{\text{Energy of one photon}}=\frac{I\lambda}{hc}$$

$$=\frac{10^{-8}\times 3650\times 10^{-10}}{6.62\times 10^{-34}\times 3\times 10^{8}}=18.35\times 10^{9}\ /m^2s$$

The number of photons absorbed N_{ab} by the surface per unit area per unit time

N_{ab} = absorption coefficient of surface × N

$= 0.8\times 18.35\times 10^{9} = 1.47\times 10^{10}/m^2s$

Now, assuming that each photon ejects one electron, the rate of electrons emitted per unit area is given by

$N-N_{ab} = 1.835\times 10^{10}-1.47\times 10^{10} = 0.37\times 10^{10}\ /m^2-s$

Power absorbed/m^2

= Absorption coefficient × Intensity of light falling on surface= $0.8\times 10^{-8} = 8\times 10^{-9}\ W/m^2$. Maximum kinetic energy is of emitted photoelectron is given by

$$(K.E.)_{max}=\frac{hc}{\lambda}-\phi$$

$$=\frac{(6.62\times 10^{-34})(3\times 10^{8})}{3650\times 10^{-10}\times 1.6\times 10^{-19}}\ eV-1.6eV$$

= 3.4 eV – 1.6 eV = 1.80 eV

Example 5: Consider two hydrogen-like atoms A and B of different masses but having equal number of protons and neutrons. The difference in the energies between the first Balmer lines emitted by A and B is 5.667 eV. When these atoms, moving with the same velocity, strike a heavy target elastically, the atom B imparts twice the momentum to the target than the atom A. Identify the atoms A and B.

Sol: The energy of hydrogen-like atom for n^{th} orbit is given by $E=-\frac{Z^2\times 13.6}{n^2}$, where Z = atomic number. First line of Balmer series corresponds to transition from orbit n = 3 to orbit n = 2. Energy emitted is $\Delta E = Z^2\times 13.6\times\left(\frac{1}{4}-\frac{1}{9}\right)$. The momentum imparted to the heavy target during elastic collision is twice the momentum of the striking particle.

Suppose Z_A and Z_B are the atomic number and m_A and m_B are the mass numbers of hydrogen like atoms A and B, respectively.

$$E_n=-\frac{Z^2Rhc}{n^2}=\frac{-Z^2\times 13.6}{n^2}\ eV$$

Energy emitted for first Balmer line of atom A

$$\Delta E_A=-Z_A^2\times 13.6\left(\frac{1}{2^2}-\frac{1}{3^2}\right)eV$$

Similarly, energy emitted for first Balmer line of atom B

$$\Delta E_B=-Z_B^2\times 13.6\times\left(\frac{1}{2^2}-\frac{1}{3^2}\right)eV$$

According to question, $\Delta E_A-\Delta E_B = 5.667$ eV

or 5.667 eV = $(Z_B^2-Z_A^2)\times 13.6\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$ eV

$$\text{or } Z_B^2-Z_A^2=\frac{5.667\times 36}{13.6\times 5}=3 \quad \text{... (i)}$$

Suppose you represent the initial velocity of each atom A and B as u.

Momentum imparted by A to target = $2m_Au$

Momentum imparted by B to target = $2m_Bu$

Then according to questions,

$$2m_Au=2m_Bu\Rightarrow 2m_A=m_B \quad \text{... (ii)}$$

In case of both the atoms A and B, number of protons and neutrons is same separately, hence $m_B=2Z_B$ and $m_A=2Z_A$

Putting m_A and m_B in equation (ii)

$$2Z_B=2(2Z_A) \text{ or } Z_B=2Z_A \quad \text{... (iii)}$$

Solving (i) and (iii) $Z_A=1$ and $Z_B=2$

i.e., atom A contains 1 proton and 1 neutron, i.e., atom A is deuterium $({}_1H^2)$.

Similarly, atom B contains 2 protons and 2 neutrons, i.e., atom B is singly ionized Helium.

Example 6: A traveling hydrogen atom in the ground state makes a head-on inelastic collision with a stationary hydrogen atom in the ground state. After collision, they move together. What is the minimum velocity of the traveling hydrogen atom if one of the atoms is to gain the minimum excitation energy after the collisions?

Sol: Here we need to consider that the kinetic energy lost in the inelastic collision will be absorbed by one of the hydrogen atoms to reach to its next excited state. As both the hydrogen atoms are initially in ground state (n=1), the minimum energy absorbed will be equal to that required by one of the atoms to reach the first excited state (n=2). If the kinetic energy of the colliding hydrogen atom is less than this minimum energy, no energy will be absorbed, i.e. inelastic collision may not take place.

Let u be the velocity of the hydrogen atom before collision and v the velocity of the two atoms moving together after collision. By the principle of conservation of momentum, we have: Mu + M × 0 = 2Mv

or $v = \frac{u}{2}$. The loss in kinetic energy ΔE due to collision is given by $\Delta E = \frac{1}{2}Mu^2 - \frac{1}{2}(2M)v^2$

As $v = \frac{u}{2}$

we have $\Delta E = \frac{1}{2}Mu^2 - \frac{1}{2}(2M)\left(\frac{u}{2}\right)^2$

$= \frac{1}{2}Mu^2 - \frac{1}{4}Mu^2 = \frac{1}{4}Mu^2$

This loss in energy is due to the excitation of one of the hydrogen atoms. The ground state (n = 1) energy of a hydrogen atom is:

$E_1 = -13.6\,eV$

The energy of the first excited level (n = 2) is:

$E_2 = -3.4\,eV$

Thus the minimum energy required to excite a hydrogen atom from ground state to first excited state is: $E_2 - E_1$ = [– 3.4 – (–13.6)] eV = 10.2 eV = 10.2 × 1.6 × 10^{-19} J

= 16.32 × 10^{-19} J

As per problem, the loss in kinetic energy in collision is due to the energy used up in exciting one of the atoms. Thus. $\Delta E = E_2 - E_1$

Or $\frac{1}{4}Mu^2$ = 16.32 × 10^{-19}

Or $u^2 = \frac{4 \times 16.32 \times 10^{-19}}{M}$

The mass of the hydrogen atom is 1.0078 amu or 1.0078 × 1.66 × 10^{-27} kg

$$u^2 = \frac{4 \times 16.32 \times 10^{-19}}{1.0078 \times 1.66 \times 10^{-27}} = 39.02 \times 10^8$$

$\Rightarrow u = 6.246 \times 10^4\,m/s$

Example 7: Assuming the potential energy between electron and proton at a distance r to be $U = \frac{ke^2}{3r^3}$, use Bohr's theory to obtain energy levels of such a hypothetical atom.

Sol: The negative of gradient of potential energy is equal to force on the electron. This force provides the necessary centripetal acceleration to the electron to move in a circular orbit around the proton. The magnitude of angular momentum of electron is quantized. The mass of the proton is very large as compared to the mass of electron, so it will not be accelerated due to the force exerted on it by the electron, hence it is assumed to be stationary.

As we know that negative of potential energy gradient is force for a conservative field.

$-\frac{dU}{dr}$ = F. It is given that $U = \frac{ke^2}{3r^3}$ ... (i)

Hence, force $F = -\frac{dU}{dr} = -\frac{d}{dr}\left(\frac{ke^2}{3r^3}\right) = \frac{ke^2}{r^4}$

According to Bohr's theory this force provides the necessary centripetal force for orbital motion.

$\frac{ke^2}{r^4} = \frac{mv^2}{r}$ ... (ii)

Also quantizing angular momentum,

$mvr = \frac{nh}{2\pi}$ ... (iii)

Hence, $v = \frac{nh}{2\pi mr}$ ... (iv)

Substituting this value in Eq.(ii), we get

$\frac{mn^2h^2}{4\pi^2m^2r^3} = \frac{ke^2}{r^4}$ or $r = \frac{4\pi^2me^2k}{n^2h^2}$

Substituting this value or r in Eq. (iv), we get

$v = \frac{n^3h^3}{8\pi^3km^2e^2}$

Total energy E = KE + PE

$= \frac{1}{2}mv^2 - \frac{ke^2}{3r^3}$

$$= \frac{m}{2}\left(\frac{n^3h^3}{8\pi^3 km^2e^2}\right)^2 - \frac{ke^2}{3}\left(\frac{n^2h^2}{4ke^2m\pi^2}\right)^3$$

$$= \frac{(n\hbar)^6}{6(ke^2)^2m^3} \text{ where } \left[\hbar = \frac{h}{2\pi}\right]$$

Example 8: When an electron in a tungsten (Z = 74) target drops from an M shell to a vacancy in the K shell, calculate the wavelength of the characteristic X-ray emitted there of.

Sol: In multi electron atoms, the nucleus is shielded from the outer most electron by the inner shell electron such that the outer most electron experience Z_{eff} charge from nucleus. The energy of this outermost electron in n^{th} shell is $E = -\frac{13.6 \times Z_{eff}^2}{n^2}$

Tungsten is a multi-electron atom. Due to the shielding of the nuclear charge by negative charge of the inner core electrons, each electron is subjected to an effective nuclear charge Z_{eff} which is different for different shells.

Thus, the energy of an electron in the nth level of a multi-electron atom is given by

$E_n = -\frac{13.6Z_{eff}^2}{n^2}$ eV For an electron in the K shell (n = 1),

$Z_{eff} = (Z - 1)$

Thus, the energy of the electron in the K shell is:

$$E_K = -\frac{(74-1)^2 \times 13.6}{1^2} \approx -72500\,\text{eV}$$

For an electron in the M shell (n = 3), the nucleus is shielded by one electron of the n = 1 state and eight electrons of the n = 2 state, a total of nine electrons, so that $Z_{eff} = Z - 9$ Thus, the energy of an electron in the M shell is:

$$E_M = -\frac{(74-9)^2 \times 13.6}{3^2} \approx -6380 \text{ eV}$$

Therefore, the emitted X-ray photon has an energy given by Hv = $E_M - E_K$

=–6380 eV –(–72500eV) = 66100 eV

Or $\frac{hc}{\lambda}$ = 66100 × 1.6 × 10^{-19} J

$$\therefore \lambda = \frac{hc}{66100 \times 1.6 \times 10^{-19}} \text{ m}$$

$$= \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8)}{66100 \times 1.6 \times 10^{-19}} \text{m} = 0.0188 \times 10^{-9} \text{ m.}$$

Example 9: Assuming that the short series limit of the Balmer series for hydrogen is 3646 Å, calculate the atomic number of the element, given X-ray wavelength down to 1.0 A. Identify the element.

Sol: Balmer series spectra is obtained when an electron transitions from higher energy orbit to the second orbit (n =2). The wave number of radiation emitted is given as $\bar{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$. The shortest wavelength will correspond to highest energy, i.e. n = ∞.

The short limit of the Balmer series is given by

$$\nu = \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R}{4}$$

$\therefore R = \frac{4}{\lambda} = \left(\frac{4}{3646}\right) \times 10^{10}\ m^{-1}$ Further the wavelengths of the k_a series are given by the relation

$$\nu = \frac{1}{\lambda} = R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

The maximum wave number correspondence to $n = \infty$ and, therefore, we must have

$$\nu = \frac{1}{\lambda} = R(Z-1)^2$$

$$\text{Or } (Z-1)^2 = \frac{1}{R\lambda} = \frac{3646 \times 10^{-10}}{4 \times 1 \times 10^{-10}}$$

= 911.5 (Z – 1) = $\sqrt{911.5}$ ≅ 30.2

Or Z = 30.2 ≅ 31

Thus, the atomic number of the element concerned is 31.

The element having atomic number Z = 30 is Gallium.

JEE Main/Boards

Exercise 1

Q.1 Define the terms: (i) work function, (ii) threshold frequency and (iii) stopping potential, with reference to photoelectric effect.

Calculate the maximum Kinetic energy of electron emitted from a photosensitive surface of work function 3.2 eV, for the incident radiation of wavelength 300nm.

Q.2 Derive the expression for the de Broglie wavelength of an electron moving under a potential difference of V volt.

Describe Davisson and Germer experiment to establish the wave nature of electrons. Draw a labelled diagram of the apparatus used.

Q.3 Two metals A and B have work functions 2eV and 5eV respectively. Which metal has lower threshold wavelength?

Q.4 de-Broglie wavelength associated with an electron accelerated through a potential difference V is λ. What will be its wavelength when the accelerating potential is increased to 4 V?

Q.5 Sketch a graph between frequency of incident radiations and stopping potential for a given photosensitive material. What information can be obtained from the value of the intercept on the potential axis?

A source of light of frequency greater than the threshold frequency is placed at a distance of 1 m from the cathode of a photo-cell. The stopping potential is found to be V. If the distance of the light source from the cathode is reduced, explain giving reasons, what change will you observe in the

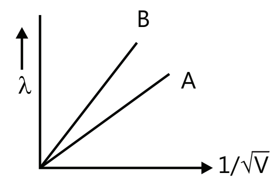
(i) Photoelectric current

(ii) Stopping potential

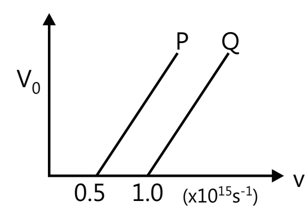
Q.6 Ultraviolet radiations of different frequencies v_1 and v_2 are incident on two photosensitive materials having work functions W_1 and W_2 $((W_1 > W_2)$ respectively. The Kinetic energy of the emitted electron is same in both cases. Which one of the two radiations will be of higher frequency?

Q.7 Draw a schematic diagram of the experimental arrangement used by Davisson and Germer to establish the wave nature of electrons. Explain briefly how the de-Broglie relation was experimentally verified in case of electrons.

Q.8 Two lines, A and B, in the plot given below show the variation of de-Broglie wavelength, λ, versus $\frac{1}{\sqrt{V}}$, where V is the accelerating potential difference. For two particles carrying the same charge, which one of the two represents a particle of smaller mass?



Q.9 The following graphs shows the variation of stopping potential V_0 with the frequency v of the incident radiation for two photosensitive metals P and Q:



(i) Explain which metal has smaller threshold wavelength.

(ii) Explain, giving reason, which metal emits photoelectron having smaller kinetic energy.

(iii) If the distance between the light source and metal P is doubled, how will the stopping potential change?

Q.10 The stopping potential in an experiment on photoelectric effect is 1.5V. What is the maximum kinetic energy of the photoelectrons emitted?

Q.11 An α -Particles and a proton are accelerated from rest by the same potential. Find the ratio of their de-Broglie wavelengths.

Q.12 Write Einstein's photoelectric equation. State clearly the three salient features observed in photoelectric effect, which can be explained on the basis of the above equation.

Q.13 Define the term 'stopping potential' in relation to photoelectric effect.

Q.14 Draw a plot showing the variation of photoelectric current with collector plate potential for two different frequencies, $v_1 > v_2$, of incident radiation having the same intensity. In which case will the stopping potential be higher? Justify your answer.

Q.15 A proton and an electron have same kinetic energy. Which one has greater de-Broglie wavelength and why?

Q.16 Define the terms (i) 'cut-off voltage' and (ii) 'threshold frequency' in relation to the phenomenon of photoelectric effect.

Using Einstein's photoelectric equation show how the cut-off voltage and threshold frequency for a given photosensitive material can be determined with the help of a suitable plot/graph.

Q.17 Derive the expression for the radius of the ground state orbit of hydrogen atom, using Bohr's postulates. Calculate the frequency of the photon, which can excite the electron to –3.4 eV from –13.6 eV.

Q.18 A stream of electrons travelling with speed 'v' m/s at right angles to a uniform electric field 'E', is deflected in a circular path of radius 'r'. Prove that $\frac{e}{m} = \frac{v^2}{rE}$.

Q.19 In a hydrogen atom, an electron of change 'e' revolves in a orbit of radius 'r' with a speed 'v'. Prove that the magnetic moment associated with the electron is given by $\frac{evr}{2}$.

Q.20 Draw a labeled diagram of experimental setup of Rutherford's alpha particle scattering experiment. Write two important inferences drawn from this experiment.

Q.21 The ground state energy of hydrogen atom is –13.6eV.

(i) What is the potential energy of an electron in the 3rd excited state?

(ii) If the electron jumps to the ground state from third excited state, calculate the wavelength of the photon emitted.

Q.22 Drawn a schematic arrangement of the Geiger–Marsden experiment. How did the scattering of α -particle by a thin foil of gold provide an important way to determine an upper limit on the size of the nucleus? Explain briefly.

Q.23 The ground state energy of hydrogen atom is –13.6eV. What are the kinetic and potential energies of electron in this state?

Q.24 In a Geiger-Marsden experiment, calculate the distance of closest approach to the nucleus of Z = 80, when an α -particle of 8 MeV energy impinges on it before it comes momentarily to rest and reverse its direction.

How will the distance of closest approach be affected when the kinetic energy of the α -particle is doubled?

Q.25 A photon and electron have got the same de-Broglie wavelength. Which has the greater total energy? Explain.

Q.26 If the intensity of incident radiation of a metal surface is doubled, what happens to the kinetic energy of the electrons emitted?

Q.27 The wavelength of a spectral line is 4000 Å. Calculate its frequency and energy. Given, c = 3×10^8 ms^{-1} and $h = 6.6\times10^{-34}$ Js.

Q.28 Calculate the longest wavelength of the incident radiation, which will eject photoelectrons from a metal surface, whose work function is 3 eV.

Exercise 2

Single Correct Choice Type

Q.1 Let n_r and n_b be respectively the number of photons emitted by a red bulb and a blue bulb of equal power in a given time.

(A) $n_r = n_b$ (B) $n_r < n_b$

(C) $n_r > n_b$ (D) Data insufficient

Q.2 In a photo-emissive cell, with exciting wavelength λ, the maximum kinetic energy of electron is K. If the exciting wavelength is changed to $\frac{3\lambda}{4}$ the Kinetic energy of the fastest emitted electron will be:

(A) $\frac{3K}{4}$ (B) $\frac{4K}{3}$

(C) Less than $\frac{4K}{3}$ (D) Greater than $\frac{4K}{3}$

Q.3 If the frequency of light in a photoelectric experiment is doubled, the stopping potential will

(A) Be doubled

(B) Be halved

(C) Become more than doubled

(D) Become less than doubled

Q.4 The stopping potential for the photoelectron emitted from a metal surface of work function 1.7 eV is 10.4 V. Identify the energy levels corresponding to the transition in hydrogen atom which will result in emission of wavelength equal to that of incident radiation for the above photoelectric effect.

(A) n = 3 to 1 (B) n = 3 to 2

(C) n = 2 to 1 (D) n = 4 to 1

Q.5 Radiation of two photon energies twice and five times the work functions of metal are incident successively on the metal surface. The ratio of the maximum velocity of photoelectrons emitted is the two cases is

(A) 1 : 2 (B) 2 : 1 (C) 1 : 4 (D) 4 : 1

Q.6 Cut off potentials for a metal in photoelectric effect for light of wavelength λ_1, λ_2 and λ_3 is found to be V_1, V_2 and V_3 volts if V_1, V_2 and V_3 are in Arithmetic Progression and λ_1, λ_2 and λ_3 will be:

(A) Arithmetic Progression

(B) Geometric Progression

(C) Harmonic Progression

(D) None

Q.7 In a photoelectric experiment, the collector plate is at 2.0V with respect to the emitter plate made of copper ϕ=4.5eV). The emitter is illuminated by a source of monochromatic light of wavelength 200nm.

(A) The minimum kinetic energy of the photoelectrons reaching the collector is 0.

(B) The maximum kinetic energy of the photoelectrons reaching the collector is 3.7ev.

(C) If the polarity of the battery is reversed then answer to part A will be 0.

(D) If the polarity of the battery is reversed then answer to part B will be 1.7eV.

Q.8 By increasing the intensity of incident light keeping frequency ($v > v_0$) fixed, on the surface of metal

(A) Kinetic energy of the photoelectrons increase

(B) Number of emitted electrons increases

(C) Kinetic energy and number of electrons increase

(D) No effect

Q.9 A proton and an electron accelerated by same potential difference have de-Broglie wavelength λ_p and λ_e

(A) $\lambda_e = \lambda_p$ (B) $\lambda_e < \lambda_p$

(C) $\lambda_e > \lambda_p$ (D) None of these

Q.10 An electron with initial kinetic energy of 100eV is accelerated through a potential difference of 50V. Now the de-Broglie wavelength of electron becomes

(A) 1Å (B) $\sqrt{1.5}$ Å (C) $\sqrt{3}$ Å (D) 12.27Å

Q.11 If h is Planck's constant in SI system, the momentum of a photon of wavelength 0.01 Å is:

(A) 10^{-2} h (B) h (C) 10^{2} h (D) 10^{12} h

Q.12 Let K_1 be the maximum kinetic energy of photoelectrons emitted by a light of wavelength λ_1 and K_2 corresponding to λ_2. If $\lambda_1 = 2\lambda_2$, then:

(A) $2K_1 = K_2$ (B) $K_1 = 2K_2$ (C) $K_1 < \frac{K_2}{2}$ (D) $K_1 > 2K_2$

Q.13 Imagine a Young's double slit interference experiment performed with waves associated with fast moving electrons produced from an electron gun. The distance between successive maxima will decrease maximum if

(A) The accelerating voltage in the electron gun is decreased

(B) The accelerating voltage is increased and the distance of the screen from the slits is decreased

(C) The distance of the screen from the slits is increased

(D) The distance between the slits is decreased.

Q.14 If the electron in a hydrogen atom was in the energy level with n = 3, how much energy in joule would be required to ionize the atom? (Ionization energy of H-atom is 2.18×10^{-18} J):

(A) 6.52×10^{-16} J (B) 2.86×10^{-10} J

(C) 2.42×10^{-19} J (D) 3.56×10^{-19} J

Q.15 In hydrogen and hydrogen like atoms, the ratio of difference of energies $E_{4n}-E_{2n}$ and $E_{2n}-E_n$ varies with its atomic number z and n as:

(A) $\frac{z^2}{n^2}$ (B) $\frac{z^4}{n^4}$ (C) $\frac{z}{n}$ (D) z^0n^0

Q.16 In a hydrogen atom, the electron is in nth excited state. It may come down to second excited state by emitting ten different wavelengths. What is the value of n?

(A) 6 (B) 7 (C) 8 (D) 5

Q.17 Monochromatic radiation of wavelength λ is incident on a hydrogen sample in ground state. Hydrogen atoms absorb the light and subsequently emit radiations of ten different wavelengths. The value of λ is

(A) 95nm (B) 103nm (C) 73nm (D) 88nm

Q.18 In a sample of hydrogen like atoms all of which are in ground state, a photon beam containing photons of various energies is passed. In absorption spectrum, five dark lines are observed. The number of bright lines in the emission spectrum will be (Assume that all transitions take place)

(A) 5 (B) 10 (C) 15 (D) None of these

Q.19 When a hydrogen atom, initially at rest emits, a photon resulting in transition n = 5 $\rightarrow$ n = 1, its recoil speed is about

(A) 10^{-4} m/s (B) 2×10^{-2} m/s

(C) 4.2 m/s (D) 3.8×10^{-2} m/s

Q.20 The electron in a hydrogen atom makes a transition from an excited state to the ground state. Which of the following statement is true?

(A) Its kinetic energy increases and its potential and total energies decrease.

(B) Its kinetic energy decreases, potential energy increases and its total energy remains the same.

(C) Its kinetic, and total energies decrease and its potential energy increases.

(D) Its kinetic, potential and total energies decrease.

Q.21 The magnitude of angular momentum, orbit radius and frequency of revolution of electron in hydrogen atom corresponding to quantum number n are L, r and respectively. Then according to Bohr's theory of hydrogen atom,

(A) fr^2L is constant for all orbits

(B) frL is constant for all orbits

(C) f^2rL is constant for all orbits

(D) frL^2 is constant for all orbits

Q.22 Radius of the second Bohr orbit of singly ionized helium atom is

(A) 0.53 Å (B) 1.06 Å (C) 0.265 Å (D) 0.132 Å

Q.23 An electron in Bohr's hydrogen atom has an energy of –3.4 eV. The angular momentum of the electron is

(A) $\frac{h}{\pi}$ (B) $\frac{h}{2\pi}$

(C) $\frac{nh}{2\pi}$ (n is an integer) (D) $\frac{2h}{\pi}$

Q.24 An electron is in an excited state in hydrogen-like atom. It has a total energy of –3.4eV. If the kinetic energy of the electron is E and its de-Broglie wavelength is λ, then

(A) E = 6.8eV, λ =6.6 × 10^{-10} m

(B) E = 3.4eV, λ = 6.6 x 10^{-10} m

(C) E = 3.4eV, λ = 6.6 x 10^{-11} m

(D) E = 6.8eV, λ = 6.6 x 10^{-11} m

Q.25 If radiation of all wavelengths from ultraviolet to infrared is passed through hydrogen a gas at room temperature, absorption lines will be observed in the:

(A) Lyman series (B) Balmer series

(C) Both (A) and (B) (D) Neither (A) nor (B)

Q.26 In the hydrogen atom, if the reference level of potential energy is assumed to be zero at the ground state level, choose the incorrect statement.

(A) The total energy of the shell increases with increase in the value of n.

(B) The total energy of the shell decrease with increase in the value of n.

(C) The difference in total energy of any two shells remains the same.

(D) The total energy at the ground state becomes 13.6 eV.

Q.27 Choose the correct statement(s) for hydrogen and deuterium atoms (considering motion of nucleus)

(A) The radius of first Bohr orbit of deuterium is less than that of hydrogen

(B) The speed of electron in the first Bohr orbit of deuterium is more than that of hydrogen.

(C) The wavelength of first Balmer line of deuterium is more than that of hydrogen

(D) The angular momentum of electron in the first Bohr orbit of deuterium is more than that of hydrogen.

Q.28 In a Coolidge tube experiment, the minimum wavelength of the continuous X-ray spectrum is equal to 66.3 pm, then

(A) Electron accelerate through a potential difference of 12.75 kV in the Coolidge tube

(B) Electrons accelerate through a potential difference of 18.75 kV in the Coolidge tube

(C) de-Broglie wavelength of the electrons reaching the anticathode is of the order of 10μm.

(D) de-Broglie wavelength of the electrons reaching the anticathode is 0.01Å.

Q.29 The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation:

(A) The intensity increases

(B) The minimum wave length increases

(C) The intensity decreases

(D) The minimum wave length decreases

Previous Years' Questions

Q.1 The shortest wavelength of X-rays emitted from an X-ray tube depends on ***(1982)***

(A) The current in the tube

(B) The voltage applied to the tube

(C) The nature of the gas in tube

(D) The atomic number of the target material

Q.2 Beta rays emitted by a radioactive material are ***(1983)***

(A) Electromagnetic radiations

(B) The electrons orbiting around the nucleus

(C) Charged particles emitted by the nucleus

(D) Neutral particles

Q.3 If elements with principal quantum number n > 4 were not allowed in nature, the number of possible elements would be ***(1983)***

(A) 60 (B) 32 (C) 4 (D) 64

Q.4 Consider the spectral line resulting from the transition n = 2 → n = 1 in the atoms and ions given below. The shortest wavelength is produced by ***(1983)***

(A) Hydrogen atom

(B) Deuterium atom

(C) Singly ionized helium

(D) Doubly ionized lithium

Q.5 Equation: $4{}_1^1H \longrightarrow {}_2^4He^{2+} + 2e^- + 26\text{ MeV}$

represents ***(1983)***

(A) β – decay (B) γ – decay

(C) Fusion (D) Fission

Q.6 For a given plate voltage, the plate current in a triode valve is maximum when the potential of ***(1985)***

(A) The grid is positive and plate is negative.

(B) The grid is zero and plate is positive.

(C) The grid is negative and plate is positive

(D) The grid is positive and plate is positive

Q.7 The X-ray beam coming from an X-ray tube will be ***(1985)***

(A) Monochromatic

(B) Having all wavelengths smaller than a certain maximum wavelength

(C) Having all wavelengths larger than a certain minimum wavelength

(D) Having all wavelengths lying between a minimum and a maximum wavelength

Q.8 **Statement-I:** If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change.

Statement-II: When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy. ***(2007)***

(A) If Statement-I is true, statement-II is true; statement-II is the correct explanation for statement-I.

(B) If Statement-I is true, statement-II is true; statement-II is not a correct explanation for statement-I.

(C) If statement-I is true; statement-II is false.

(D) If statement-I is false; statement-II is true.

Q.9 To produce characteristic X-rays using a tungsten target in an X-ray generator, the accelerating voltage should be greater than...... V and the energy of the characteristic radiation is .... eV. ***(1983)***

(The binding energy of the innermost electron in tungsten is 40 keV).

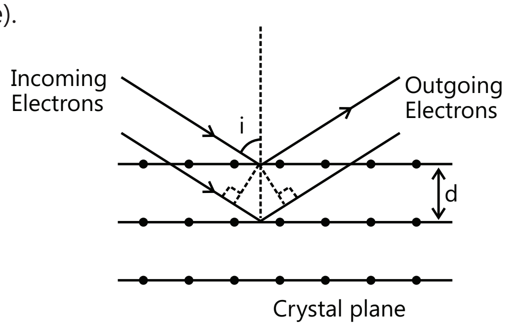
Q.10 The radioactive decay rate of a radioactive element is found to be 10^3 disintegration/second at a certain time. If the half-life of the element is one second, the decay rate after one second is .... and after three seconds is......... ***(1983)***

Q.11 The maximum kinetic energy of electrons emitted in the photoelectric effect is linearly dependent on the .... of the incident radiation. ***(1984)***

Q.12 In the uranium radioactive series the initial nucleus is ${}^{238}_{92}U$ and the final nucleus is ${}^{206}_{82}Pb$. When the uranium nucleus decays to lead, the number of α-particles emitted is .... and the number of β-particles emitted is......... ***(1985)***

Directions : Q.13, Q.14 and Q.15 are based on the following paragraph.

Wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see in figure).



Q.13 Electrons accelerated by potential V are diffracted from a crystal. If d = 1Å and i = 30°, V should be about ($h = 6.6 \times 10^{-34}$ Js, $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C) ***(2008)***

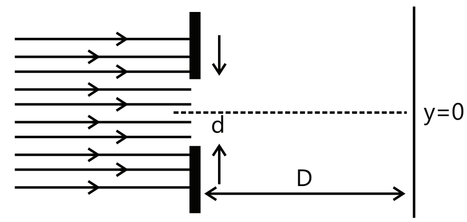
(A) 2000 V (B) 50 V (C) 500 V (D) 1000 V

Q.14 If a strong diffraction peak is observed when electrons are incident at an angle 'i' from the normal to the crystal planes with distance 'd' between them (see figure), de Broglie wavelength λ_{dB} of electrons can be calculated by the relationship (n is an integer) ***(2008)***

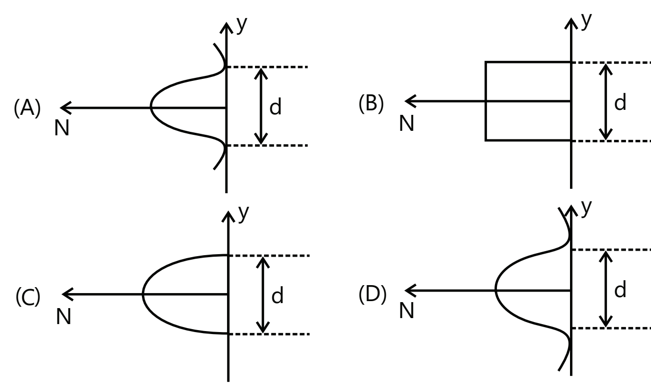
(A) $d \sin i = n\lambda_{dB}$ (B) $2d \cos i = n\lambda_{dB}$

(C) $2d \sin i = n\lambda_{dB}$ (D) $d \cos i = n\lambda_{dB}$

Q.15 In an experiment, electrons are made to pass through a narrow slit of width 'd' comparable to their de Broglie wavelength. They are detected on a screen at a distance 'D' from the slit (see figure). ***(2008)***



Which of the following graph can be expected to represent the number of electrons 'N' detected as a function of the detector position 'y'(y = 0 corresponds to the middle of the slit)?



Q.16 Two points P and Q are maintained at the potentials of 10V and -4V respectively. The work done in moving 100 electrons from P to Q is ***(2009)***

(A) -19×10^{-17} J (B) 9.60×10^{-17} J

(C) -2.24×10^{-16} J (D) 2.24×10^{-16} J

Q.17 The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photo electrons was found to be 1.68 eV. The work function of the metal is (hc = 1240 eV nm) ***(2009)***

(A) 3.09 eV (B) 1.41 eV

(C) 151 eV (D) 1.68 ev

Q.18 Statement-I: When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement-II: Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light. ***(2010)***

(A) Statement-I is true, statement-II is true; statement-II is the correct explanation of statement-I.

(B) Statement-I is true, statement-II is true; statement-II is not the correct explanation of statement-I.

(C) Statement-I is false, statement-II is true.

(D) Statement-I is true, statement-II is false.

Q.19 If a source of power 4 kW produces 10^{20} photons/second, the radiation belong to a part of the spectrum called ***(2010)***

(A) X–rays (B) Ultraviolet rays

(C) Microwaves (D) γ – rays

Q.20 This question has Statement-I and Statement-II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-I: A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface doubled, both the K_{max} and V_0 are also doubled ***(2011)***

Statement-II: The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(A) Statement-I is true, statement-II is true; statement-II is the correct explanation of statement-I.

(B) Statement-I is true, statement-II is true; statement-II is not the correct explanation of statement-I.

(C) Statement-I is false, statement-II is true.

(D) Statement-I is true, statement-II is false.

Q.21 This question has statement-I and statement-II. Of the four choices given after the statements, choose the one that best describes the two statements

Statement-I: Davisson – germer experiment established the wave nature of electrons.

Statement-II: If electrons have wave nature, they can interfere and show diffraction. ***(2012)***

(A) Statement-I is false, statement-II is true

(B) Statement-I is true, statement-II is false

(C) Statement-I is true, statement-II is the correct explanation for statement-I

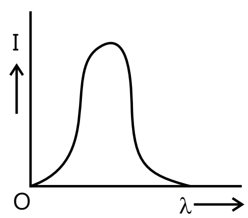
(D) Statement-I is true, statement-II is true, statement-II is not the correct explanation for statement-I.

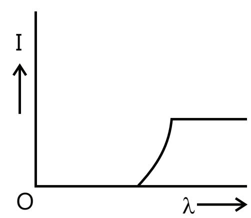
Q.22 A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by (n is an integer) ***(2012)***

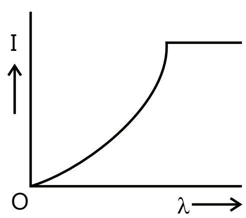
(A) $\frac{(m_1+m_2)^2 n^2h^2}{2m_1^2m_2^2r^2}$ (B) $\frac{n^2h^2}{2(m_1+m_2)r^2}$

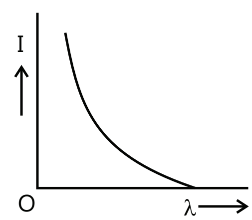
(C) $\frac{2n^2h^2}{(m_1+m_2)r^2}$ (D) $\frac{(m_1+m_2)n^2h^2}{2m_1m_2r^2}$

Q.23 The anode voltage of a photocellis kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows : ***(2013)***

(A) 

(B) 

(C) 

(D) 

Q.24 In a hydrogen like atom electron make transition from an energy level with quantum number n to another with quantum number (n – 1). If n>>1, the frequency of radiation emitted is proportional to : ***(2013)***

(A) $\frac{1}{n}$ (B) $\frac{1}{n^2}$ (C) $\frac{1}{n^3/2}$ (D) $\frac{1}{n^3}$

Q.25 The radiation corresponding to 3 → 2 transition of hydrogen atoms falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to ***(2014)***

(A) 1.8 eV (B) 1.1 eV (C) 0.8 eV (D) 1.6 eV

Q.26 Hydrogen ($_1H^1$), Deuterium ($_1H^2$), singly ionised Helium $(_2He^4)^+$ and doubly ionised lithium $(_3Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from n= 2 to n= 1. If the wave lengths of emitted radiation are λ_1, λ_2, λ_3 and λ_4 respectively then approximately which one of the following is correct? ***(2014)***

(A) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (B) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

(C) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (D) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

Q.27 Match List-I (Fundamental Experiment) with List-II (its conclusion) and select the correct option from the choices given below the list : ***(2015)***

	List - I		List - II
(i)	Franck-Hertz experiment	(p)	Particle nature of light
(ii)	Photo-electric experiment	(q)	Discrete energy levels of atom
(iii)	Davison, Germer experiment	(r)	Wave nature of electron
		(s)	Structure of atom

(A) (i) → (p), (ii) → (s), (iii) → (r)

(B) (i) → (q), (ii) → (s), (iii) → (r)

(C) (i) → (q), (ii) → (p), (iii) → (r)

(D) (i) → (s), (ii) → (r), (iii) → (q)

Q.28 Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be : ***(2016)***

(A) $< v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (B) $= v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

(C) $= v\left(\frac{3}{4}\right)^{\frac{1}{2}}$ (D) $> v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

JEE Advanced/Boards

Exercise 1

Q.1 When a monochromatic point source of light is at a distance of 0.2 m from a photoelectric cell, the cut off voltage and the saturation current are respectively 0.6 V and 18.0 mA. If the same source is placed 0.6 m away from the photoelectric cell, then find

(a) The stopping potential

(b) The saturation current

Q.2 663 mW of light from of 540 nm source is incident on the surface of a metal. If only 1 of each 5 x 10^9 incidents photons is absorbed and causes an electron to be ejected from the surface, the total photocurrent in the circuit is _______.

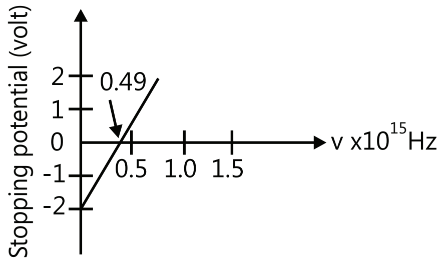
Q.3 Light of Wavelength 330 nm falling on a piece of metal ejects electrons with sufficient energy which requires voltage V_0 to prevent a electron from reading collector. In the same setup, light of wavelength 220 nm, ejects electron which require twice the voltage V_0 to stop them in reaching a collector. Find the numerical value of voltage V_0. (Take plank's constant, $h = 6.6\times10^{-34}$ J s and 1 eV = 1.6×10^{-19} J)

Q.4 A small 10W source of ultraviolet light of Wavelength 99 nm is held at a distance 0.1m from a metal surface. The radius of an atom of the metal is approximately 0.05 nm. Find

(i) The average number of photons striking an atom per second.

(ii) The number of photoelectrons emitted per unit area per second if the efficiency of liberation of photoelectrons is 1%.

Q.5 The surface of cesium is illuminated with monochromatic light of various wavelengths and the stopping potentials for the wavelengths are measured. The results of this experiment is plotted as shown in the figure. Estimate the value of work function of the cesium and Planck's constant.



Q.6 A small plate of a metal (work function = 1.17 eV) is placed at a distance of 2m from a monochromatic light source of wave length 4.8×10^{-7} m power 1.0 watt. The light falls normally on the plate. Find the number of photons striking the metal plate per square meter per second. If a constant uniform magnetic field of strength 10^{-4} tesla is applied parallel to the metal surface, find the radius of the largest circular path followed by the emitted photoelectrons.

Q.7 Electrons in hydrogen like atoms (Z = 3) make transition from the fifth to the fourth orbit & from the fourth to the third orbit. The resulting radiations are incident normally on a metal plate & eject photoelectrons. The stopping potential for the photoelectrons ejected by the shorter wavelength is 3.95 V. Calculate the work function of the metal, & the stopping potential for the photoelectrons ejected by the longer wavelength. (Rydberg constant = $1.094 \times 10^{7}\ m^{-1}$).

Q.8 A beam of light has three wavelength 4144 Å, 4972 Å & 6216Å with a total intensity of $3.6 \times 10^{-3}\ W.m^{-2}$ equally distributed amongst the three wavelengths. The beam falls normally on an area 1.0 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in time t = 2 s.

Q.9 A small 10 W source of ultraviolet light of wavelength 99nm is held at a distance 0.1 m from a metal surface. The radius of an atom of the metal is approximately 0.05 nm. Find:

(i) The number of photons striking an atom per seconds.

(ii) The number of photoelectrons emitted per seconds if the efficiency of liberation of photoelectrons is 1%.

Q.10 In a photoelectric effect set-up, a point source of light of power 3.2×10^{-3} W emits mono energetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the center of a stationary metallic sphere of work function 3.0 eV & of radius 8.0×10^{-3}. The efficiency of photoelectrons emission is one for every 10^6 incident photons. Assume that the sphere is isolated and initially neutral, and that photoelectrons are instantly swept away after emission.

(a) Calculate the number of photoelectrons emitted per seconds.

(b) Find the ratio of the wavelength of incident light to the de-Broglie wave length of the fastest photoelectrons emitted.

(c) It is observed that the photoelectron emissions stops at a certain time t after the light source is switched on. Why?

(d) Evaluate the time t.

Q.11 When photons of energy 4.25eV strike the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_a eV and de Broglie wavelength λ_a. The maximum kinetic energy of photoelectrons liberated from another metal B by photons of energy 4.7eV is $T_b = (T_a - 1.5)$ eV. If the de Broglie wavelength of these photoelectrons is $\lambda_b = 2\lambda_a$, then find

(a) The work function of a

(b) The work function of b

(c) T_a and T_b

Q.12 An electron of mass "m" and charge "e" initially at rest gets accelerated by a constant electric field E. The rate of change of de Broglie wavelength of this electron at time t is..................

Q.13 A hydrogen atom in a state having a binding energy 0.85eV makes a transition to a state of excitation energy 10.2eV. The wave length of emitted photon is ....nm.

Q.14 A hydrogen atom is in 5^{th} excited state. When the electrons jump to ground state the velocity of recoiling hydrogen atom is ....m/s and the energy of the photon is ....eV.

Q.15 The ratio of series limited wavelength of Balmer series to wavelength of first line of

Paschen series is ....

Q.16 A neutron with kinetic energy 25 eV strikes a stationary deuteron. Find the de Broglie wavelengths of both particles in the frame of their center of mass.

Q.17 Assume that the de Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance 'd' between the atoms of the array is 2 Å. A similar standing wave is again formed if 'd' is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electrons in electron volts and the least value of d for which the standing wave of the type described above can form.

Q.18 A stationary He^+ ion emitted a photon corresponding to the first line its Lyman series. That photon liberated a photoelectron from a stationary hydrogen atom in the ground state. Find the velocity of the photoelectron.

Q.19 A gas of identical hydrogen like atom has some atom in the lowest (ground) energy level A & some atoms in a particular upper (excited) energy level B & there are no atoms in any other energy level. The atoms of the gas make transition to a higher energy level by the absorbing monochromatic light of photon energy 2.7eV. Subsequently, the atom emit radiation of only six different photon energies. Some of the emitted photons have energy 2.7eV. Some have energy more and some have less than 2.7eV.

(i) Find the principle quantum of the initially excited level B.

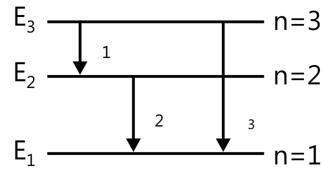
(ii) Find the ionization energy for the gas atoms.

(iii) Find the maximum and the minimum energies of the emitted photons.

Q.20 A hydrogen atom in ground state absorbs a photon of ultraviolet radiation of wavelength 50nm. Assuming that the entire photon energy is taken up by the electron, with what kinetic energy will the electron be ejected?

Q.21 An electron joins a helium nucleus to form a He^+ ion in ground state. The wavelength of the photon emitted in this process if the electron is assumed to have had no kinetic energy when it combines with nucleus is ________ nm.

Q.22 Three energy levels of an atom are shown in the figure. The wavelength corresponding to three possible transition are λ_1, λ_2 and λ_3. The value of λ_3 in terms of λ_1 and λ_2 is given by ______.



Q.23 Imagine an atom made up of a proton and a hypothetical particle of double the mass of an electron but having the same charge as the electron. Apply the Bohr atom model and consider a possible transition of this hypothetical particle to the first excited level. Find the longest wavelength photon that will be emitted λ (in terms of the Rydberg constant R.)

Q.24 In a hydrogen atom, the electron moves in an orbit of radius 0.5 Å making 10^{16} revolutions per second. The magnetic moment associated with the orbital motion of the electron is _________.

Q.25 A hydrogen like atom has its single electron orbiting around its stationary nucleus. The energy excite the electron from the second Bohr orbit to the third Bohr orbit is 47.2 eV. The atomic number of this nucleus is ______________.

Q.26 A single electron orbits a stationary nucleus of charge Ze where Z is a constant and e is the electronic charge. It requires 47.2eV to excite the electron from the 2^{nd} Bohr orbit to 3^{rd} Bohr orbit. Find

(i) The value of Z

(ii) Energy required to excite the electron from third to the fourth orbit

(iii) The wavelength of radiation required to remove the electron from the first orbit to

(iv) Infinity the Kinetic energy, potential energy and angular momentum in the first Bohr

(v) Orbit the radius of the first Bohr orbit.

Q.27 A hydrogen like atom (atomic number Z) is in higher excited state of quantum number n. This excited atom can make a transition to the first excited state by successive emitting two photons of energy 22.95eV and 5.15eV respectively. Alternatively, the atom from the same excited state can make transition to the second excited state by successive emitting two photons of energies 2.4eV and 8.7eV respectively. Find the value of n and Z.

Q.28 Find the binding energy of an electron in the ground state of a hydrogen like atom in whose spectrum the third of the corresponding Balmer series is equal to 108.5 nm.

Q.29 Which level of the doubly ionized lithium has the same energy as the ground state energy of the hydrogen atom? Find the ratio of the two radii of corresponding orbits.

Q.30 A 20 KeV energy electron is brought to rest in an X-ray tube, by undergoing two successive bremsstrahlung events, thus emitting two photons. The wavelength of the second photon is 130 x 10^{-12} m greater than the wavelength of the first emitted photon. Calculate the wavelength of the two photons.

Exercise 2

Single Correct Choice Type

Q.1 10^{-3} W of 5000 Å light is directed on a photoelectric cell. If the current in the cell is 0.16 μA, the percentage of incident photons which produce photoelectrons, is

(A) 0.4% (B) 0.04% (C) 20% (D) 10%

Q.2 Photons with energy 5eV are incident on a cathode C, on a photoelectric cell. The maximum energy of the emitted photoelectrons is 2 eV. When photons of energy 6 eV are incident on C, no photoelectrons will reach the anode A if the stopping potential of A relative to C is

(A) 3V (B) –3V (C) –1 V (D) 4 V

Q.3 In a hydrogen atom, the binding energy of the electron of the nth state is E_n, then the frequency of revolution of the electron in the nth orbit is:

(A) $\frac{2E_n}{nh}$ (B) $\frac{2E_n n}{h}$ (C) $\frac{E_n}{nh}$ (D) $\frac{E_n n}{h}$

Q.4 Difference between nth and $(n+1)^{th}$ Bohr's radius of 'H' atom is equal to it's $(n-1)^{th}$ Bohr's radius. The value of n is:

(A) 1 (B) 2 (C) 3 (D) 4

Q.5 Electron in a hydrogen atom is replaced by an identically charged particle muon with mass 207 times that of electron. Now the radius of K shell will be

(A) 2.56×10^{-3} Å (B) 109.7 Å

(C) 1.21×10^{-3} Å (D) 22174.4 Å

Q.6 An electrons collides with a fixed hydrogen atom in its ground state. Hydrogen atom gets excited and the colliding electron loses all its kinetic energy. Consequently the hydrogen atom may emit a photon corresponding to the largest wavelength of the Balmer series. The min. K.E. of colliding electron will be

(A) 10.2eV (B) 1.9 eV

(C) 12.1 eV (D) 13.6 eV

Q.7 A neutron collides head on with a stationary hydrogen atom in ground state

(A) If kinetic energy of the neutron is less than 13.6 eV, collision must be elastic.

(B) If kinetic energy of the neutron is less than 13.6 eV, collision may be inelastic.

(C) Inelastic collision takes place when initial kinetic energy of neutron is greater than 13.6 eV.

(D) Perfectly inelastic collision cannot take place.

Q.8 An electron in hydrogen atom first jumps from second excited state to first excited state and then, from first excited state to ground state. Let the ratio of wavelength, momentum and energy of photons in the two cases be x, y and z, then select the wrong answer(s):

(A) $z = \frac{1}{x}$ (B) $x = \frac{9}{4}$ (C) $y = \frac{5}{27}$ (D) $z = \frac{5}{27}$

Multiple Correct Choice Type

Q.9 In photoelectric effect, stopping potential depends on

(A) Frequency of the incident light

(B) Intensity of the incident light by varying source

(C) Emitter's properties

(D) Frequency and intensity of the incident light

Q.10 Two electrons are moving with the same speed v. One electron enters a region of uniform electric field while the other enters a region of uniform magnetic field, then after sometime if the de-Broglie wavelengths of the two are λ_1 and λ_2, then:

(A) $\lambda_1 = \lambda_2$ (B) $\lambda_1 > \lambda_2$

(C) $\lambda_1 < \lambda_2$ (D) $\lambda_1 > \lambda_2$ or $\lambda_1 < \lambda_2$

Q.11 A neutron collides head-on with a stationary hydrogen atom in ground state. Which of the following statements are correct (Assume that the hydrogen atom and neutron has same mass):

(A) If kinetic energy of the neutron is less than 20.4 eV collision must be elastic.

(B) If kinetic energy of the neutron is less than 20.4 eV collision may be inelastic.

(C) Inelastic collision may be take place only when initial kinetic energy of neutron is greater than 20.4 eV.

(D) Perfectly inelastic collision cannot take place.

Q.12 A free hydrogen atom in ground state is at rest. A neutron of kinetic energy 'K' collides with the hydrogen atom. After collision hydrogen atom emits two photons in succession one of which has energy 2.55 eV. (Assume that the hydrogen atom and neutron has same mass)

(A) Minimum value of 'K' is 25.5 eV.

(B) Minimum value of 'K' is 12.75 eV.

(C) The other photon has energy 10.2eV.

(D) The upper energy level is of excitation energy 12.75 eV.

Q.13 A particular hydrogen like atom has its ground state binding energy 122.4eV. It is in ground state. Then:

(A) Its atomic number is 3

(B) An electron of 90eV can excite it.

(C) An electron of kinetic energy nearly 91.8eV can be almost brought to rest by this atom.

(D) An electron of kinetic energy 2.6eV may emerge from the atom when electron of kinetic energy 125eV collides with this atom.

Q.14 A beam of ultraviolet light of all wavelengths pass through hydrogen gas at room temperature, in the x-direction. Assuming all photons emitted due to electron transition inside the gas emerge in the y-direction. Let A and B denote the lights emerging from the gas in the x and y directions respectively.

(A) Some of the incident wavelengths will be absent in A.

(B) Only those wavelengths will be present in B which are absent in A.

(C) B will contain some visible light.

(D) B will contain some infrared light.

Q.15 X-rays are produced by accelerating electrons across a given potential difference to strike a meta target of high atomic number. If the electrons have same speed when they strike the target, the X-ray spectrum will exhibit

(A) A minimum wavelength

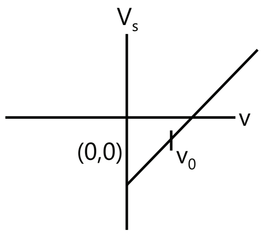
(B) A continuous spectrum

(C) Some discrete comparatively prominent wavelength

(D) Uniform density over the whole spectrum

Assertion Reasoning Type

Q.16 Statement-I: Figure shows graph of stopping potential and frequency of incident light in photoelectric effect. For values of frequency less than threshold frequency (v_0) stopping potential is negative.



Statement-II: Lower the value of frequency of incident light (for $v > v_0$) the lower is the maxima of kinetic energy of emitted photoelectrons.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Q.17 Statement-I: Two photons having equal wavelengths have equal linear momenta.

Statement-II: When light shows its photons character, each photon has a linear momentum $\lambda = \frac{h}{p}$.

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Q.18 Statement-I: In the process of photoelectric emission, all the emitted photoelectrons have same K.E.

Statement-II: According to Einstein's photoelectric equation

$KE_{max} = hv - \phi$.

(A) Statement-I is True, statement-II is True, statement-II is a correct explanation for statement-I.

(B) Statement-I is True, statement-II is True, statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is True, statement-II is False

(D) Statement-I is False, statement-II is True

Q.19 Statement-I: Work function of aluminum is 4.2 eV. If two photons each of energy 2.5 eV strikes on a piece of aluminum, the photoelectric emission does not occur.

Statement-II: In photoelectric effect a single photon interacts with a single electron and electron is emitted only if energy of each incident photon is greater than the work function.

(A) Statement-I is True, statement-II is True, statement-II is a correct explanation for statement-I.

(B) Statement-I is True, statement-II is True, statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is True, statement-II is False

(D) Statement-I is False, statement-II is True

Q.20 Statement-I: An electron and a proton are accelerated through the same potential difference. The de-Broglie wavelength associated with the electron is longer.

Statement-II: de-Broglie wavelength associated with a moving particle is $\lambda = \frac{h}{p}$ where, p is the linear momentum and both have same K.E.

(A) Statement-I is True, statement-II is True, statement-II is a correct explanation for statement-I.

(B) Statement-I is True, statement-II is True, statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is True, statement-II is False

(D) Statement-I is False, statement-II is True

Q.21 Statement-I: In a laboratory experiment, on emission from atomic hydrogen in a discharge tube, only a small number of lines are observed whereas a large number of lines are present in the hydrogen spectrum of a star.

Statement-II: The temperature of discharge tube is much smaller than that of the star.

(A) Statement-I is True, statement-II is True, statement-II is a correct explanation for statement-I.

(B) Statement-I is True, statement-II is True, statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is True, statement-II is False

(D) Statement-I is False, statement-II is True

Previous Years' Questions

Q.1 A single electron orbits around a stationary nucleus of charge +Ze where Z is a constant and e is the magnitude of the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit. ***(1981)***

Find:

(a) The value of Z.

(b) The energy required to excite the electron from the third to the fourth Bohr orbit.

(c) The wavelength of the electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.

(d) The kinetic energy, potential energy and the angular momentum of the electron in the first Bohr orbit.

(e) The radius of the first Bohr orbit.

(The ionization energy of hydrogen atom = 13.6 eV, Bohr radius = $5.3\times10^{-11}\,m$, velocity of light = $3\times10^{8}\,m/s$. Planck's constant = 6.6×10^{-34} J s).

Q.2 Hydrogen atom in its ground state is excited by means of monochromatic radiation of wavelength 975° Å. How many different lines are possible in the resulting spectrum? Calculate the longest wavelength amongst them. You may assume the ionization energy for hydrogen atom as 13.6 eV. ***(1982)***

Q.3 How many electrons, protons and neutrons are there in a nucleus of atomic number 11 and mass number 24? ***(1982)***

(a) Number of electrons =

(b) Number of protons =

(c) Number of neutrons =

Q.4 A uranium nucleus (atomic number 92, mass number 238) emits an alpha particle and the resulting nucleus emits β -particle. What are the atomic number and mass number of the final nucleus? ***(1982)***

(a) Atomic number =

(b) Mass number =

Q.5 Ultraviolet light of wavelengths 800 Å and 700 Å when allowed to fall on hydrogen atoms in their ground state is found to liberate electrons with kinetic energy 1.8 eV and 4.0 eV respectively. Find the value of Planck's constant. ***(1983)***

Q.6 the ionization energy of a hydrogen like Bohr atom is 4 Rrydberg.

(a) What is the wavelength of the radiation emitted when the electron jumps from the first excited state to the ground state? ***(1984)***

(b) What is the radius of the first orbit for this atom?

Now, as $r \propto \frac{1}{Z}$ Radius of first orbit of this atom, $r_1 = \frac{r_{H_1}}{Z}$ $= \frac{0.529}{2} = 0.2645$ Å

Q.7 A doubly ionized lithium atom is hydrogen-like with atomic number 3.

(a) Find the wavelength of the radiation required to excite the electron in Li^{2+} from the first to the third Bohr orbit. (Ionization energy of the hydrogen atom equals 13.6 eV.)

(b) How many spectral lines are observed in the emission spectrum of the above excited system? ***(1985)***

Q.8 There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half-life of neutrons is 700s, what fraction of neutrons will decay before they travel a distance of 10 m? ***(1986)***

Q.9 A particle of charge equal to that of an electron –e, and mass 208 times of the mass of the electron (called a mu-meson) moves in a circular orbit around a nucleus of charge +3e. (Take the mass of the nucleus to be infinite). Assuming that the Bohr model of the atom is applicable to this system. ***(1988)***

(a) Derive an expression for the radius of the n^{th} Bohr orbit.

(b) Find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for the hydrogen atom.

(c) Find the wavelength of the radiation emitted when the mu-meson jumps from the third orbit to the first orbit. (Rydberg's constant = $1.097 \times 10^7\ m^{-1}$)

Paragraph 1: (Q.10–Q.12) In a mixture of $H-He^+$ gas (He^+ is single ionized He atom), H atoms and He^+ ions excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy of He^+ ions (by collisions). Assume that the Bohr model of atom is exactly valid.

Q.10 The quantum number n of the state finally populated in He^+ ions is ***(2008)***

(A) 2 (B) 3 (C) 4 (D) 4

Q.11 The wavelength of light emitted in the visible region by He^+ ions after collisions with H-atoms is ***(2008)***

(A) 6.5×10^{-7} m (B) 5.6×10^{-7} m

(C) 4.8×10^{-7} m (D) 4.0×10^{-7} m

Q.12 The ratio of the kinetic energy of the n = 2 electron for the H atom to that of He^+ ion is **(2008)**

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

Q.13 Some laws/processes are given in column I. Match these with the physical phenomena given in column II. ***(2006)***

Column I		**Column II**	
(A)	Nuclear fusion	(p)	Converts some matter into energy.
(B)	Nuclear fission	(q)	Generally possible for nuclei with low atomic number
(C)	β -decay	(r)	Generally possible for nuclei with higher atomic number
(D)	Exothermic nuclear reaction	(s)	Essentially proceeds by weak nuclear forces

Q.14 The threshold wavelength for photoelectric emission from a material is 5200 Å. Photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a ***(1982)***

(A) 50W infrared lamp (B) 1 W infrared lamp

(C) 50W ultraviolet lamp (D) 1W ultraviolet lamp

Q.15 The allowed energy for the particle for a particular value of n is proportional to ***(2009)***

(A) a^{-2} (B) $a^{-3/2}$ (C) a^{-1} (D) a^2

Q.16 If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and a = 6.6 nm, the energy of the particle in its ground state is closest to ***(2009)***

(A) 0.8 MeV (B) 8 MeV (C) 80 MeV (D) 800 MeV

Q.17 The speed of the particle that can take discrete values is proportional to ***(2009)***

(A) $n^{-3/2}$ (B) n^{-1} (C) $n^{1/2}$ (D) n

Q.18 An α -particle and a proton are accelerated from rest by a potential difference of 100V. After this, their de-Broglie wavelength are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is : ***(2010)***

Q.19 The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 $\mathring{A}$. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is ***(2010)***

(A) 1215 $\mathring{A}$ (B) 1640 $\mathring{A}$ (C) 2430 $\mathring{A}$ (D) 4687 $\mathring{A}$

Q.20 A proton is fired from very far away towards a nucleus with charge Q = 120 e, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is: (take the proton mass, mp = (5/3) × 10^{-27} kg; h/e = 4.2 × 10^{-15} J.s / C; $\frac{1}{4\pi\varepsilon_0}$ = 9 × 10^9 m/F; 1 fm = 10^{-15}) ***(2012)***

Q.21 A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is 3 × 10^8 m/s. The final momentum of the object is ***(2013)***

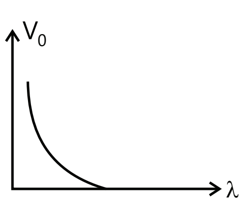
(A) 0.3 × 10^{-17} kg ms^{-1} (B) 1.0 × 10^{-17} kg ms^{-1}

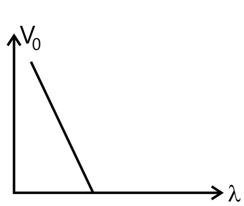
(C) 3.0 × 10^{-17} kg ms^{-1} (D) 9.0 × 10^{-17} kg ms^{-1}

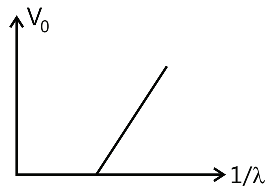
Q.22 The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is ***(2013)***

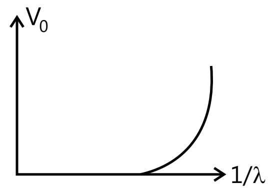
Q.23 Consider a hydrogen atom with its electron in the n^{th} orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is (hc = 1242 eV nm) ***(2015)***

Q.24 For photo-electric effect with incident photon wavelength λ, the stopping potential is V_o. Identify the correct variation(s) of V_0 with λ and $1/\lambda$. ***(2015)***

(A)


(B)


(C)


(D)


Q.25 In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below: ***(2016)***

$\lambda(\mu m)$	V_0 (Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that c = 3 × 10^8 m s^{-1} and e = 1.6 × 10^{-19} C, Planck's constant (in units of J s) found from such an experiment is

(A) 6.0 × 10^{-34} (B) 6.4 × 10^{-34}

(C) 6.6 × 10^{-34} (D) 6.8 × 10^{-34}

Q.26 Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where n >> 1. Which of the following statement(s) is(are) true? ***(2016)***

(A) Relative change in the radii of two consecutive orbitals does not depend on Z

(B) Relative change in the radii of two consecutive orbitals varies as 1/n

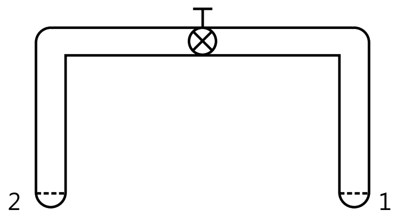
(C) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(D) Relative change in the angular momenta of two consecutive orbitals varies as 1/n

Q.27 A hydrogen atom in its ground state is irradiated by light of wavelength 970A. Taking hc/e = 1.237 × 10^{-6} eVm and the ground state energy of hydrogen atom as –13.6 eV, the number of lines present in the emission spectrum is ***(2016)***

Q.28 A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure. Just after opening the valve, ***(2008)***

Figure:



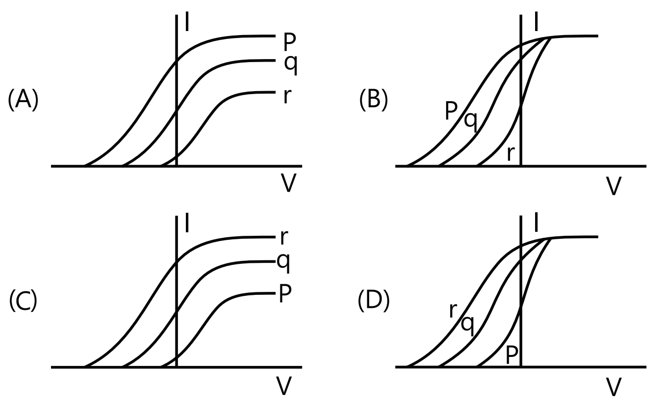
(A) Air from end 1 flows towards end 2. No change in the volume of the soap bubbles

(B) Air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases

(C) No changes occurs

(D) Air from end 2 flows towards end 1. volume of the soap bubble at end 1 increases

Q.29 Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV, respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is (Take hc = 1240 eV nm) ***(2009)***



Paragraph for questions 30 to 32 ***(2010)***

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

Q.30 A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the n^{th} level (n = 0 is not allowed) is

(A) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (B) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

(C) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (D) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

Q.31 It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi}\times 10^{11}$Hz. Then the moment of inertia of CO molecule about its centre of mass is close to (Take h = $2\pi\times 10^{-34}$ Js)

(A) 2.76×10^{-46} kg m^2 (B) 1.87×10^{-46} kg m^2

(C) 4.67×10^{-47} kg m^2 (D) 1.17×10^{-47} kg m^2

Q.32 In a CO molecule, the distance between C (mass = 12 a.m.u) and O (mass = 16 a.m.u.), where 1 a.m.u. = $\frac{5}{3}\times 10^{-27}$ kg, is close to

(A) 2.4×10^{-10} m (B) 1.9×10^{-10} m

(C) 1.3×10^{-10} m (D) 4.4×10^{-11} m

Q.33 A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is A $\times 10^z$ (where 1 < A < 10). The value of 'Z' is ***(2011)***

Q.34 Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The g ravitational constant is G. The correct statement(s) is (are) ***(2013)***

(A) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$

(B) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$

(C) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

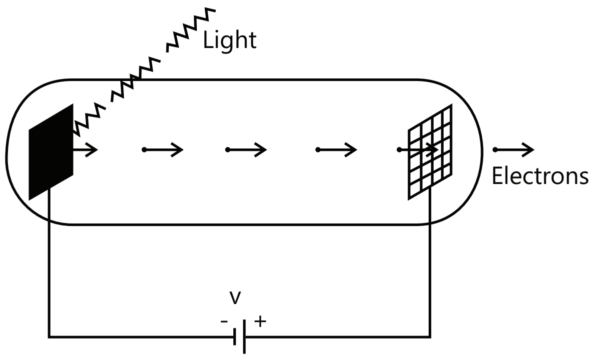
(D) The energy of the mass m remains constant.

Q.35 A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2, respectively. If the ratio $u_1 : u_2 = 2 : 1$and hc = 1240 eV nm, the work function of the metal is nearly ***(2014)***

(A) 3.7 eV (B) 3.2 eV

(C) 2.8 eV (D) 2.5 eV

Q.36 Light of wavelength λ_{ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e, which of the following statement(s) is(are) true? ***(2016)***



(A) For large potential difference (V >> ϕ/e), λ_e is approximately halved if V is made four times

(B) λ_e decreases with increase in ϕ and λ_{ph}

(C) λ_e increases at the same rate as λ_{ph} for $\lambda_{ph} < hc/\phi$

(D) λ_e is approximately halved, if d is doubled

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q.10 Q.17 Q.23

Q.27 Q.28

Exercise 2

Q.2 Q.4 Q.7

JEE Advanced/Boards

Exercise 1

Q.4 Q.8 Q.10

Q.19 Q.22 Q.30

Exercise 2

Q.1 Q.6 Q.8

Q.12 Q.13 Q.14

Q.16

Previous Years' Questions

Q.10

Answer Key

JEE Main/Boards

Exercise 1

Q.1 (iii)1.49×10^{-19} J

Q.4 $\frac{\lambda}{2}$

Q.8 Line B represent a particle of smaller mass.

Q.9 (i) Metal Q, (ii) Metal P, (iii) Stopping potential remains unchanged.

Q.10. 1.5 eV

Q.11 $1 : 2\sqrt{2}$

Q.17 2.46×10^{15} Hz

Q.21 (i) –1.7 eV (ii) 972.54 Å

Q.23 +13.6 eV, –27.2 eV

Q.24 28.8×10^{-15} m

Q.27 7.5×10^{14} Hz, 3.094 eV

Q.28 4137.5 Å

Exercise 2

Single Correct Choice Type

Q.1 C	**Q.2** D	**Q.3** C	**Q.4** A	**Q.5** A	**Q.6** C
Q.7 B	**Q.8** B	**Q.9** C	**Q.10** A	**Q.11** D	**Q.12** C
Q.13 B	**Q.14** C	**Q.15** D	**Q.16** A	**Q.17** A	**Q.18** C
Q.19 C	**Q.20** A	**Q.21** B	**Q.22** B	**Q.23** A	**Q.24** B
Q.25 A	**Q.26** B	**Q.27** A	**Q.28** B	**Q.29** D	

Previous Years' Questions

Q.1 B	**Q.2** C	**Q.3** A	**Q.4** D	**Q.5** C	**Q.6** D
Q.7 C	**Q.8** B	**Q.9** 30×10^3V, 30×10^3eV		**Q.10** 500 dps, 125 dps	
Q.11 Frequency	**Q.12** 8, 6	**Q.13** B	**Q.14** B	**Q.15** D	**Q.16** D
Q.17 B	**Q.18** D	**Q.19** A	**Q.20** C	**Q.21** C	**Q.22** D
Q.23 D	**Q.24** D	**Q.25** B	**Q.26** C	**Q.27** C	**Q.28** D

JEE Advanced/Boards

Exercise 1

Q.1 (a) 0.6 V, (b) 2.0 mA

Q.2 5.76×10^{-11} A

Q.3 $\frac{15}{8}$V

Q.4 $\frac{5}{16}, \frac{10^{19}}{8\pi}$

Q.5 2eV, 6.53×10^{-34} J s

Q.6 4.8×10^{16}, 4.0 cm

Q.7 2 eV, 0.754 V

Q.8 1.1×10^{12}

Q.9 (i) $\frac{5}{16}$ photon/s, (ii) $\frac{5}{1600}$ electrons/s

Q.10 (a) $10^5\ s^{-1}$; (b) 286.76 ; (d) 111.1s

Q.11 (a) 2.25 eV, (b) 4.2 eV, (c) 2.0 eV, 0.5 eV **Q.12** $-\frac{h}{eEt^2}$

Q.13 487 nm **Q.14** 4.26 m/s, 13.2 eV **Q.15** 7 : 36

Q.16 $\lambda_{deutron} = \lambda_{neutron} = 8.6$ pm **Q.17** KE $\cong$ 148.4 eV, d_{least} = 0.5 Å **Q.18** 3.1×10^6 m / s

Q.19 (i) 2 ; (ii) 23.04 × 10^{-19} J ; (iii) $4 \to 1$, $4 \to 3$ **Q.20** 11.24 eV

Q.21 22.8 nm **Q.22** $\frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$ **Q.23** $\frac{18}{5R}$

Q.24 1.257 × 10^{-23} Am^2 **Q.25** 5

Q.26 (i) Z = 5, (ii) E= 16.5 eV, (iii) λ=36.4 A, (iv) K.E = 340 eV, P.E = –680 eV, (v) Radius r = 0.1058 Å

Q.27 Z = 3, n = 7 **Q.28** 54.4 eV **Q.29** n = 3, 3 : 1

Q.30 $\lambda_1 = 0.871\,\mathring{A}$ and $\lambda_2 = 2.17\,\mathring{A}$

Exercise 2

Single Correct Choice Type

Q.1 B **Q.2** B **Q.3** A **Q.4** D **Q.5** A **Q.6** C

Q.7 A **Q.8** B

Multiple Correct Choice Type

Q.9 A, C **Q.10** A, D **Q.11** A,C **Q.12** A, C, D **Q.13** A, C, D

Q.14 A, C, D **Q.15** A, B, C

Assertion Reasoning Type

Q.16 D **Q.17** C **Q.18** D **Q.19** D **Q.20** A **Q.21** A

Previous Years' Questions

Q.1 (a) 5 (b) 16.53 eV (c) 36.4 Å (d) 340 eV, –680 eV, –340 eV, 1.05 × 10^{-34} $\frac{kgm^2}{s}$ (e) 1.06 × 10^{-11} m

Q.2 Six, 1.875 μm **Q.3** 0, 11, 13 **Q.4** (a) 91 (b) 234

Q.5 6.6 × 10^{-34} J s **Q.6** (a) 300 Å (b) 0.2645 Å **Q.7** (a) 113.74 Å (b) 3

Q.8 3.96 × 10^{-6} **Q.9** (a) $r_n = \frac{n^2 h^2 \varepsilon_0}{624\pi m_e e^2}$ (b) n = 25 (c) 0.546 $\mathring{A}$

Q.10 C **Q.11** C **Q.12** A **Q.13** $A \to p, q$; $B \to p, r$; $C \to p, s$; $D \to q$

Q.14 C,D **Q.15** A **Q.16** B **Q.17** D **Q.18** 3 **Q.19** A

Q.20 7 fm **Q.21** B **Q.22** 1 **Q.23** 2 **Q.24** A **Q.25** B

Q.26 A, B, D **Q.27** 6 **Q.28** B **Q.29** A **Q.30** D **Q.31** B

Q.32 C **Q.33** 7 **Q.34** B **Q.35** A **Q.36** A

Solutions

JEE Main/Boards

Exercise 1

Sol 1: (i) Work function – It is the minimum energy of incident photon below which no ejection of photoelectron from a metal surface will take place is known as work function or thresholds energy for that metal

$\phi = hV_0$

(ii) Threshold frequency – it is the minimum frequency of incident photon below which no ejection of photoelectron from a metal surface will take place is known as threshold frequency for that metal.

(iii) Stopping potential – The negative potential (V_0) applied to the anode at which the current gets reduced to zero is called stopping potential.

$KE_{max} = E - \phi$

$$E = \frac{12400}{3000} = 4.13 \text{ eV}$$

$KE_{max} = 4.13 - 3.2 = 0.93 \text{ eV} = 1.49 \times 10^{-19}$ J

Sol 2: KE = eV

$$\lambda_d = \frac{h}{p} = .\frac{h}{\sqrt{2mKE}}. = \frac{h}{\sqrt{2meV}}$$

$$= .\frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.67\times10^{-19}\times V}} = \frac{12.27}{\sqrt{V}}\text{Å}$$

For Davisson and Germer's experiment, refer theory.

Sol 3: $\phi_A = 2 \text{ eV} = \dfrac{hc}{\lambda_A}$

$\phi_B = 5 \text{ eV} = \dfrac{hc}{\lambda_B}$

$\lambda_A > \lambda_B$

Sol 4: K.E. = eV

$$\lambda_1 = \frac{h}{\sqrt{2mKE_1}} = \frac{h}{\sqrt{2meV}} = \lambda$$

$$\lambda_2 = \frac{h}{\sqrt{2mKE_2}} = \frac{h}{\sqrt{2me\times4v}} = \frac{\lambda_1}{2} = \frac{\lambda}{2}$$

Sol 5: For graph, refer theory.

If the distance is reduced, intensity of light will increase and number of electrons will increase and current will increase. Stopping potential will not be affected by distance.

Sol 6: $h\nu_1 = W_1 + KE$

$h\nu_2 = W_2 + KE$

$W_1 > W_2$

$\Rightarrow \nu_1 > \nu_2$

Sol 7: Refer theory

Sol 8: $\lambda = \dfrac{h}{\sqrt{2meV}}$

$$\text{Slope} = \frac{h}{\sqrt{2me}}$$

Slope of B > Slope of A

$\Rightarrow m_B < m_A$

Sol 9: (i) Metal P has greater threshold wavelength because P has lower threshold frequency.

(ii) $KE_{max} = h\nu - h\nu_0$

Metal P emits electrons with less kinetic energy as P has less threshold frequency.

(iii) If distance is doubled, there is no change in stopping potential.

Sol 10: $KE_{max} = eV = 1.5$ eV

Sol 11: $\dfrac{\lambda_\alpha}{\lambda_p} = \dfrac{h}{\sqrt{2m_\alpha(KE)_\alpha}} \div \dfrac{h}{\sqrt{2m_p(KE)_p}}$

$$= \sqrt{\frac{m_p}{m_\alpha}\frac{(KE)_p}{(KE)_\alpha}} = \sqrt{\frac{1}{4}\cdot\frac{eV}{2eV}} = \frac{1}{2\sqrt{2}}$$

Sol 12: $h\nu = h\nu_0 + KE$

Refer theory

Sol 13: Refer theory

Sol 14: Refer theory for the graph

$h\nu_1 = h\nu_0 + eV$; $h\nu_2 = h\nu_0 + eV$

Stopping potential V is higher for ν_1 frequency by the above equations.

Sol 15: $\lambda_p = \frac{h}{\sqrt{2m_p(KE)}}$; $\lambda_e = \frac{h}{\sqrt{2m_e(KE)}}$

$m_p > m_e$

$\lambda_p < \lambda_e$

Sol 16: (i) Cut-off voltage is the negative potential applied to the anode at which the current gets reduced to zero.

Refer theory for second part of question.

Sol 17: Refer theory

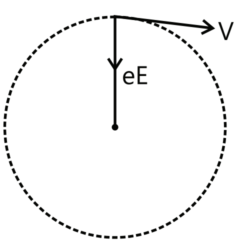
E = – 3.4 eV – (– 13.6 eV)

E = 10.2 eV

$h\nu = 10.2 \times 10^{-19} \times 1.6$ J

$\nu = 2.46 \times 10^{15}$ Hz

Sol 18: $eE = .\frac{mv^2}{r}.$



$$\frac{e}{m} = \frac{v^2}{rE}$$

Sol 19: Magnetic moment of a charged particle moving in a circle is given by

$\mu = IA$

I = charge flowing per sec $= \frac{e}{2\pi}\omega$

$$\mu = \frac{e\omega}{2\pi} \times \pi r^2 = \frac{evr}{2}$$

Sol 20: Refer theory

Sol 21: (i) PE = 2 T.E.

$$PE = \frac{-13.6 \times 2}{16} = -0.85 \times 2 = -1.7 \text{ eV}$$

$$\text{(ii) } E = 13.6\left[\frac{1}{1^2} - \frac{1}{4^2}\right] = \frac{15}{16} \times 13.6 \text{ eV}$$

E = 12.75 eV

$$\lambda = \frac{12400}{12.75} = 972.54 \text{ Å}$$

Sol 22: Refer theory

Sol 23: T.E. = – 13.6 eV

KE = |T.E.| = 13.6 eV

P.E = 2T.E = – 13.6 × 2 = –27.2 eV

Sol 24: $8 \times 10^6 = \frac{K \times (2e)(80e)}{r}$

$$r = \frac{9 \times 10^9 \times 160 \times 1.6 \times 10^{-19}}{8 \times 10^6} = 28.8 \times 10^{-15} \text{ m}$$

If kinetic energy is doubled then closest distance will become half of the original.

Sol 25: $\lambda = \frac{h}{\sqrt{2m(KE)}}$

$$\lambda_e = \frac{h}{\sqrt{2m_e(KE)_e}} = \frac{h}{\sqrt{2m_p(KE)_p}}$$

$m_e(KE)_e = m_p(KE)_p$

$m_p > m_e$

$\Rightarrow (KE)_e > (KE)_p$

Sol 26: Kinetic energy = $h\nu - h\nu_0$

It is independent of the intensity of light.

Sol 27: $\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{4 \times 10^{-7}} = 0.75 \times 10^{15} = 7.5 \times 10^{14}$ Hz

Energy = $h\nu$

$= 6.6 \times 10^{-34} \times 7.5 \times 10^{14} = 49.5 \times 10^{-20}$ J= 3.094 eV

Sol 28: Longest wavelength $\Rightarrow$ minimum frequency photon = ν_0

$$\frac{12400}{3} = 4137.5 \text{ Å}$$

Exercise 2

Single Correct Choice Type

Sol 1: (C) Intensity of both bulb is same i.e. $I_1 = I_2$

E_r = Energy of red colour photon

E_b = Energy of blue colour photon

$E_b > E_r$

$n_r = \frac{I_1}{E_r}$

$n_b = \frac{I_2}{E_b}$

$\Rightarrow n_b < n_r$

Sol 2: (D) $\frac{hc}{\lambda} = \phi + K$ ... (i)

$\frac{4hc}{3\lambda} = \phi + K_2$ ... (ii)

By (i) and (ii)

$\frac{4}{3}[\phi + K] = \phi + K_2$

$\frac{4\phi}{3} + \frac{4k}{3} = \phi + K_2$

$K_2 = \frac{4K}{3} + \frac{\phi}{3}$

Sol 3: (C) $h\nu = \phi + KE$

KE = eV

V = stopping potential

$h\nu = \phi + eV$ ... (i)

$h2\nu = \phi + eV_2$ ... (ii)

By (i) and (ii)

$2(\phi + eV) = \phi + eV_2$

$\phi + 2eV = eV_2$

$V_2 = 2V + \frac{\phi}{e}$

Sol 4: (A) $E = \phi + KE$

KE = eV, where V is stopping potential

E = 1.7 eV + 10.4 eV

E = 12.1 eV

$12.1 = 13.6 \times (1)^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

[As energy is greater than 12.1 eV so 1 state has to be ground state]

$12.1 = 13.6\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right]$

$\frac{12.1}{13.6} = 1 - \frac{1}{n_2^2}$

$\frac{1}{n_2^2} = 1 - \frac{12.1}{13.6} = \frac{1.5}{13.6}$

$n_2^2 = \frac{13.6}{1.5}$

$n_2 = 3$

Sol 5: (A) $E_1 = 2\phi$

$E_2 = 5\phi$

$E_1 = \phi + KE_1$

$2\phi - \phi = KE_1$

$KE_1 = \phi = \frac{1}{2}mv_1^2$

$E_2 = \phi + KE_2$

$5\phi - \phi = KE_2$

$KE_2 = 4\phi = \frac{1}{2}mv_2^2$

$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{2\phi}{m}}}{\sqrt{\frac{8\phi}{m}}} = \frac{1}{2}$$

Sol 6: (C) $\frac{hc}{\lambda_1} = \phi + KE_1$

$KE_1 = eV_1$

$\frac{hc}{\lambda_1} = \phi + eV_1 \Rightarrow V_1 = \left(\frac{hc}{\lambda_1} - \phi\right)\frac{1}{e}$

$\frac{hc}{\lambda_2} = \phi + eV_2 \Rightarrow V_2 = \left(\frac{hc}{\lambda_2} - \phi\right)\frac{1}{e}$

$\frac{hc}{\lambda_3} = \phi + eV_3 \Rightarrow V_3 = \left(\frac{hc}{\lambda_3} - \phi\right)\frac{1}{e}$

V_1 V_2 and V_3 are in A.P.

$\Rightarrow V_1 + V_3 = 2V_2$

$$\left(\frac{hc}{\lambda_1} - \phi\right)\frac{1}{e} + \left(\frac{hc}{\lambda_3} - \phi\right)\frac{1}{e} = 2\left(\frac{hc}{\lambda_2} - \phi\right)\frac{1}{e}$$

$$\frac{hc}{\lambda_1}+\frac{hc}{\lambda_3}=\frac{2hc}{\lambda_2}$$

$$\frac{1}{\lambda_1}+\frac{1}{\lambda_3}=\frac{2}{\lambda_2}$$

Sol 7: (B) ϕ = 4.5 eV

Wavelength of light = 2000Å

Energy of photon = $\frac{12400}{2000}=\frac{124}{20}=6.2\,\text{eV}$

K.E. of emitted electron = $h\nu-\phi$

= 6.2 – 4.5 = 1.7 eV

As electrons are further accelerated by 2V

so final kinetic energy = 1.7 eV + 2eV

= 3.7 eV

Sol 8: (B) $h\nu = h\nu_0$ + KE

Kinetic energy depends only on the energy of incident photon. Number of emitted electrons $\propto$ intensity of light.

Sol 9: (C) $\lambda_p=\frac{h}{p_p}=\frac{h}{\sqrt{2m_p(KE)_p}}$

$$\lambda_e=\frac{h}{p_e}=\frac{h}{\sqrt{2m_e(KE)_e}}$$

$m_p > m_e$

As proton and electron both are accelerated by same potential difference so $KE_p = KE_e$

$\lambda_p < \lambda_e$

Sol 10: (A) Initial KE is 100 eV

After accelerating through potential difference of 50 v final KE is 150 eV.

$$\lambda_d=\sqrt{\frac{150}{V}}=\sqrt{\frac{150}{150}}=1\,\text{Å}$$

Sol 11: (D) $\lambda=\frac{h}{p}$; $p=\frac{h}{0.01\times10^{-10}}=10^{12}h$

Sol 12: (C) $\frac{hc}{\lambda_1}=\phi+K_1$ ... (i)

$\frac{hc}{\lambda_2}=\phi+K_2$ ... (ii)

$$\lambda_2=\frac{\lambda_1}{2}$$

$$\frac{2hc}{\lambda_1}=\phi+K_2$$

By (i) and (ii)

$2(\phi + K_1) = \phi + K_2 \Rightarrow \phi + 2K_1 = K_2$

$$K_1=\frac{K_2}{2}-\frac{\phi}{2}$$

Sol 13: (B) Distance between two successive maxima in Young's double slit experiment is $\frac{\lambda D}{d}$

Distance will decrease if D will decreases.

Sol 14: (C)

Energy required = $\frac{2.18\times10^{-18}}{9}$ = 2.42×10^{-19} J

Sol 15: (D) In some Hydrogen like atom

$$E_{4n}-E_{2n}=\left(-\frac{13.6}{(4n)^2}+\frac{13.6}{(2n)^2}\right)Z^2$$

$$=\frac{13.6Z^2}{4n^2}\left[-\frac{1}{4}+1\right]$$

$$=\frac{3\times13.6Z^2}{16n^2}$$

$$E_{2n}-E_n=\left(-\frac{13.6}{(en)^2}+\frac{13.6}{n^2}\right)Z^2$$

$$=\frac{13.6z^2}{n^2}\left[-\frac{1}{4}+1\right]=\frac{3\times13.6Z^2}{4n^2}$$

Ratio = $\frac{E_{4n}-E_{2n}}{E_{2n}-E_n}=\frac{1}{4}$

Ratio is independent of Z and n.

Sol 16: (A)

n′ = n + 1

n′ = 3

No. of lines = $\frac{(n+1-3)\big((n+1-3)+1\big)}{2}$

$$=\frac{(n-2)(n-1)}{2}=10$$

(n – 2) (n – 1) = 20,

n = 6

Sol 17: (A) Ten different wavelengths are emitted so

$\frac{n(n-1)}{2} = 10 \Rightarrow n\ (n-1) = 20$

$\Rightarrow n = 5$

Energy of incident radiation is $13.6\left[\frac{1}{1^2} - \frac{1}{5^2}\right]$

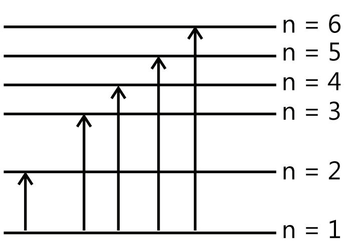
$= \frac{24}{25} \times 13.6 = 13.056$ eV

$\lambda = \frac{12400}{13.056} = 949.75$ Å

Sol 18: (C) Five dark line corresponds to transitions so highest state of electron is n = 6

So no of lines in emission spectrum $= \frac{n(n-1)}{2}$

$= \frac{6 \times 5}{2} = 15$



Sol 19: (C) Energy of photon $= 13.6\left(\frac{1}{1^2} - \frac{1}{25}\right)$

$= \frac{24}{25} \times 13.6$ eV

Momentum of photon

$= \frac{h}{\lambda} = \frac{h}{\frac{hc}{E}} = \frac{E}{c} = \frac{24}{25} \times \frac{13.6 \times 1.6 \times 10^{-19}}{3 \times 10^8}$

By momentum conservation

$mv = 6.96 \times 10^{-27}$

$1.67 \times 10^{-27} \times v = 6.96 \times 10^{-27}$

v = 4.169 m/s

Sol 20: (A) Velocity $\propto \frac{1}{n}$

So kinetic energy will increase

P.E. $= \frac{-2 \times 13.6 Z^2}{n^2}$

So P.E. will decrease

T.E. $= \frac{-13.6 Z^2}{n^2}$

T.E. will decrease with decrease in n.

Sol 21: (B) Angular momentum $= \frac{nh}{2\pi} = mvr$

$f \propto \frac{1}{n^3}$

$r \propto n^2$

$f r L \propto \frac{1}{n^3} \times n^2 \times n = 1 \Rightarrow$ independent of n.

Sol 22: (B) $r = \frac{0.529 n^2}{Z} = \frac{0.529 \times 4}{2} = 1.058$ Å

Sol 23: (A) Energy $E_n = \frac{-13.6}{n^2} = -3.4$ (n = 2)

angular momentum $= \frac{nh}{2\pi} = \frac{2h}{2\pi} = \frac{h}{\pi}$

Sol 24: (B) $E_n = \frac{-13.6}{n^2} = -3.4 =$ T.E.

n = 2

Kinetic energy = |T.E.| = 3.4 eV

$\lambda = \frac{h}{\sqrt{2mKE}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}}$

$= \frac{6.6 \times 10^{-34}}{9.9} \times 10^{25} = 6.6 \times 10^{-10}$ m

Sol 25: (A) Since some photons have energy greater than 13.6 eV so electrons in hydrogen atoms will come out of hydrogen atom. So only Lyman series absorbtion will be observed. Electron will not excite in other excited states as there are only few photons of required energy for the transition. So Balmer series will not be observed.

Sol 26: (B) Difference of energy between any shell is independent of the reference level.

T.E. = K.E. + P.E.

KE at ground state = 13.6 eV

So T.E. at ground state = 13.6 + 0 = 13.6 eV

Sol 27: (A) $r \propto \frac{1}{m}$

Mass of dueterium > mass of hydrogen

$\Rightarrow r_d < r_h$

Velocity is same for both.

Energy of dueterium > Energy of hydrogen

$\Rightarrow$ Wavelength of dueterium < wavelength of hydrogen

Angular momentum $= mvr = \frac{nh}{2\pi}$

is independent of mass.

Sol 28: (B) $\lambda_{min} = \frac{hc}{eV} = \frac{12420}{V}$ Å

$$V = \frac{12420}{0.663} = 18.75\,kV$$

Sol 29: (D) $\lambda_{min} = \frac{12420}{V}$

If V increase λ_{min} will decrease.

Previous Years' Questions

Sol 1: (B) Shortest wavelength or cut-off wavelength depends only upon the voltage applied in the Coolidge tube.

Sol 2: (C) Beta particles are fast moving electrons which are emitted by the nucleus.

Sol 3: (A) The maximum number of electrons in an orbit are $2n^2$. If n > 4, is not allowed, then the number of maximum electrons that can be in first four orbit are:

$$2(1)^2 + 2(2)^2 + 2(3)^2 + 2(4)^2$$

$$= 2 + 8 + 18 + 32 = 60$$

Therefore, possible elements can be 60.

Sol 4: (D) Shortest wavelength will correspond to maximum energy. As value of atomic number (Z) increases, the magnitude of energy in different energy states gets increased. Value of Z is maximum for doubly ionized lithium atom (Z = 3) among the given elements. Hence, wavelength corresponding to this will be least.

Sol 5: (C) During fusion process two or more lighter nuclei combine to form a heavy nucleus.

Sol 6: (D) For a given plate voltage, the plate current in a triode valve is maximum when the potential of the grid is positive and plate is positive

Sol 7: (C) The X-ray beam coming from an X-ray tube will be having all wavelengths larger than a certain minimum wavelength

Sol 8: (B) Cut-off wavelength depends on the accelerating voltage, not the characteristic wavelengths. Further, approximately 2% kinetic energy of the electrons is utilized in producing X-rays. Rest 98% is lost in heat.

Sol 9: Minimum voltage required is corresponding to n = 1 to n = 2. Binding energy of the innermost electron is given as 40 keV i.e., ionization potential is 40 kV. Therefore,

$$V_{min} = \frac{40 \times 10^3 \left(\frac{1}{1^2} - \frac{1}{2^2}\right)}{\left(\frac{1}{1^2} - \frac{1}{\infty}\right)} = 30 \times 10^3\,V$$

The energy of the characteristic radiation will be $30 \times 10^3\,eV$.

Sol 10: $R = R_0\left(\frac{1}{2}\right)^n$

Here R_0 = initial activity

= 1000 disintegration/s

and n = number of half-lives.

At t = 1 s, n = 1

$\therefore\ R = 10^3\left(\frac{1}{2}\right) =$ 500 disintegration/s

At t = 3 s, n = 3

$R = 10^3\left(\frac{1}{2}\right)^3$ = 125 disintegration/s

Sol 11: $K_{max} = h\nu - W$

Therefore, K_{max} is linearly dependent on frequency of incident radiation.

Sol 12: Number of α-particles emitted

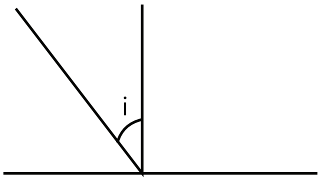
$$n_1 = \frac{238 - 206}{4} = 8$$

and number of β-particles emitted are say n_2, then

$92 - 8 \times 2 + n_2 = 82$

$\therefore\ n_2 = 6$

Sol 13: (B)



$2d \cos i = n\lambda$

$$2d \cos i = \frac{h}{\sqrt{2meV}}$$

v = 50 volt

Sol 14: (B) $2d \cos i = n\lambda_{dB}$

Sol 15: (D) Diffraction pattern will be wider than the slit.

Sol 16: (D) $W = QdV = Q(V_q - V_p)$

$= -100 \times (1.6 \times 10^{-19}) \times (-4 - 10)$

$= +100 \times 1.6 \times 10^{-19} \times 14 = +2.24\ 10^{-16}$ J.

Sol 17: (B)

$$\frac{1}{2}mv^2 = eV_o = 1.68\text{ eV} \Rightarrow h\nu = \frac{hc}{\lambda} = \frac{1240\text{evnm}}{400\text{nm}}$$

$= 3.1\text{ eV} \Rightarrow 3.1\text{ eV} = W_o + 1.6\text{ eV}$

$\therefore W_o = 1.42$ eV

Sol 18: (D) Since the frequency of ultraviolet light is less than the frequency of X–rays, the energy of each incident photon will be more for X–rays.

$K.E_{photoelectron} = hv - \phi$

Stopping potential is to stop the fastest photoelectron

$$V_0 = \frac{hv}{e} - \frac{\phi}{e}$$

So, $K.E_{max}$ and V_0 both increases.

But K.E ranges from zero to $K.E_{max}$ because of loss of energy due to subsequent collisions before getting ejected and not due to range of frequencies in the incident light.

Sol 19: (A) 4 x 10^3 = 10^{20} x hf

$$f = \frac{4 \times 10^3}{10^{20} \times 6.023 \times 10^{-34}}$$

f = 6.03 x 10^{16} Hz

The obtained frequency lies in the band of X–rays.

Sol 20: (C) $KE_{max} = h\upsilon - h\upsilon_0$

$h\upsilon - h\upsilon_0 = e \times \Delta v$

$$V_0 = \frac{h\upsilon}{e} - \frac{h\upsilon_0}{e}$$

'υ' is doubled

$KE_{max} = 2h\upsilon - h\upsilon_0$

$$V_0' = (\Delta V)' = \frac{2h\upsilon}{e} - \frac{h\upsilon_0}{e}$$

$\frac{KE_{max}}{KE_{max}}$ may not be equal to 2

$\Rightarrow \frac{V_0'}{V_0}$ may not be equal to 2

Ke = max = $h\upsilon - h\upsilon_0$

$$V = \frac{h\upsilon}{e} - \frac{h\upsilon_0}{e}$$

Sol 21: (C) Davisson – Germer experiment showed that electron beams can undergo diffraction when passed through atomic crystals. This shows the wave nature of electrons as waves can exhibit interference and diffraction.

Sol 22: (D) $r_1 = \frac{m_2 f}{m_1 + m_2}; r_2 = \frac{m_1 r}{m_1 + m_2}$

$$(I_1 + I_2)\omega = \frac{nh}{2\pi} = n\hbar$$

$$K.E. = \frac{1}{2}(I_2 + I_2)\omega^2 = \frac{(m_1 + m_2)n^2h^2}{2m_1m_2r^2}$$

Sol 23: (D) As λ is increased, there will be a value of λ above which photoelectrons will be cease to come out so photocurrent will become zero. Hence, (D) is correct answer .

Sol 24: (D) $\Delta E = h\nu$

$$\nu = \frac{\Delta E}{h} = k\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \frac{k2n}{n^2(n-1)^2}$$

$$\approx \frac{2k}{n^3} \propto \frac{1}{n^3}$$

Sol 25: (B) $r = \frac{m\upsilon}{qB}$

$$= \frac{\sqrt{2meV}}{eB}$$

$$= \frac{1}{B} = \sqrt{\frac{2m}{e}V} \Rightarrow V = \frac{B^2r^2e}{2m} = 0.8\text{ V}$$

For transition between 3 to 2,

$$E = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{13.6\times 5}{36} = 1.88\text{ eV}$$

Work function = 1.88 eV – 0.8 eV

= 1.08 eV = 1.1 eV

Sol 26: (C) $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n^2}\right]$

$\Rightarrow \lambda \propto \frac{1}{Z^2}$ for given n_1 & n_2

$$\Rightarrow \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$$

Sol 27: (C) (i) Frants – Hertz Experiment is associated with Discrete energy levels of atom

(ii) Photo electric experiment is associated with particle nature of light.

(iii) Davison – Germer experiment is associated with wave nature of electron.

Sol 28: (D) $\frac{hc}{\lambda} = w + \frac{1}{2}mv^2$ ... (i)

$$\frac{hc}{\lambda'} = w + \frac{1}{2}m(v')^2$$

$$\frac{hc}{\left(\frac{3\lambda}{4}\right)} = w + \frac{1}{2}m(v')^2 \quad \text{... (ii)}$$

Equation $\left[(i)\times\frac{4}{3}\right] - (ii)$

$$\frac{4hc}{3\lambda} - \frac{4}{3}\frac{hc}{\lambda} = \frac{4}{3}w + \frac{4}{3}\left(\frac{1}{2}mv^2\right) - w - \frac{1}{2}m(v')^2$$

$$\Rightarrow \frac{4}{3}w + \frac{4}{3}\left(\frac{1}{2}mv^2\right) = w + \frac{1}{2}m(v')^2$$

$$\Rightarrow \frac{1}{2}m(v')^2 = \frac{w}{3} + \frac{4}{3}\frac{1}{2}mv^2$$

$$\Rightarrow \frac{1}{2}m(v')^2 > \frac{4}{3}\left(\frac{1}{2}mv^2\right)$$

$$\Rightarrow v' > \sqrt{\frac{4}{3}}v$$

JEE Advanced/Boards

Exercise 1

Sol 1: (a) Stopping potential is a property of material so it will remain same.

(b) Saturation current $\propto \frac{1}{r^2}$

r is thrice of initial distance so

Saturation current = $\frac{1}{9}\times 18\text{ mA} = 2\text{mA}$

Sol 2: λ = 540 nm

Energy of photon $E = \frac{12400}{5400} = \frac{62}{27}\text{eV}$

Power = 663 mW

No. of photon per sec $= \frac{663\times 10^{-3}}{\frac{6.2}{27}\times 1.6\times 10^{-19}} = \frac{27\times 663}{62\times 1.6}\times 10^{16}$

No. of it e^- per sec $= \frac{27\times 663\times 10^{16}}{62\times 1.6\times 5\times 10^9}$

= 3.61×10^8 e^-/sec = $3.61 \times 10^8 \times 1.6 \times 10^{-19}$ A

= 5.776×10^{-11} A

Sol 3: λ = 330 nm

$h\nu_1 = h\nu_0 + KE$

$KE = eV_0$

$h\nu_1 = h\nu_0 + eV_0$

$h\nu_2 = h\nu_0 + 2eV_0$

$h(\nu_2 - \nu_1) = eV_0$

$$V_0 = \frac{h(\nu_2 - \nu_1)}{e} = \frac{E_2 - E_1}{e}$$

$$E_2 = \frac{12400}{2200} eV \;;\; E_1 = \frac{12400}{3300} eV$$

$$E_2 = \frac{62}{11} eV \quad ; E_1 = \frac{41.3}{11} eV$$

$$V_0 = \frac{62-41.3}{11e} eV = 1.88 \text{ V}$$

Sol 4: λ = 990 Å

d = 0.1 m

r = 0.05 nm = 5×10^{-11} m

(i) Intensity of light = $\frac{10}{4\pi(0.1)^2} \times \pi(5\times10^{-11})^2$

= $250 \times 25 \times 10^{-22}$

= 6250×10^{-22}

= 6.25×10^{-19} J

Energy of photon = $\frac{12400}{990}$ = 12.52 eV

= 20×10^{-19} J

Average no. of photon = $\frac{6.25\times10^{-19}}{20\times10^{-19}} = \frac{5}{16}$

(ii) No. of electron = $\frac{10}{4\pi(0.1)^2} \times \frac{1}{(20\times10^{-19}\text{ J})} \times \frac{1}{100}$

$$= \frac{10\times10^{18}}{4\pi\times2} = \frac{5}{4\pi}\times10^{18} = \frac{10^{19}}{8\pi}$$

Sol 5: $h\nu = h\nu_0 + KE$

$KE = eV_0$

$h\nu = h\nu_0 + eV_0$

$eV_0 = h\nu - h\nu_0$

$y = V_0$; $x = n$

$ey = hx - h\nu_0$

$$y = \frac{hx}{e} - \frac{\phi}{e}$$

Work function ϕ = 2eV

Slope = $\frac{h}{e} = \frac{2}{0.49\times10^{15}}$

$$h = \frac{2\times1.6\times10^{-19}}{0.49\times10^{15}} = 6.53 \times 10^{-34} \text{ J-s}$$

Sol 6: ϕ = 1.17 eV

d = 2m

$\lambda = 4.8 \times 10^{-7}$ m = 4800 Å

P = 1W

Intensity of light = $\frac{p}{4\pi d^2} = \frac{1}{4\pi\times4} = \frac{1}{16\pi}$

Energy of 1 photon = $\frac{12400}{4800} = 2.58\,eV$

Number of photons striking per square meter per sec

$$= \frac{1}{16\pi\times2.58\times1.6\times10^{-19}} = 4.81 \times 10^{16}$$

$h\nu = h\nu_0 + KE$

$E = h\nu$

$E = h\nu_0$

$\frac{1}{2}mv^2 = h\nu - h\nu_0 = E - E_0$

$\frac{12400}{4800} - 1.17$

$\frac{1}{2}m_e v^2 = 2.58 - 1.17 = 1.41$ eV .... (i)

$$v^2 = \frac{2.82\times1.6\times10^{-19}}{9.1\times10^{-31}}$$

$v^2 = 0.495 \times 10^{12}$

$v = 7.04 \times 10^5$ m/s

Magnetic force = eVB = $\frac{mv^2}{R}$

By (i)

mv^2 = 2.82 eV

$eVB = \frac{2.82\,eV}{R}$

$7.04 \times 10^5 \text{ m/s} \times 10^{-4} = \frac{2.82}{R}$

R = 4 cm

Sol 7: Z = 3

Energy of $E_1 = 13.6 \times 9 \left[\frac{1}{16} - \frac{1}{25}\right]$

E_1 = 2.754 eV

Energy $E_2 = 13.6 \times 9 \left[\frac{1}{9} - \frac{1}{16}\right]$

$E_2 = 5.95$ eV

$E_2 = \phi + KE_2$

$KE_2 = 3.95$ eV

$\phi = E_2 - KE_2$

$= 5.95 - 3.95$

$\phi = 2$ eV

$E_1 = \phi + KE_1$

$KE_1 = 2.754 - 2$

$eV = 0.754$ eV

$V = 0.754$ Volts

Sol 8: $\lambda_1 = 4144$ Å; $E_1 = \dfrac{12400}{4144}$eV $= 2.99$ eV

$\lambda_2 = 4972$ Å; $E_2 = \dfrac{12400}{4972}$eV $= 2.49$ eV

$\lambda_3 = 6216$ Å; $E_3 = \dfrac{12400}{6216}$eV $= 1.99$ eV

$\phi = 2.3$ eV

Intensity $I_1 = I_2 = I_3 = \dfrac{3.6\times10^{-3}}{3}$ $Wm^{-2} \times 10^{-4}$

$= 1.2 \times 10^{-7}$ W

No electrons will be emitted by 6216 Å wavelength photons as $E_3 < \phi$.

No. of photons in light of wavelength λ_2 is

$\dfrac{1.2\times10^{-7}}{2.49\times1.6\times10^{-19}} = 3 \times 10^{11}$ photons / sec

No of photons in light wavelength λ_1 is

$\dfrac{1.2\times10^{-7}}{2.99\times1.6\times10^{-19}} = 2.5 \times 10^{11}$ photons / sec

No of electrons liberated in 2 seconds

$= 2\,(3 + 2.5) \times 10^{11}$

$= 11 \times 10^{11}$ electrons.

Sol 9: (i) Refer Sol 4 Exercise-I JEE Advanced

(ii) No. of photons $= \dfrac{5}{16}$ per second

No. of electrons $= \dfrac{5}{16} \times \dfrac{1}{100}$ per second

$= \dfrac{5}{1600}$ per second

Sol 10: $P = 3.2 \times 10^{-3}$ W

(a) Energy of photons = 5 eV;

$\therefore \lambda = \dfrac{12400}{5} = 2480$ Å

Distance = 0.8 m

$\phi = 3$ eV

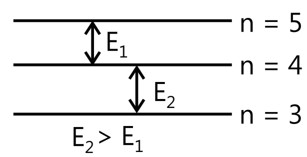
Radius $= 8 \times 10^{-3}$ m

Efficiency $= \dfrac{1}{10^6}$ electrons per photon

Power incident on atom

$$= \frac{3.2\times10^{-3}}{4\pi(0.8)^2}\times\pi(8\times10^{-3})^2$$

$$= \frac{10^{-3}}{0.8}\times0.8\times0.8\times10^{-4}$$



$= 0.8 \times 10^{-7} = 8 \times 10^{-8}$ W

No. of photons $= \dfrac{8\times10^{-8}}{5\times1.6\times10^{-19}}$

N = No. of electrons

$$= \frac{8\times10^{-8}}{10^6\times5\times1.6\times10^{-19}} = 10^5\ s^{-1}$$

(b) $\lambda_d = \dfrac{h}{mv}$

$KE = 2eV = \dfrac{p^2}{2m}$

$p^2 = 2 \times 1.6 \times 10^{-19} \times 2 \times 9.1 \times 10^{-31}$

$p^2 = 58.24 \times 10^{-50}$

$p = 7.63 \times 10^{-25}$

$$\lambda_d = \frac{6.6\times10^{-34}}{7.63\times10^{-25}} = 0.86\times10^{-9}$$

$$\frac{\lambda}{\lambda_d} = \frac{2480\times10^{-10}}{0.86\times10^{-9}} = \frac{248}{0.86} = 286.76$$

(c) After some time sphere gets positively charged and it will create electric field which will stop the further emission of electrons.

(d) KE = $h\nu - h\phi$ = 5 eV – 3 eV = 2 eV

K.E. = eV = 2 eV

V = 2 volts

Potential at the surface of sphere

$$= \frac{Kq}{r} = \frac{K(Nt)\times1.6\times10^{-19}}{8\times10^{-3}} = \frac{9\times10^{9}\times10^{5}t\times1.6\times10^{-19}}{8\times10^{-3}}$$

= 9 × 0.2 × 10^{-2} × t

2 = 1.8 × 10^{-2} × t

So time required $t = \frac{2}{1.8}\times100$ = 111.1 sec

Sol 11: For metal A

Energy of photons = 4.25 eV

Maximum $KE_A = T_a$

de Broglie wavelength = λ_a

For metal B

$KE_{B\,max} = T_b = T_a - 1.5$

Energy of photons = 4.7 eV

De-Broglie wavelength = $\lambda_b = 2\lambda_a$

$$KE = \frac{p^2}{2m} = \left(\frac{h}{\lambda_d}\right)^2\times\frac{1}{2m}$$

$$T_a = \left(\frac{h}{\lambda_a}\right)^2\times\frac{1}{2m}$$

$$T_b = \left(\frac{h}{2\lambda_a}\right)^2\times\frac{1}{2m}$$

$T_b = T_a - 1.5$

$$\left(\frac{h}{2\lambda_a}\right)^2\times\frac{1}{2m} - \frac{h^2}{\lambda_a^2}\times\frac{1}{2m} = -1.5$$

$$\left(\frac{h}{\lambda_a}\right)^2\times\frac{1}{2m}\times\frac{3}{4} = -1.5$$

$T_a \div 2 = 1$

T_a = 2 eV

T_b = 2 – 1.5 = 0.5 eV

From metal A

$E_A = \phi_A + T_a$

ϕ_A = 4.25 – 2 = 2.25 eV

For metal B

$E_B = \phi_B + T_b$

ϕ_B = 4.7 – 0.5 = 4.2 eV

Sol 12: Force on electron = eE

Acceleration = $\frac{eE}{m}$

Velocity = $\frac{eE}{m}t$;   p = eEt

$$\lambda_d = \frac{h}{p}$$

$$\frac{d\lambda_d}{dt} = -\frac{h}{p^2}\frac{dp}{dt} = \frac{-heE}{(eEt)^2} = \frac{-heE}{e^2E^2t^2} = \frac{-h}{eEt^2}$$

Sol 13: B.E. = 0.85 eV = $\frac{13.6}{n^2}$

n = 4

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = R\left(\frac{1}{4}-\frac{1}{16}\right)$$

$$\frac{1}{\lambda} = \frac{12R}{4\times16} = \frac{3R}{16} \Rightarrow \lambda = \frac{16R}{3} = 487\text{ nm}$$

Sol 14: 5th excited state

$\Rightarrow$ n = 6

m = mass of atom

v = velocity of atom

$$\frac{(mv^2)}{2m} + \frac{hc}{\lambda} = E_6 - E_0 = -\frac{13.6}{36} + 13.6$$

$$= \frac{35}{36}\times13.6$$

Momentum conservation, $\frac{h}{\lambda} = mv$

$$\frac{1}{2m}\left(\frac{h}{\lambda}\right)^2 + \frac{hc}{\lambda} = \frac{35}{36}\times13.6\times\left(10^{-19}\times1.6\right)J$$

$\Rightarrow \lambda$ = 939.4 Å

Energy = 13.2 eV

$v = \frac{h}{m\lambda} \Rightarrow v$ = 4.26 m/s

Sol 15: Energy of series limit of Balmer is $\frac{13.6}{4} = 3.4$

$$\lambda_B = \frac{hc}{3.4}$$

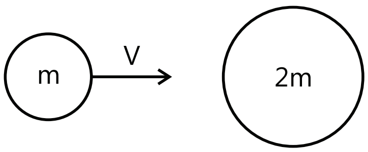
Energy of first line of Paschen is $13.6\left[\frac{1}{3^2}-\frac{1}{4^2}\right]$

$$\frac{hc}{\lambda_p}=\frac{13.6\times 7}{9\times 16}=0.661$$

$$\lambda_P=\frac{hc}{0.661}$$

Ration $\frac{\lambda_B}{\lambda_P}=\frac{0.661}{3.4}=\frac{7}{36}$

Sol 16: 25 eV = $\frac{1}{2}m_n v^2$



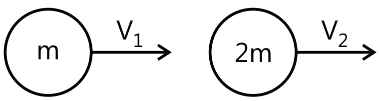
$$v^2=\frac{50\times 1.6\times 10^{-19}}{1.67\times 10^{-27}}=47.9\times 10^8$$

$V = 6.92 \times 10^4$ m/s

$$V_{cm}=\frac{m_1v_1+m_2v_2}{m_1+m_2}=\frac{m\times V+0}{3m}=\frac{V}{3}$$

$= 2.3 \times 10^4$ m/s

By momentum conservation



$mV = mV_1 + 2mV_2$

$V = V_1 + 2V_2$

By energy conservation

$$\frac{1}{2}mV^2=\frac{1}{2}mV_1^2+\frac{1}{2}2mV_2^2$$

$V^2 = V_1^2 + 2V_2^2 \Rightarrow V^2 = (V-2V_2)^2 + 2V_2^2$

$V^2 = V^2 + 6V_2^2 - 4V_2V \Rightarrow 6V_2 = 4V$

$$V_2=\frac{2V}{3}$$

$$V_1 = V-2V_2 = V-\frac{4V}{3}=-\frac{V}{3}$$

Velocity of 1 w.r.t. C.M. is $V_{1cm}=\frac{-2V}{3}=-4.6\times 10^4$ m/s

Velocity of 2 w.r.t. CM is $V_{2cm}=\frac{V}{3}=2.3\times 10^4$ m/s

$$\lambda_1=\frac{h}{mv_{1cm}}=\frac{6.6\times 10^{-34}}{1.67\times 10^{-27}\times 4.6\times 10^4}=8.6\text{ pm}$$

$$\lambda_2=\frac{h}{2mv_{2cm}}=\frac{6.6\times 10^{-34}}{2\times 1.67\times 10^{-27}\times 2.3\times 10^{-4}}=8.6\text{ pm}$$

Sol 17: $\frac{n\lambda}{2}=2\text{Å}$

$$\frac{(n+1)\lambda}{2}=2.5\text{Å}\Rightarrow\frac{\lambda}{2}=0.5\text{Å}$$

$$\Rightarrow\lambda=1\text{Å}\Rightarrow\lambda=\frac{h}{p}$$

$$p=\frac{6.6\times 10^{-34}}{1\times 10^{-10}}=6.6\times 10^{-24}$$

$$\text{Energy}=\frac{p^2}{2m}=\frac{(6.6)^2\times 10^{-48}}{2\times 9.1\times 10^{-31}}=\frac{2.39\times 10^{-17}}{1.61\times 10^{-19}}=148.4\text{eV}$$

Sol 18: (D) Energy of photon from He^+

$$=13.6\,Z^2\left[\frac{1}{1^2}-\frac{1}{2^2}\right]$$

$$=13.6\times 4\times\frac{3}{4}=13.6\times 3\text{ eV}$$

Energy of photon from H = 13.6 eV

Energy of photoelectron = 13.6×3-13.6 = 13.6×2 eV

K.E. = $\frac{1}{2}mv^2 = 13.6\times 2\times 1.6\times 10^{-19}$

$$v^2=\frac{4\times 13.6\times 1.6\times 10^{-19}}{9.1\times 10^{-31}}\Rightarrow v^2=9.56\times 10^{12}$$

$v = 3.09 \times 10^6$ m/s

Sol 19: (i) $\frac{n(n-1)}{2}=6$

$\Rightarrow n(n-1) = 12 \Rightarrow n = 4$

since emitted photons are of energy less, equal and more than 2.7 eV

So level B must be n = 2

(ii) $2.7 = 13.6\,Z^2\left[\frac{1}{2^2}-\frac{1}{4^2}\right]$

$$2.7=\frac{13.6Z^2}{4}\left[\frac{3}{4}\right]$$

Ionisation energy $\frac{2.7\times16}{3}$ = 14.4 eV = $14.4\times1.6\times10^{-19}$ J

= 23.04×10^{-19} J

(iii) Maximum energy= 13.6 $Z^2 \left[\frac{1}{1^2}-\frac{1}{4^2}\right]$; $4 \to 1$

Minimum energy = 13.6 $Z^2 \left[\frac{1}{3^2}-\frac{1}{4^2}\right]$; $4 \to 3$

Sol 20: λ = 500 Å

Energy = $\frac{12400}{500} = 24.8\text{eV}$

Energy required to take out electron from atom = 13.6 eV

KE = 24.8 – 13.6 = 11.2 eV

Sol 21: Energy of photon = 13.6 Z^2

= 13.6 × 4 = 54.4

Wavelength = $\frac{12400\,\text{eV}}{54.4\,\text{eV}}$ = 227.94 Å = 22.8 nm

Sol 22: $E_3 - E_2 + E_2 - E_1 = E_3 - E_1$

$$\frac{12400}{\lambda_1}+\frac{12400}{\lambda_2}=\frac{12400}{\lambda_3}$$

$$\frac{1}{\lambda_1}+\frac{1}{\lambda_2}=\frac{1}{\lambda_3} \Rightarrow \lambda_3=\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$$

Sol 23: Energy of new atom = 2 × energy of hydrogen atom

$$\frac{hc}{\lambda}=13.6\times2\left[\frac{1}{2^2}-\frac{1}{3^2}\right]$$

$$\frac{1}{\lambda}=2R\left[\frac{1}{2^2}-\frac{1}{3^2}\right] \Rightarrow \lambda=\frac{18}{5R}$$

Sol 24: r = 0.5 Å ; $\omega = 2\pi\times10^{16}$ rad / sec.

Magnetic moment = $\frac{qVr}{2}$

$$=\frac{e\omega r^2}{2} = \frac{1.6\times10^{-19}\times2\pi\times10^{16}}{2}\times\frac{1}{4}\times10^{-20} = 0.4\times10^{-23}$$

= 1.25×10^{-23} Am^2

Sol 25: 47.2 = 13.6 $Z^2 \left[\frac{1}{2^2}-\frac{1}{3^2}\right]$

$$47.2\times2=\frac{13.6Z^2\times5}{36}$$

$Z^2 = 25 \Rightarrow Z = 5$

Sol 26: 47.2 = 13.6 $Z^2 \left[\frac{1}{2^2}-\frac{1}{3^2}\right]$

(i) Z = 5

(ii) E = 13.6 × 25 $\left[\frac{1}{9}-\frac{1}{16}\right]$ = 16.5 eV

(iii) E = 13.6 × 25 $\left[\frac{1}{1^2}-\frac{1}{\infty^2}\right]$

E = 13.6 × 25 eV

E = 340 eV

$\lambda = \frac{12400}{340}$ = 36.4 Å

(iv) KE = |T.E.|

KE = 13.6 Z^2 = 13.6 × 25 = 340 eV

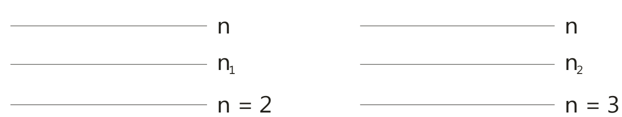
P.E. = – 2 |T.E.|

P.E. = – 2 × 340 = – 680 eV

(v) Angular momentum = mvr = $I\omega = n\frac{h}{2\pi}$

Radius = $\frac{0.529n^2}{z}$ = 0.1058 Å

Sol 27:



Energy gap between quantum states n and 2 is

22.95 + 5.15 = 28.1 eV

Energy gap between quantum state n and 3 is

2.4 + 8.7 = 11.1 eV

Energy gap between n = 2 and n = 3 is

28.1 – 11.1 = 17 eV

$$17 = 13.6\,Z^2\left[\frac{1}{2^2}-\frac{1}{3^2}\right] = \frac{13.6Z^2\times5}{36}$$

$Z^2 = 9 \quad \Rightarrow Z = 3$

$$28.1 = 13.6\times9\left[\frac{1}{2^2}-\frac{1}{n^2}\right]$$

$$\frac{1}{4}-\frac{1}{n^2}=0.229 \Rightarrow \frac{1}{n^2}=0.25-0.229$$

$\Rightarrow n^2 = 48.96 \Rightarrow n = 7$

Sol 28: $E = 13.6\ Z^2\left[\frac{1}{2^2}-\frac{1}{5^2}\right]$

$$\frac{12400}{1085} = 13.6Z^2\left[\frac{1}{2^2}-\frac{1}{5^2}\right]$$

$$\frac{12400}{1085} = 13.6Z^2\left[\frac{21}{100}\right]$$

$\Rightarrow Z^2 = 4 \Rightarrow Z = 2$

Binding energy = $13.6\ Z^2 = 13.6 \times 4 = 54.4$ eV

Sol 29: Energy of n^{th} state of lithium ion = $\frac{-13.6\times 3^2}{n^2}$

= – 13.6 eV (n = 3)

Radius = $\frac{0.529n^2}{2}$

$$\frac{R_{Li^{2+}}}{R_H} = \frac{\frac{0.529\times 9}{3}}{0.529} = 3$$

Sol 30: Total energy of the photons is 20 KeV

Let their wavelength be λ_1, λ_2 Å

$$\frac{12400}{\lambda_1}+\frac{12400}{\lambda_2} = 20000$$

$$\frac{1}{\lambda_1}+\frac{1}{\lambda_2} = 1.613 \quad \text{... (i)}$$

$$\lambda_2 - \lambda_1 = 1.3 \quad \text{... (ii)}$$

By (i) and (ii)

$$\frac{1}{\lambda_1}+\frac{1}{1.3+\lambda_1} = 1.613$$

$$\frac{1.3+2\lambda_1}{\lambda_1(1.3+\lambda_1)} = 1.613$$

$1.3 + 2\lambda_1 = 1.613 \times 1.3\ \lambda_1 + 1.613\ \lambda_1^2$

$1.6\lambda_1^2 + 0.096\ \lambda_1 - 1.3 = 0$

$$\lambda_1 = \frac{-0.096\pm\sqrt{(0.096)^2+4\times 1.3\times 1.6}}{3.2}$$

$\lambda_1 = 0.871$ Å

$\lambda_2 = 2.17$ Å

Exercise 2

Single Correct Choice Type

Sol 1: (B) No. of photons = $\frac{10^{-3}}{\text{Energy of 1 photon}}$

Energy of 1 photon = $\frac{12400}{5000}$ = 2.48 eV

n_p = No. of photons = $\frac{10^{-3}}{2.48\times 1.6\times 10^{-19}}$

= $0.25 \times 10^{16} = 2.5 \times 10^{15}$ photon

n_e = No. of electron = $\frac{0.16\,\mu A}{1.6\times 10^{-19}} = \frac{1.6\times 10^{-7}}{1.6\times 10^{-19}}$

= 10^{12} electron

Efficiency = $\frac{n_e}{n_p}\times 100 = \frac{10^{12}}{2.5\times 10^{15}} \times 100$

$= \frac{1}{2.5\times 10} = \frac{4}{100} = 0.04\%$

Sol 2: (B) KE_{max} = 2eV

$E_1 = \phi + KE_{max\,1}$

5eV = ϕ + 2eV

ϕ = 3eV

$E_2 = \phi + KE_{max\,2}$; $KE_{max\,2} = eV_2$

6 eV = 3 eV + $eV_2 \Rightarrow$ 3 eV = eV_2

V_2 = 3V

So stopping potential of A w.r.t. C is – 3V

Sol 3: (A) $E_n = \frac{2\pi^2 mk^2Z^2e^4}{n^2h^2}$

Frequency $\nu = \frac{Z^2.4\pi^2 mk^2e^4}{n^3.h^3}$

$$\frac{\nu}{E_n} = \frac{Z^2\times 4\pi^2 mk^2e^4\ n^2h^2}{n^2h^3\ 2\pi^2\ mk^2Z^2e^4} \Rightarrow \frac{\nu}{E_n} = \frac{2}{nh}$$

$$\Rightarrow \nu = \frac{2E_n}{nh}$$

Sol 4: (D) Bohr radius = $\frac{n^2}{Z}\times 0.529$

$r_{n+1} - r_n = [(n + 1)^2 - n^2]\ 0.529 = r_{n-1}$

$= (2n + 1)\, 0.529 = (n - 1)^2 \times 0.529$

$\Rightarrow (n - 1)^2 = 2n + 1$

$n^2 + 1 - 2n = 2n + 1 \Rightarrow n^2 = 4n$

$\Rightarrow n = 4$

Sol 5: (A) $r_n = \dfrac{n^2h^2e_0}{pme^2Z} = \dfrac{0.529n^2}{Z}$

n =1, Z = 1

For mean $r'_n = \dfrac{0.529n^2}{207Z} = 2.56 \times 10^{-3}$ Å

Sol 6: (C) Hydrogen emit a photon corresponding to the largest wavelength of the Balmer series. This implies electron was excited to n = 3

Energy required for transition $n = 1 \to 3$ is

$$13.6\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{13.6 \times 8}{9} = 12.088\,\text{eV}$$

Minimum kinetic energy = 12.1 eV

Sol 7: (A) Refer Q-11 (in Exercise II JEE Advanced)

Sol 8: (B) $n = 3 \to 2$; $E_1 = 13.6\left[\frac{1}{4} - \frac{1}{9}\right] = \frac{5}{36} \times 13.6$

$n = 2 \to 1$; $E_2 = 13.6\left[1 - \frac{1}{4}\right] = \frac{3}{4} \times 13.6$

$$\lambda_1 = \frac{hc}{E_1}; \quad \lambda_2 = \frac{hc}{E_2}$$

$$x = \frac{\lambda_1}{\lambda_2} = \frac{E_2}{E_1} = \frac{3}{4 \times 5} \times 36 = \frac{27}{5}$$

$$Z = \frac{E_1}{E_2} = \frac{5}{27}$$

$$y = \frac{p_1}{p_2} = \frac{h/\lambda_1}{h/\lambda_2} = \frac{\lambda_2}{\lambda_1} = \frac{5}{27}$$

Multiple Correct Choice Type

Sol 9: (A, C) Stopping potential $\propto$ kinetic energy

Kinetic energy depends on the frequency of light

$h\nu = h\nu_0 + KE$

Sol 10: (A, D) $\lambda_d = \dfrac{h}{p}$

de Broglie wavelength λ_1 will not change in magnetic field as in magnetic field kinetic energy does not change. Kinetic energy of electron in electric field may increase, remain same or decrease that's why λ_2 can increase or decrease.

$\lambda_1 > \lambda_2$ or $\lambda_2 < \lambda_1$ or $\lambda_1 = \lambda_2$

Sol 11: (A, C) Minimum energy required for transition to happen from ground state is 10.2 eV.

If the total loss in energy is less than 10.2 eV no transition will occur. Either there can be loss of energy greater than 10.2 eV or no loss in energy since the energy of neutron is 20.4 eV the maximum loss in energy due to inelasticity will be less then 10.2 eV. Therefore only option is no loss in energy which means elastic collision. So (A and C).

Sol 12: (A, C, D) Photon of energy 2.55 eV is emitted when transition is from n = 4 to n = 2

So other photon corresponds to $n = 2 \to n = 1$

Energy absorbed by hydrogen atom = 10.2 + 2.55 = 12.75 eV

Minimum Kinetic energy of photon is when collision is perfectly inelastic i.e. when K = 25.5 eV

Refer Q. 11

Sol 13: (A, C, D) $13.6\, Z^2 = 122.4$

Z = 3

For n = 1, $E_1 = -122.4$ eV

n = 2 $\quad E_2 = -30.6$ eV

$E_2 - E_1 = 91.8$ eV

If 125 eV energy electron collides with this atom then 122.4 eV will be used to take out the electron and kinetic energy of electron will be 125 – 122.4 = 2.6 eV

Sol 14: (A, C, D) Some incident wavelengths will be absent in A as some of them will be absorbed by the hydrogen atom. B will emit photons of Energy Corresponding to transitions in the hydrogen atom. This energy will lie in visible and infrared region.

Sol 15: (A, B, C) Having electrons of same speed won't matter because electrons get decelerated to different velocities (just like electrons with random velocities) giving photons of different wavelength. (Read theory).

Assertion Reasoning Type

Sol 16: (D) For frequency less than ν_o no electrons are emitted. so Statement-I is/false.

Sol 17: (C) Momentum of photon is $p = \frac{h}{\lambda}$

Sol 18: (D) All emitted electrons do not have same K.E.

There K.E. range from 0 to ($h\nu - \phi$).

Sol 19: (D) If electron will not emit as only one single photon should have energy more than work function.

Sol 20: (A) $\lambda_e = \frac{h}{\sqrt{2m_e(KE)}}$; $\lambda_p = \frac{h}{\sqrt{2m_p(KE)}}$

$m_p > m_e$

$\Rightarrow \lambda_e > \lambda_p$

Sol 21: (A) By Boltzmann's law (randomization increases with temperature) electron's occupy more number of excited levels at higher temperature.

Previous Years' Questions

Sol 1: (a) Given $E_3 - E_2$ = 47.2 eV

Since $E_n \propto \frac{Z^2}{n^2}$ (for hydrogen like atoms)

$$\text{or}(-13.6)\left(\frac{Z^2}{9}\right) - \left[-(13.6)\left(\frac{Z^2}{4}\right)\right] = 47.2$$

Solving this equation, we get Z = 5

(b) Energy required to excite the electron from 3rd to 4th orbit:

$E_{3-4} = E_4 - E_3$

$$= (-13.6)\left(\frac{25}{16}\right) - \left[(-13.6)\left(\frac{25}{9}\right)\right] = 16.53\text{eV}$$

(c) Energy required to remove the electron from first orbit to infinity (or the ionization energy) will be:

E = (13.6) $(5)^2$ = 340 eV

The corresponding wavelength would be,

$$\lambda = \frac{hc}{E} = \frac{6.6\times10^{-34}\times3\times10^{8}}{340\times1.6\times10^{-19}}$$

$= 0.0364 \times 10^{-7}$ m = 36.4 Å

(d) In first orbit, total energy = –340eV

Kinetic energy = +340 eV

Potential energy = –2 × 340 eV = –680 eV

and angular momentum = $\frac{h}{2\pi}$

$$= \frac{6.6\times10^{-34}}{2\pi} = 1.05 \times 10^{-34}\ \text{kg–m}^2/\text{s}$$

(e) $r_n \propto \frac{n^2}{Z}$

Radius of first Bohr orbit

$$r_1 = \frac{r_1^H}{Z} = \frac{5.3\times10^{-11}}{5} = 1.06\times10^{-11}\text{m}$$

Sol 2: Energy corresponding to given wavelengths:

$$E(\text{in eV}) = \frac{12375}{\lambda(\text{in Å})} = \frac{12375}{975} = 12.69\text{eV}$$

Now, let the electron excites to nth energy state. Then, $E_n - E_1 = 12.69$

$$\text{or}\frac{(-13.6)}{(n^2)} - (-13.6) = 12.69$$

$\therefore n \approx 4$

i.e., electron excites to 4th energy state. Total number of lines in emission spectrum would be:

$$\frac{n(n-1)}{2} = \frac{4\times3}{2} = 6$$

Longest wavelength will correspond to the minimum energy and minimum energy is released in transition from n = 4 to n = 3.

$$E_{4-3} = E_4 - E_3 = \frac{-13.6}{(4)^2} - \frac{-13.6}{(3)^2} = 0.66\text{ eV}$$

$\therefore$ Longest wavelength will be,

$$\lambda_{max} = \frac{12375}{E(\text{in eV})} = \frac{12375}{0.66}\text{Å} = 1.875\times10^{-6}\text{m} = 1.875\ \mu\text{m}$$

Sol 3: Number of proton = atomic number = 11

Number of neutron = mass number – atomic number = 13

But note that in the nucleus number of electron will be zero.

Sol 4: ${}_{92}U^{238} \xrightarrow{\alpha-\text{decay}} {}_{90}X^{234} \xrightarrow{\beta-\text{decay}} {}_{91}Y^{234}$

During an α-decay atomic number decreases by 2 and mass number by 4. During a β-decay, atomic number increases by 1 while mass number remains unchanged.

Sol 5: When 800 Å wavelength falls on hydrogen atom (in ground state) 13.6 eV energy is used in liberating the electron. The rest goes to kinetic energy of electron.

Hence, K = E – 13.6 (in eV) or

$$(1.8\times1.6\times10^{-19}) = \frac{hc}{800\times10^{-10}} - 13.6\times1.6\times10^{-19} \quad ....(i)$$

Similarly for the second wavelength:

$$(4.0\times1.6\times10^{-19}) = \frac{hc}{700\times10^{-10}} - 13.6\times1.6\times10^{-19} \quad ...(ii)$$

Solving these two equations, we get

$$h \approx 6.6\times10^{-34}\,\text{Js}$$

Sol 6: (a) 1 Rydberg = 2.2 × 10^{-18} J = Rhc

Ionisation energy is given as 4 Rydberg

$$= 8.8\times10^{-18}\ \text{J} = \frac{8.8\times10^{-18}}{1.6\times10^{-19}} = 55\ \text{eV}$$

$\therefore$ Energy in first orbit $E_1 = -55\,\text{eV}$

Energy of radiation emitted when electron jumps from first excited state (n = 2) to ground state (n = 1):

$$E_{21} = \frac{E_1}{(2)^2} - E_1 = -\frac{3}{4}E_1 = 41.25\,\text{eV}$$

$\therefore$ Wavelength of photon emitted in this transition would be,

$$\lambda = \frac{12375}{41.25} = 300\ \text{Å}$$

(b) Let Z be the atomic number of given element. Then

$$E_1 = (-13.6)(Z^2) \text{ or } -55 = (-13.6)(Z^2) \text{ or } Z \approx 2$$

Now, as $r \propto \frac{1}{Z}$

Radius of first orbit of this atom,

$$r_1 = \frac{r_{H_1}}{Z} = \frac{0.529}{2} = 0.2645\ \text{Å}$$

Sol 7: Given Z = 3 : $E_n \propto \frac{Z^2}{n^2}$

(a) To excite the atom from n = 1 to n = 3, energy of photon required is,

$$E_{1-3} = E_3 - E_1 = \frac{(-13.6)(3)^2}{(3)^2} - \left[\frac{(-13.6)(3)^2}{(1)^2}\right]$$

= 108.8 eV

Corresponding wavelength will be,

$$\lambda\,(\text{in Å}) = \frac{12375}{E(\text{in eV})} = \frac{12375}{108.8}$$

= 113.74 Å

(b) From n^{th} orbit total number of emission lines can be , $\frac{n(n-1)}{2}$.

$\therefore$ Number of emission lines = $\frac{3(3-1)}{2}$ = 3

Sol 8: Speed of neutrons = $\sqrt{\frac{2K}{m}}$

$$\left(\text{From } K = \frac{1}{2}mv^2\right)$$

$$\text{or } v = \sqrt{\frac{2\times0.0327\times1.6\times10^{-19}}{1.675\times10^{-27}}} \approx 2.5\times10^3\ \text{m/s}$$

Time taken by the neutrons to travel a distance of 10 m:

$$t = \frac{d}{v} = \frac{10}{2.5\times10^3} = 4.0\times10^{-3}$$

Number of neutrons decayed after time t

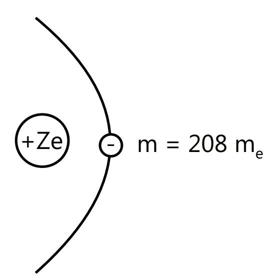
$$N = N_0(1-e^{-\lambda t})$$

$\therefore$ Fraction of neutrons that will decay in this time interval

$$= \frac{N}{N_0} = (1-e^{-\lambda t}) = 1 - e^{-\frac{\ln(2)}{700}\times4.0\times10^{-3}} = 3.96\times10^{-6}$$

Sol 9: If we assume that mass of nucleus >> mass of mu-meson, then nucleus will be assumed to be at rest, only mu-meson is revolving round it.

(a) In n^{th} orbit, the necessary centripetal force to the mu-meson will be provided by the electrostatic force between the nucleus and the mu-meson.



Hence, $\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2}$ ... (i)

Further, it is given that Bohr model is applicable to this system also. Hence,

Angular momentum in n^{th} orbit $= \frac{nh}{2\pi}$

or $mvr = n\frac{h}{2\pi}$ ... (ii)

We have two unknowns v and r (in n^{th} orbit). After solving these two equations, we get $r = \frac{n^2h^2\varepsilon_0}{Z\pi me^2}$

Substituting Z = 3 and m = $208\,m_e$, we get

$$r_n = \frac{n^2h^2\varepsilon_0}{624\pi m_e e^2}$$

(b) The radius of the first Bohr orbit for the hydrogen atom is: $\frac{h^2\varepsilon_0}{\pi m_e e^2}$

Equating this with the radius calculated in part (a), we

$$n^2 \approx 624 \text{ or } n \approx 25$$

(c) Kinetic energy of atom $= \frac{mv^2}{2} = \frac{Ze^2}{8\pi\varepsilon_0 r}$

and the potential energy $= -\frac{Ze^2}{4\pi\varepsilon_0 r}$

$\therefore$ Total energy $E_n = \frac{-Ze^2}{8\pi\varepsilon_0 r}$

Substituting value of r, calculate in part (a),

$$E_n = \frac{1872}{n^2}\left[-\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right]$$

But $\left[-\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right]$ is the ground state energy of hydrogen atom and hence is equal to – 13.6 eV.

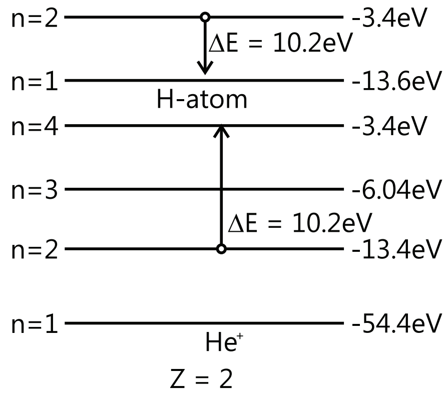
$$\therefore E_n = \frac{-1872}{n^2}(13.6)\text{ eV} = -\frac{25459.2}{n^2}\text{eV}$$

$$\therefore E_3 - E_1 = -25459.2\left[\frac{1}{9} - \frac{1}{1}\right] = 22630.4 \text{ eV}$$

$\therefore$ The corresponding wavelength,

$$\lambda(\text{In})\,\mathring{A} = \frac{12375}{22630.4} = 0.546\,\mathring{A}$$

Sol 10: (C)



Energy given by H-atom in transition from n = 2 to n = 1 is equal to energy taken by He^+ atom in transition from n = 2 to n = 4.

Sol 11: (C) Visible light lies in the range, λ_1 = 4000 Å to λ_2 = 7000 Å. Energy of photons corresponding to these wavelength (in eV) would be:

$$E_1 = \frac{12375}{4000} = 3.09 \text{ eV}$$

$$E_2 = \frac{12375}{7000} = 1.77 \text{ eV}$$

From energy level diagram of He^+ atom we can see that in transition from n = 4 to n = 3, energy of photon released will lie between E_1 and E_2.

$$\Delta E_{43} = -3.4 - (-6.04) = 2.64 \text{ eV}$$

Wavelength of photon corresponding to this energy,

$$\lambda = \frac{12375}{264}\text{ Å} = 4687.5 \text{ Å} = 4.68\times10^{-7} \text{ m}$$

Sol 12: (A) Kinetic energy $K \propto Z^2$

$$\therefore \frac{K_H}{K_{He^+}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Sol 13: A $\rightarrow$ p, q; B $\rightarrow$ p, r; C $\rightarrow$ p, s; D $\rightarrow$ q

Sol 14: (C, D) For photoemission to take place, wavelength of incident light should be less than the threshold wavelength. Wavelength of ultraviolet light < 5200Å while that of infrared radiation > 5200 Å.

Sol 15: (A) $a = \frac{n\lambda}{2} \Rightarrow \lambda = \frac{2a}{n}$

$$\lambda_{deBroglie} = \frac{h}{p}$$

$$\frac{2a}{n} = \frac{h}{p} \Rightarrow p = \frac{nh}{2a}$$

$$E = \frac{p^2}{2m} = \frac{n^2h^2}{8a^2m}$$

$$\Rightarrow E \propto 1/a^2$$

Sol 16: (B)

$$E = \frac{h^2}{8a^2m} = \frac{(6.6\times10^{-34})^2}{8\times(6.6\times10^{-9})^2\times10^{-30}\times1.6\times10^{-19}} = 8\text{ meV}.$$

Sol 17: (D) $mv = \dfrac{nh}{2a}$

$$v = \frac{nh}{2am} \Rightarrow v \propto n$$

Sol 18: $P_1 = \sqrt{2m(100\text{ eV})}$

$$\lambda_P = \frac{h}{\sqrt{2m(100\text{ eV})}} \Rightarrow \lambda_\alpha = \frac{h}{\sqrt{2(4m)(100\text{ eV})}}$$

$$\frac{\lambda_P}{\lambda_\alpha} = \sqrt{8}$$

$\Rightarrow$ The ratio $\dfrac{\lambda_P}{\lambda_\alpha}$ to the nearest integer, is equal to 3.

Sol 19: (A) $\dfrac{1}{6561} = R\left(\dfrac{1}{4}-\dfrac{1}{9}\right) = \dfrac{5R}{36}$

$$\frac{1}{\lambda} = 4R\left(\frac{1}{4}-\frac{1}{16}\right) = \frac{3R\times4}{16}$$

$$\lambda = 1215\,\mathring{A}$$

Sol 20: $0+\dfrac{1}{2}mv^2 = \dfrac{K(Q)e}{10\times10^{-15}} = \dfrac{K(120e)e}{10\times10^{-15}}$

$$\frac{1}{2}\times\frac{5}{3}\times10^{-27}v^2 = \frac{9\times10^9\times120\times(1.6\times10^{-19})^2}{10\times10^{-15}}$$

$$\Rightarrow v = \frac{9\times6\times10^9\times120\times2.56\times10^{-38}}{50\times10^{-42}}$$

$$\Rightarrow v = \sqrt{331.776\times10^{13}}$$

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{4.2\times10^{-15}\times1.6\times10^{-19}}{\frac{5}{3}\times10^{-27}\times\sqrt{331.776\times10^{13}}} = \frac{4.2\times4.8\times10^{-34}}{57.6\times5\times10^{-21}} = 0.07\times10^{-13}$$

$$\lambda = 7\times10^{-15} = 7\text{ fm}$$

Sol 21: (B) $t = 100\times10^{-9}$ sec, $P = 30\times10^{-3}$ Watt, $C = C\times10^8$ m/s

$$\text{Momentum} = \frac{Pt}{C} = \frac{30\times10^{-3}\times100\times10^{-9}}{3\times10^8} = 1.0\times10^{-17}\text{ kg ms}^{-1}$$

Sol 22: Slope of graph is h/e = constant $\Rightarrow$ 1

Sol 23: $E_{photon} = E_{ionize\ atom} + E_{kinetic\ energy}$

$$\frac{1242}{90} = \frac{13.6}{n^2}+10.4$$

from this, n = 2

Sol 24: (A) For photoelectric emission

$$V_0 = \left(\frac{hc}{e}\right)\frac{1}{\lambda}-\frac{\phi}{e}$$

Sol 25:(B) $KE_{max} = \dfrac{hc}{\lambda}-\phi = eV_0$

$$\Rightarrow \frac{hc}{\lambda_1}-\frac{hc}{\lambda_2} = e(V_1-V_2)$$

$$\Rightarrow hc\left(\frac{1}{0.3}-\frac{1}{0.4}\right) = 1.6\times10^{-19}\times10^{-6}$$

$$\Rightarrow hc\left(\frac{0.1}{0.12}\right) = 1.6\times10^{-25}$$

$$\Rightarrow h = \frac{1.6\times10^{-25}\times1.2}{3\times10^8} = 0.64\times10^{-33} = 6.4\times10^{-34}$$

$$\Rightarrow hc\left(\frac{1}{0.4}-\frac{1}{0.5}\right) = (1.6\times10^{-19})\times0.6\times10^{-6}$$

$$\Rightarrow hc = (0.96\times10^{-25})\times\frac{0.20}{0.10}\times\frac{1}{3\times10^8}$$

$$\Rightarrow h = \frac{1.92}{3}\times10^{-33} = 6.4\times10^{-34}$$

Sol 26: (A), (B), (D) Orbital radius $r_{n} = n^2c$ [c = constant]

Angular momentum = nh = L

$$\frac{\Delta r}{r_n} = \frac{(n+1)^2 - n^2}{n^2} = \frac{2}{n} \text{ .....[B]; } \frac{\Delta L_n}{L_n} = \frac{1}{n} \text{ .... [D]}$$

(A) is correct since it will get cancelled in calculation of relative charge.

Sol 27: [6] Photon Energy

$$= \frac{hc}{\lambda} = \frac{1.237\times10^{-6}}{970\times10^{-10}} = \frac{1237}{970}\times 10\text{eV}$$

Absorption of this photon changes the energy to = – 13.6 + 12.75 = – 0.85 eV

Number of possible transitions from the 4th quantum state = ${}^4C_2 = 6$

Sol 28: (B) P_1 = pressure just inside the bubble at the end 2 = $P_0 + \frac{4T}{R}$

P_2 = pressure just inside the bubble at the end 1 = $P_0 + \frac{4T}{r}$

R > r $\Rightarrow P_2 < P_1 \Rightarrow$ Air will flow from end 1 to end 2

Sol 29: (A) $V_B = (1/e)[(hc/\lambda) - \phi]$

$V_P = (1/e)[(1240/550) - 2]\text{eV} = 0.2545$ V

$V_q = (1/e)[(1240/450) - 2.5]\text{eV} = 0.255$ V

$V_r = (1/e)[(1240/350) - 3]\text{eV} = 0.5428$ V

If n is the number of photons in unit time then $nhc/\lambda = I$

$\Rightarrow i_P : i_q : i_r = n_P : n_q : n_r = \lambda_P : \lambda_q : \lambda_r$

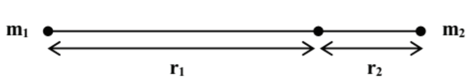
Sol 30: (D) $L = \frac{nh}{2\pi}$

$$\text{K.E.} = \frac{L^2}{2I} = \left(\frac{nh}{2\pi}\right)^2 \frac{1}{2I}$$

Sol 31: (B) $h\nu = k._{En=2} - kE_{n=1}$

$I = 1.87 \times 10^{-46}$ kg m^2

Sol 32: (C)



$$r_1 = \frac{m_2 d}{m_1 + m_2} \text{ and } r_2 = \frac{m_1 d}{m_1 + m_2}$$

$$I = m_1 r_1^2 + m_2 r_2^2$$

$\therefore$ d = 1.3 ×10^{-10} m.

Sol 33: (7) Stopping potential = $\frac{hc}{\lambda} - W$

= 6.2 eV −4.7 eV = 1.5 eV

$$V = \frac{Kq}{r} = 1.5$$

$$n = \frac{1.5\times10^{-2}}{9\times10^9\times1.6\times10^{-19}} = 1.05\times10^7$$

Z = 7

Sol 34: (B) $\frac{-2GMm}{L} + \frac{1}{2}mv^2 = 0$

$$\Rightarrow v = 2\sqrt{\frac{GM}{L}}$$

Note: The energy of mass 'm' means its kinetic energy (KE) only and not the potential energy of interaction between m and the two bodies (of mass M each) – which is the potential energy of the system.

Sol 35: (A) $\dfrac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} = \dfrac{u_1^2}{u_2^2}$

ϕ = 3.7 eV

Sol 36: (A) $K_{max} = \frac{hc}{\lambda_{ph}} - \phi + eV$ [K_{max} = maximum energy e^- reaching the anode]

$$\Rightarrow \frac{h^2}{2m\lambda_e^2} = \left(\frac{hc}{\lambda_{ph}} - \phi\right) + eV \quad \text{... (i)}$$

From Equation (i) (A) follows

if ϕ increases and λ_{ph} increases then $\left(\frac{hc}{\lambda_{ph}} - \phi\right)$ decreases

As a result λ_c increases λ_e is independent of 'd' and clearly λ_e and λ_{ph} do not increase at the same rate.