16. GEOMETRICAL OPTICS

1. INTRODUCTION

Light is a form of radiant energy; that is, energy is emitted by the excited atoms or molecules that can cause the sensation of vision by a normal human eye.

The branch of physics that deals with the phenomena of light is called optics. There are two branches of optics: (a) ray optics and (b) wave optics.

SOME DEFINITIONS

(a) **Ray:** The 'path' along which the light travels is called a ray. The rays are represented by straight lines with arrows directed towards the direction of travel of light.

(b) **Beam:** A bundle of rays is called a beam. A beam is parallel when its rays are parallel, it is divergent when its rays spread out from a point, and it is convergent when its rays meet at a point.

Object and image

If the rays from a point on an object actually diverge from it and fall on the mirror, then the object is the real object of the mirror. If the rays incident on the mirror does not start from a point but appear to converge at a point, then that point is the virtual object of the mirror.

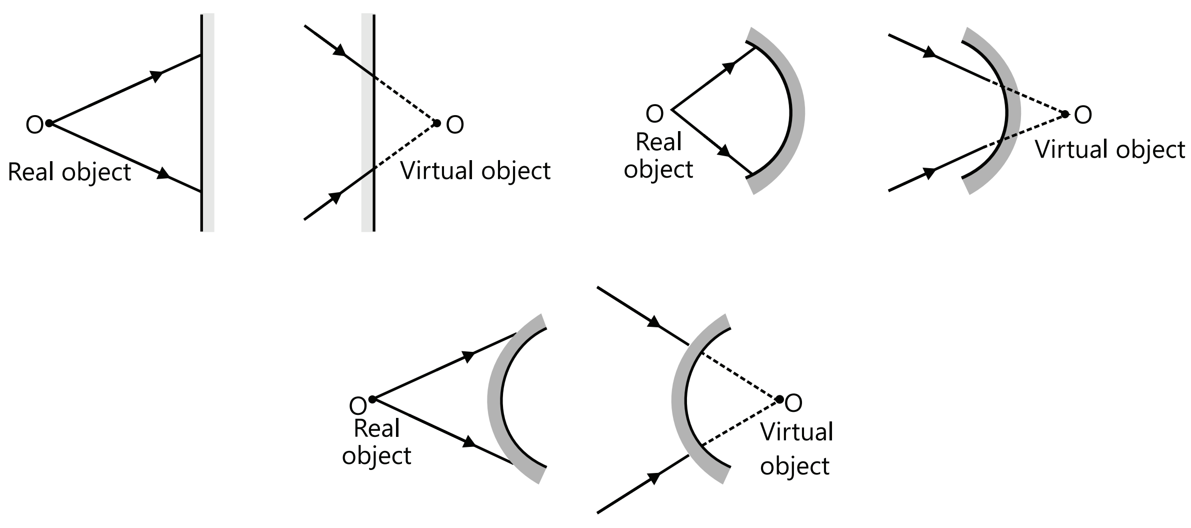


Figure 16.1

If the rays converge at a point after an interaction with a surface, then a real image will be formed, and if the rays diverge after an interaction with a surface, a virtual image will be formed.

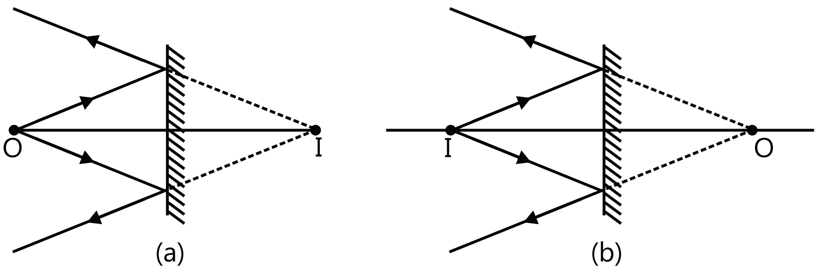


Figure 16.2

Real object, virtual object, real image, virtual image: In Fig. 16.2 (a), the object is real, while the image is virtual. In Fig. 16.2 (b), the object is virtual, while the image is real.

2. REFLECTION OF LIGHT

Definition

When the light falling on a surface turns back into the same medium, it means it is reflected. The angle made by the incident ray with the normal to the reflecting or refracting surface is called the angle of incidence, and the angle made by the reflected or refracted ray with normal is called the angle of reflection or refraction.

2.1 Laws of Reflection

(a) When the incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence lie in the same plane, it is called the plane of incidence.

(b) The angle of incidence is equal to the angle of reflection $\boxed{\angle i = \angle r}$.

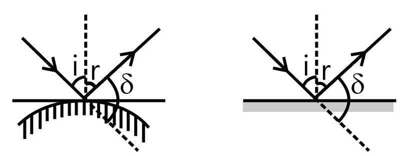


Figure 16.3

2.2 Deviation of Ray

The deviation is defined as the angle between the directions of the incident ray and the reflected ray (or the emergent ray). It is generally denoted by δ.

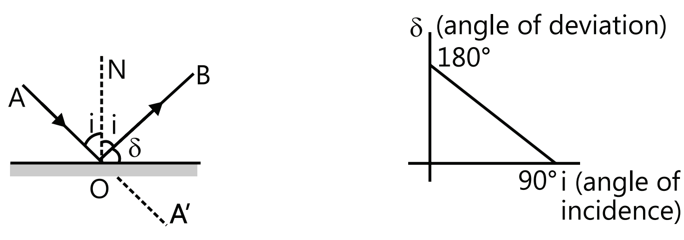


Figure 16.4

Here, $\angle A'OB = \delta = \angle AOA' - \angle AOB = 180^0 - 2i$

or $\delta = 180^0 - 2i$

PLANCESS CONCEPTS

The above two laws of reflection can be applied to the reflecting surfaces that are not even horizontal. The following Fig. 16.5 illustrates this point.

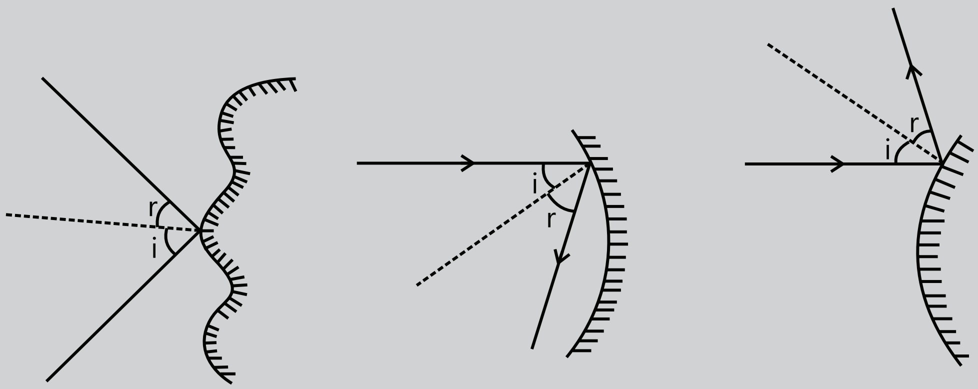


Figure 16.5

Vaibhav Gupta (JEE 2009 AIR 54)

3. REFLECTION FROM A PLANE SURFACE (OR PLANE MIRROR)

Almost everybody is familiar with the image formed by a plane mirror. If the object is real, the image formed by a plane mirror is virtual, erect, of same size of the original object and in the same distance from the mirror.

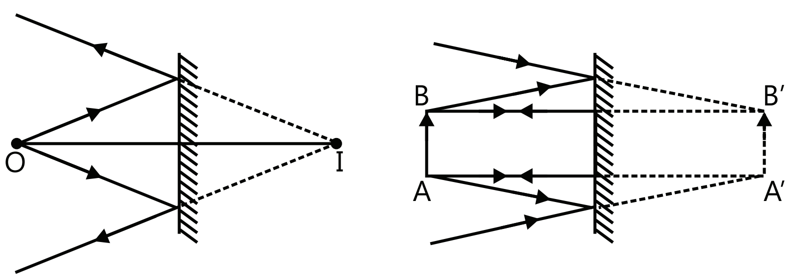


Figure 16.6

If an object is placed in front of a mirror as shown in Fig. 16.6, we get its image in the mirror due to the reflection of light.

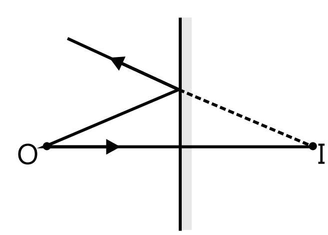


Figure 16.7

(a) The distance between the object and the mirror = the distance between the image and the mirror.

(b) The line joining the object point with its image is normal to the reflecting surface.

(c) The image is laterally inverted (left–right inversion).

(d) The size of the image is same as that of the object.

(e) For a real object, the image is virtual, and for a virtual object, the image is real.

(f) For a fixed incident light ray, if the mirror is rotated by an angle θ, the reflected ray turns through an angle 2θ. If plane mirror is rotated through about an axis perpendicular to plane of mirror, then the reflected ray image spot does not rotate.

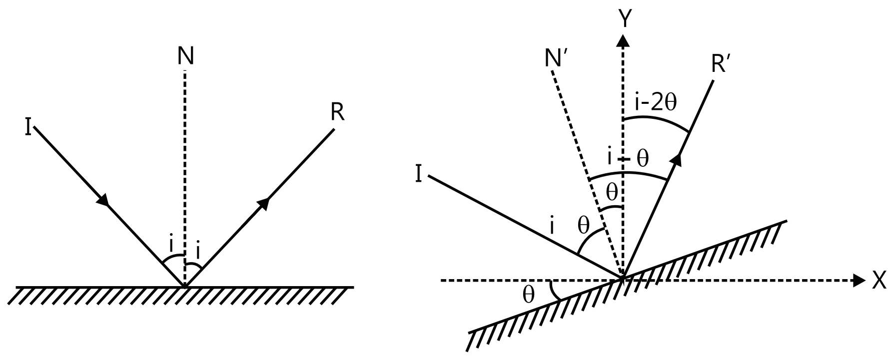


Figure 16.8

(g) The minimum size of a plane mirror required to see the full-size image of a person by himself is half the size of that person.

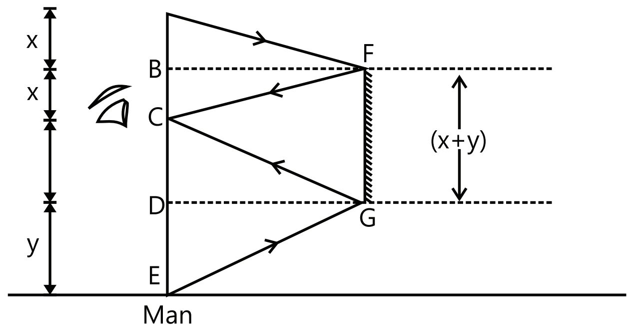


Figure 16.9

(h) A plane mirror behaves like a window to the virtual world.

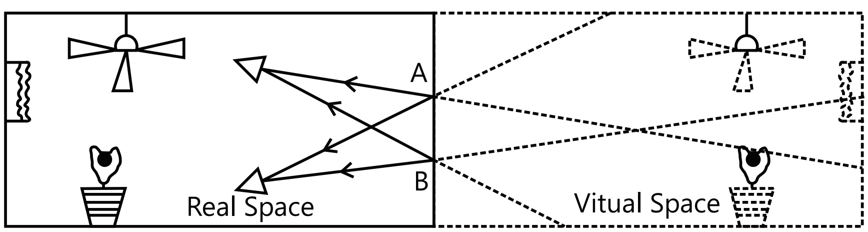


Figure 16.10

PLANCESS CONCEPTS

To find the location of the image of an object from an inclined plane mirror, you have to see the perpendicular distance of that object from the mirror.

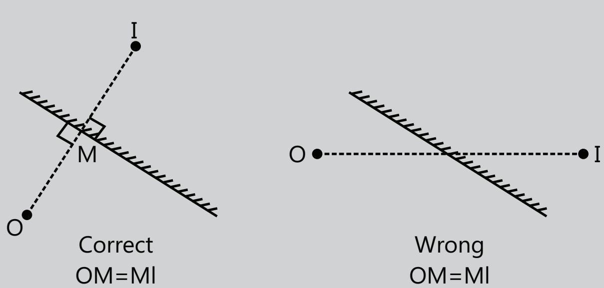


Figure 16.11

Vaibhav Krishnan (JEE 2009 AIR 22)

Illustration 1: A point source of light S, placed at a distance *L* in front of the center of a mirror of width *d*, hangs vertically on the wall. Assume that a man walks in front of the mirror along a line parallel to the mirror at a distance 2*L* from it as shown in the Fig. 16.12. The greatest distance over which he can see the image of the light source in the mirror is

(A) d/2 (B) d (C) 2d (D) 3d **(JEE MAIN)**

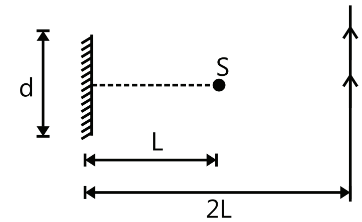


Figure 16.12

Sol: As the man is walking parallel to the mirror, the image of the point object S thus formed will also move relative to the man. We construct the ray diagram to obtain the position of the image from the man.

The ray diagram is shown in the Fig. 16.13.

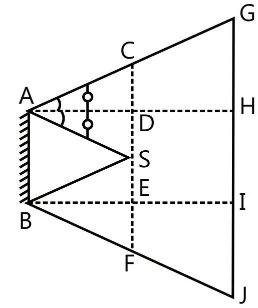


Figure 16.13

$HI = AB = d$; $DS = CD = \frac{d}{2}$

since $AH = 2AD$; $\therefore$ $GH = 2CD = 2\frac{d}{2} = d$

Similarly $IJ = d$; $\therefore$ $GJ = GH + HI + IJ$; $= d + d + d = 3d$

Illustration 2: Two plane mirrors M_1 and M_2 are inclined at an angle θ as shown in the Fig. 16.14. A ray of light 1, which is parallel to M_1, strikes M_2, and after two reflections, ray 2 becomes parallel to M_2. Find the angle θ.

(JEE MAIN)

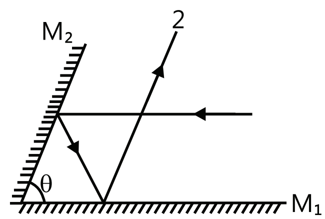


Figure 16.14

Sol: The angle of reflection is equal to angle of incidence about the normal. If ray makes angle α with the normal then the angle made with the surface is $\theta = 90 - \alpha$. Completing the ray diagram for multiple reflection we get the angle θ.

Different angles are as shown in the following Fig. 16.15. In triangle ABC,

$\theta + \theta + \theta = 180°$

$\theta = 60°$

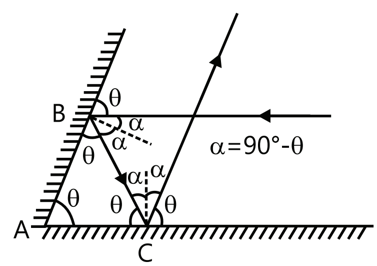


Figure 16.15

Illustration 3: A ray of light is travelling at an angle of 20° above the horizontal plane. At what angle with the horizontal plane must a plane mirror be placed in its path so that it becomes vertically upward after the reflection? **(JEE MAIN)**

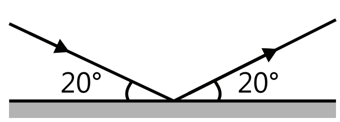


Figure 16.16

Sol: The angle of incidence and reflection are similar with respect to the normal. To make the ray reflect vertically upwards, we need to incline the mirror at an angle $\theta = \left(\frac{90 - i}{2}\right)$ where I is the angle of incidence.

$$\Rightarrow \theta = \frac{90 - 20}{2} = 35°$$

3.1 Velocity of Image Formed by a Plane Mirror

$X_{OM} \rightarrow$ *x* coordinate of the object relative to the mirror.

$X_{IM} \rightarrow$ *x* coordinate of the image relative to the mirror.

Differentiating, $\dfrac{dx_{IM}}{dt} = -\dfrac{dx_{OM}}{dt} \Rightarrow v_{IM} = -V_{OM}$

$\Rightarrow$ Velocity of the image relative to the mirror

= - velocity of the object relative to the mirror.

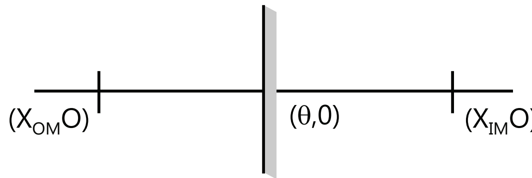


Figure 16.17

Illustration 4: Find the velocity of the image when the object and mirror both are moving toward each other with the velocities 2 m/s and 3 m/s, respectively. **(JEE MAIN)**

Sol: As both the object and mirrors are moving towards each other with a constant speed. The velocity of object with respect to the mirror and velocity of image with respect to the mirror are equal in magnitude but opposite in direction.

Here, $v_{OM} = -v_{IM}$

$$v_O - v_M = -(v_I - v_M)$$

$$\Rightarrow (+2m/s) - (-3m/s) = -v_I + (-3)$$

$$\Rightarrow v_I = -8m/s$$

3.2 Image Formation by Multiple Reflection

Case I: When the mirrors are parallel to each other Figure 16.18 shows an image formed by an object placed at a distance *y*, from M_1 and at a distance *x* from M_2. The number of images formed by the parallel plane mirrors is infinite.

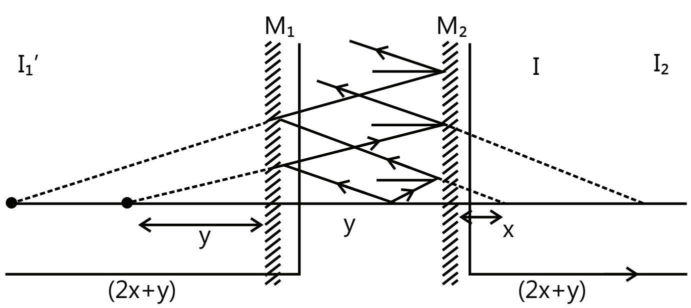


Figure 16.18

Case II: When the mirrors are inclined at an angle θ.

(a) All the images formed by the two mirrors lie on a circle with center C (an intersection point of the two mirrors). Here, if the angle between these two mirrors is θ, then an image will be formed on a circle at an angle $2\pi - \theta$. If the angle θ is less, the number of images formed is high.

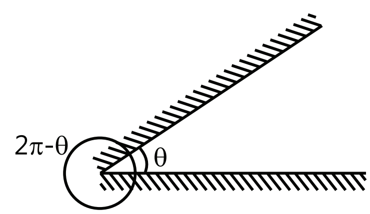


Figure 16.19

(b) If n is the number of image, then

$$\left.\begin{array}{l}\text{If (i) } 360^0/\theta \text{ is even} \\ \text{(ii) } 360^0/\theta \text{ is odd and object is kept symmetrically}\end{array}\right\} \text{then } n = (360^0/\theta) - 1$$

and $n = \left(\dfrac{360^0}{\theta}\right)$ for all other conditions.

PLANCESS CONCEPTS

The number of images formed by two mutually perpendicular ($\theta=90^o$) mirrors is three. All these three images will lie on a circle with center at C, the point of intersection of mirrors M_1 and M_2, and whose radius is equal to the distance between C and object O.

In fact, whatever be the angle, all the images lie on a circle.

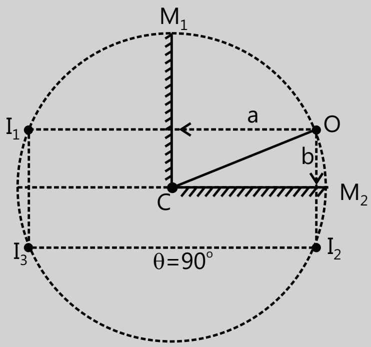


Figure 16.20

Vijay Senapathi (JEE 2011 AIR 71)

4. SPHERICAL MIRRORS

A spherical mirror is a smooth reflecting surface that forms a part of a spherical surface. If reflection takes place from the inner reflecting surface, then the mirror is called a concave mirror. If the reflection takes place from the outer surface, it is called a convex mirror. The reflection of a light from a concave and a convex mirror is shown in the Fig. 16.21.

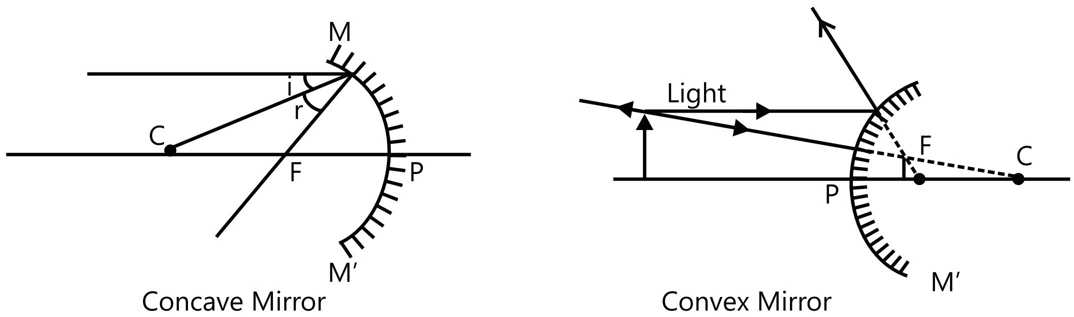


Figure 16.21

4.1 Parameters Associated with the Spherical Mirror

A pole (P) or vertex is the geometrical center of a reflecting surface. Center of curvature C is the center of the sphere, of which the mirror is a part. Radius of curvature R is equal to the distance between P and C of the mirror and is the radius of the sphere, of which the mirror is a part. The principal axis is the line CP that passes through P and C. If a ray of light is emitted from an object at infinity so that the beam of light is parallel to the principal axis, an image is formed at principal focus F after reflection. Focal length, f, is the distance between P and F along the principal axis. When a beam of light is incident parallel to the principal axis, the reflected rays converge on F in a concave mirror and diverge from F in a convex mirror after reflection. Aperture of a spherical mirror is the effective diameter MM′ of the light-reflecting area in the mirror. When the aperture of a mirror is small, the focal length is equal to half the radius of curvature.

4.2 Sign Convention

The following sign conventions based on coordinate geometry are used:

(a) The rays of light travel from the left to the right direction.

(b) All the distances measured from the pole and in the direction of light toward the right of the pole are positive. The distances measured in the opposite direction of light toward the left of the pole are negative.

(c) The transverse distances, above the principal axis, are positive and the principal axis are negative.

(d) If condition (1) in sign conventions is followed, this sign convention follows the right-hand Cartesian coordinate system.

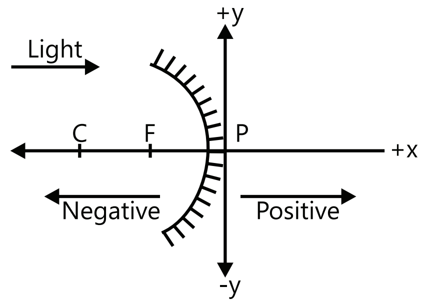


Figure 16.22

4.3 Rules for Image Formation

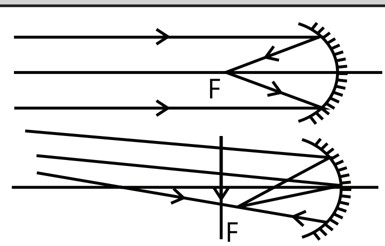
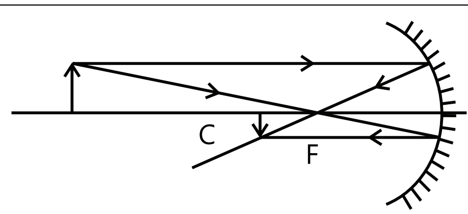
The following rules are used for locating the image of an object by considering the reflection of three types of rays based on laws of reflection:

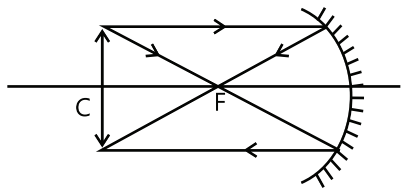
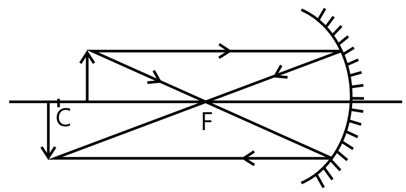
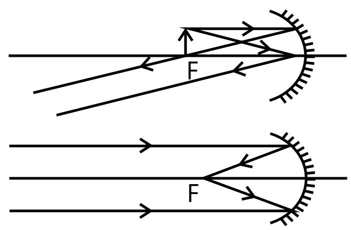
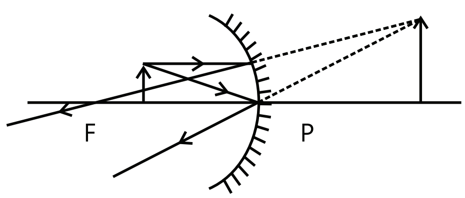
(a) A ray incident parallel to the principal axis will pass through the principal focus after reflection in the case of a concave mirror and will appear to originate from the focus in the case of a convex mirror.

(b) A ray that passes through the principal focus of a concave mirror, or that passes toward the principal focus of a convex mirror, travels parallel to the principal axis after reflection.

(c) A ray that passes through the center of curvature of a concave mirror or toward the center of curvature of a convex mirror is reflected from the mirror along the same path.

4.4 Image Formation by a Concave Mirror (for Real Object)

S. No.	Position of Object	Diagram	Position of Image	Nature of Image
1	Infinity	 **Figure 16.23**	At the principal focus F or in the focal plane Image problem	Real, inverted and diminished
2	Between infinity and C	 **Figure 16.24**	Between F and C	Real, inverted and diminished

S. No.	Position of Object	Diagram	Position of Image	Nature of Image
3	C	 **Figure 16.25**	C	Real, inverted and of same size as the object.
4	Between F and C	 **Figure 16.26**	Between C and infinity	Real, inverted and magnified
5	At F or in the focal plane	 **Figure 16.27**	At infinity	Real, inverted and highly magnified
6	Between F and P	 **Figure 16.28**	Behind the mirror	Virtual, erect and magnified.

4.5 Image Formation by a Convex Mirror

An image is formed between the pole and the focus for all the positions of the real object except when the objects are at infinity in which case the image is formed at F in the focal plane. The image formed is virtual, erect and diminished. The ray diagram for the formation of image I of object O after reflection from a convex mirror is shown in the Fig. 16.29.

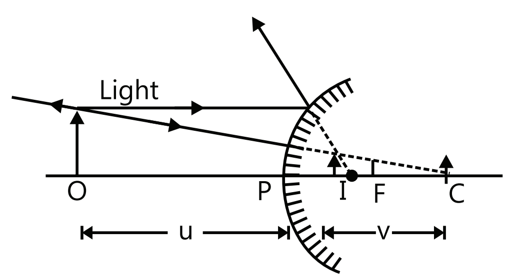


Figure 16.29

PLANCESS CONCEPTS

Image formed by a convex mirror is always virtual, erect and diminished; no matter where the object is placed (except for virtual objects).

Anurag Saraf (JEE 2011 AIR 226)

4.6 Mirror Formula

If an object is placed at a distance *u* from the pole of a mirror, its image is formed at a distance *v* from the pole, and its focal length *f* is given by $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ where *f* = *R*/2 (only for paraxial rays).

According to the sign conventions, *f* and *R* are negative for a concave mirror and are positive for a convex mirror. The power of a mirror, *P*, measured in units of dioptres is given by

$$P=-\frac{1}{f}\text{ Where } f \text{ is in metres }=-\frac{100}{f}\text{ Where } f \text{ is in centimetres}$$

Illustration 5: A convex mirror has a radius of curvature of 20 cm. Find the position of the image of an object placed at a distance of 12 cm from the mirror. **(JEE MAIN)**

Sol: The position of the image is found using formula $\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$.

According to sign convention, if the object is placed to the left side of the pole, object distance is considered to be negative.

The situation is shown in the Fig. 16.30. Here, *u* = −12 cm and *R* = +20 cm.

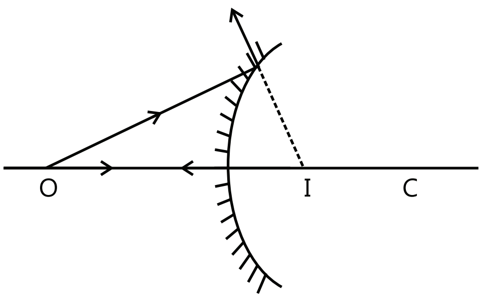


Figure 16.30

We have, $\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$ or $\frac{1}{v}=\frac{2}{R}-\frac{1}{u}=\frac{2}{20\,cm}-\frac{1}{-12cm}=\frac{11}{60\,cm}$

$$\Rightarrow v=\frac{60}{11}cm.$$

The positive sign of *v* shows that the image is formed on the right side of the mirror and is a virtual image.

4.7 Magnification by Mirror

If an object of linear size *O* is placed vertically on the axis of a concave or convex mirror at a distance *u* from the pole and its image of size *I* is formed at a distance *v* from the pole, then the lateral or transverse magnification, *m*, is given by $m=\frac{I}{0}=-\left(\frac{v}{u}\right)$

A negative magnification indicates that the image is inverted with respect to the object, whereas a positive magnification implies that the image is erect with respect to the object.

Illustration 6: A concave and a convex mirror of focal lengths 10 cm and 15 cm, respectively, are placed at a distance of 70 cm from each other. An object AB of height 2 cm is placed at a distance of 30 cm from the concave mirror. First, a ray is incident on a concave mirror and then on the convex mirror. Find the size, position and nature of the image. **(JEE MAIN)**

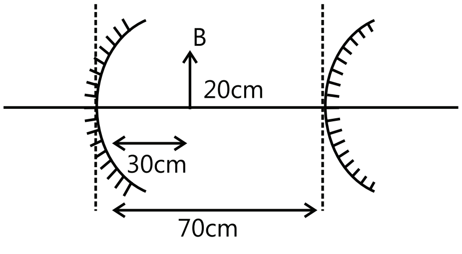


Figure 16.31

Sol: The position of the image is found using formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$.

The image formed by concave mirror is the object for the convex mirror.

For a concave mirror, $u=-30cm,\ \ f=-10cm$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ $\Rightarrow$ $\frac{1}{v}-\frac{1}{30}=\frac{-1}{10}$; $\Rightarrow$ $v=-15cm$

Now, $\frac{A'B'}{AB}=\frac{-v}{u}=\frac{-15}{-30}$; $\Rightarrow$ $A'B'=-1cm$

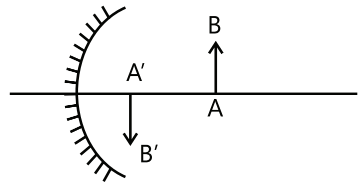


Figure 16.32

An image formed by the first reflection will be real, inverted and diminished.

For a convex mirror,

$u'=-55cm,\ f=+15cm$ Using $\frac{1}{v'}+\frac{1}{u'}=\frac{1}{f'}$

$\Rightarrow$ $\frac{1}{v'}-\frac{1}{55}=\frac{1}{15}$ $\Rightarrow$ $v'=165/14cm$

Now, $\frac{A"B"}{A'B'}=-\frac{v'}{u'}=-\frac{\left(\frac{165}{14}\right)}{(-55)}$ $\Rightarrow$ $A"B"=\left(\frac{3}{14}\right)(-1)=-0.2cm$

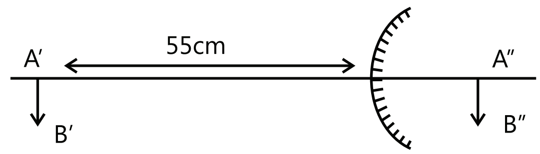


Figure 16.33

The final image will be virtual and diminished.

Illustration 7: An object ABED is placed in front of a concave mirror beyond the center of curvature C as shown in the Fig. 16.34. State the shape of the image. **(JEE ADVANCE)**

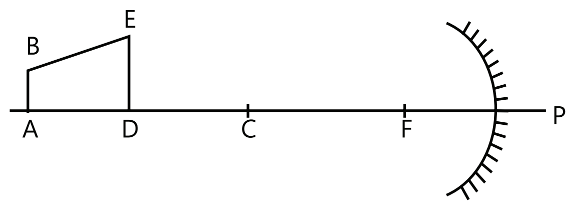


Figure 16.34

Sol: As the object is placed beyond center of the curvature, the image thus formed will lie between center of curvature and pole. The position of the image is found using formula

$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

The object is placed beyond C. Hence, the image will be real and it will lie between C and F. Furthermore, *u*, *v* and *f* all are negative; hence, the mirror formula will become

$-\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$ or $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{u-f}{uf}$ or $v=\frac{f}{1-\frac{f}{u}}$.

Now $u_{AB}>u_{ED}$; $\therefore$ $v_{AB}<v_{ED}$

and $|m_{AB}|<|m_{ED}|$ $\left(\text{as } m=\frac{-v}{u}\right)$.

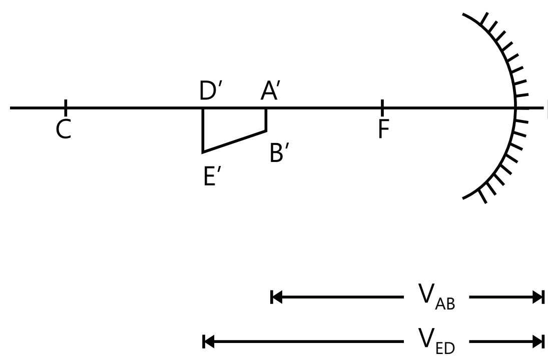


Figure 16.35

Therefore, the shape of the image will be as shown in the Fig. 16.39. Also note that

$v_{AB}<u_{AB}$ and $v_{ED}<u_{ED}$, So, $|m_{AB}|<1$ and $|m_{ED}|<1$.

4.8 Relation between Object and Image Velocity

Differentiating equation $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.

$\Rightarrow -\frac{1dv}{v^2dt}-\frac{1du}{u^2dt}=0$ $\Rightarrow -\frac{1}{v^2}V_{IM}-\frac{1}{u^2}V_{OM}=0$ $V_{IM}\Rightarrow$ velocity of image w.r.t. mirror

$\Rightarrow V_{IM}=-\frac{v^2}{u^2}V_{OM}$ $V_{OM}\Rightarrow$ velocity of object w.r.t. mirror

$\Rightarrow V_{IM}=-m^2V_{OM}$

Illustration 8: A mirror with a radius of curvature of 20 cm and an object that is placed at a distance 15 cm from the mirror both are moving with the velocities 1 m/s and 10 m/s as shown in the Fig. 16.36. Find the velocity of the image. **(JEE MAIN)**

Sol: As the object and the mirror are moving away from each other with different speed, the magnification of the image will also change. The velocity of image will be $V_{im} = -\frac{v^2}{u^2}V_{om}$.

The position of the image is found using formula $\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$.

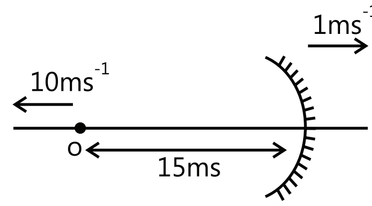


Figure 16.36

Using $\frac{1}{v}+\frac{1}{u}=\frac{2}{R} \quad \Rightarrow \quad \frac{1}{v}-\frac{1}{15}=-\frac{1}{10} \quad \Rightarrow v=-30\text{cm}$

Now, using $V_{im} = -\frac{v^2}{u^2}V_{om} \quad \Rightarrow \quad \left(V_i - V_m\right) = -\frac{v^2}{u^2}\left(V_0 - V_m\right)$

$$\Rightarrow \qquad V_i - (1) = \frac{(-30)^2}{(-15)^2}\left[(-10)-(-1)\right] \Rightarrow V_i = 45\text{ cm/s}.$$

So the image will move with the velocity of 45 cm/s.

Illustration 9: A gun of mass m_1 fires a bullet of mass m_2 with a horizontal speed v_0. The gun is fitted with a concave mirror of focal length *f* facing toward a receding bullet. Find the speed of separations of the bullet and the image just after the gun was fired. **(JEE ADVANCED)**

Sol: The bullet when leave the gun it moves in direction opposite the motion of gun. As there are no external forces acting on the bullet, then the momentum of the system can be conserved. The velocity of image will be $V_{im} = -\frac{v^2}{u^2}V_{om}$.

Let v_1 be the speed of the gun (or the mirror) just after the firing of bullet. From conservation of linear momentum, $m_2 v_0 = m_1 v_1 \Rightarrow v_1 = \frac{m_2 v_0}{m_1}$ ... (i)

Now, $\frac{du}{dt}$ is the rate at which the distance between mirror and bullet is increasing

$= v_1 + v_0$ ... (ii)

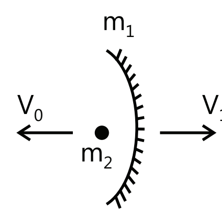


Figure 16.37

We know that $\therefore \quad \frac{dv}{dt} = \left(\frac{v^2}{u^2}\right)\frac{du}{dt}$.

Here, $\frac{v^2}{u^2} = m^2 = 1$ (as at the time of firing bullet is at pole).

$\therefore \quad \frac{dv}{dt} = \frac{du}{dt} = v_1 + v_0$. ... (iii)

Here, $\frac{dv}{dt}$ is the rate at which the distance between the image (of bullet) and the mirror is increasing. Hence, if v_2 is the absolute velocity of image (toward right), then

$v_2 - v_1 = \frac{dv}{dt} = v_1 + v_0 \quad \Rightarrow \quad v_2 = 2v_1 + v_0$ ... (iv)

Therefore, the speed of separation of the bullet and the image will be

$v_r = v_2 + v_0 = 2v_1 + v_0 + v_0$

$v_r = 2(v_1 + v_0)$.

Substituting the value of v_1 from Eq. (i), we have $\quad v_r = 2\left(1+\frac{m_2}{m_1}\right)v_0$.

5. REFRACTION OF LIGHT AND LAWS OF REFRACTION

(a) The deviation or bending of light rays from their original path while travelling from one medium to another is called refraction.

(b) If the refracted ray bends away from the normal, then the second medium is said to be RARER as compared to the first medium, and the speed increases.

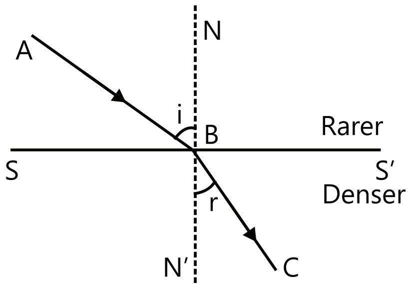


Figure 16.38

If the refracted ray bends toward the normal, then the second medium is said to be DENSER compared to the first, and the speed decreases.

Deviation due to refraction $\delta = i - r$.

PLANCESS CONCEPTS

In general, light will travel in straight lines and the deviation occurs only when there is a change of medium (or refractive index (RI)).

B Rajiv Reddy (JEE 2012 AIR 11)

5.1 Laws of Refraction

The incident ray (AB), the normal (NN') to the refracting surface (SS') at the point of incidence (B), and the refracted ray (BC) all lie in the same plane called the plane of incidence or the plane of refraction.

5.2 Snell's Law

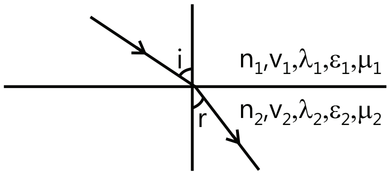


Figure 16.39

For any two given media and for light of given wavelength,

$$\frac{\sin i}{\sin r} = \text{constant}\ ; \qquad \frac{\sin i}{\sin r} = {}^1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}.$$

${}^1\mu_2$ = RI of the second medium with respect to the first medium;

μ_1 = RI of the first medium with respect to air or absolute

RI = c/v_1 ;

μ_2 = RI of the second medium with respect to air or absolute RI = c/v_2 ;

v_1, v_2 are the speeds of light in the first and the second medium, respectively;

λ_1, λ_2 are the wavelengths of light in the first and the second medium, respectively;

c=the speed of light in air (or vacuum) = $3\text{x}10^8$ m/s.

Note:

(i) The higher the value of RI, the denser (optically) the medium is.

(ii) The frequency of light does not change during refraction.

(iii) The refractive index of the medium relative to $\text{air} = \sqrt{\mu_r \varepsilon_r}$.

5.3 Refraction through a Transparent Sheet

Let the ray is incident at face AB.

Apply the Snell's law at faces AB and CD.

$\mu_1 \sin i_1 = \mu_2 \sin i_2$ ... (i)

$\mu_2 \sin i_2 = \mu_3 \sin i_3$ ... (ii)

(The angle of incidence for face CD is i_2)

From Eqs (i) & (ii), $\mu_1 \sin i_1 = \mu_3 \sin i_3$

or $\sin i_3 = \dfrac{\mu_1}{\mu_3} \sin i_1$ ... (iii)

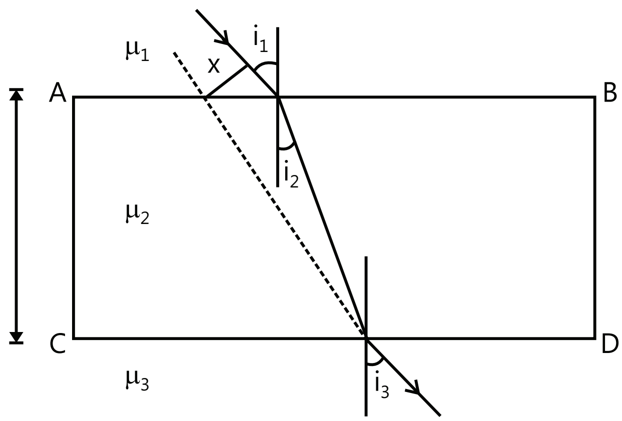


Figure 16.40

The incident ray and emerging ray are parallel. It shows that the deviation of ray is not affected by the refractive index of the sheet; it depends by μ_1 and μ_3. μ_2 only causes lateral displacement which is given by $x = \dfrac{t \sin(i_1 - i_2)}{\cos i_2}$.

At a glance:

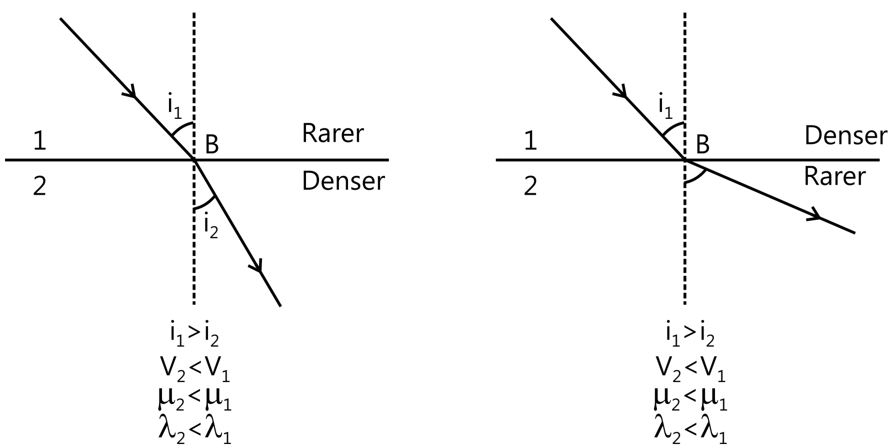


Figure 16.41

PLANCESS CONCEPTS

In general, the speed of light in any medium is less than that in vacuum. The refractive index μ of a medium is defined as,

$$\mu = \frac{\text{Speed of light in vaccum}}{\text{Speed of light in medium}} = \frac{c}{v}.$$

Anand K (JEE 2011 AIR 47)

Illustration 10: A light beam passes from medium 1 to medium 2. Show that the emerging beam is parallel to the incident beam. **(JEE MAIN)**

Sol: When a ray of light enters from one medium to other medium of different refractive index, then according to Snell's law we get $\frac{\mu_1}{\mu_2} = \frac{\sin i}{\sin r}$ where i is the angle of incidence and r is the angle of refraction. The first refracted ray is incident on other surface. So the angle o incidence on the second surface is equal to the angle of refraction from first surface.

Applying the Snell's law at A and B, $\mu_1 \sin i_1 = \mu_2 \sin i_2$

Or $\frac{\mu_1}{\mu_2} = \frac{\sin i_2}{\sin i_1}$ ... (i)

Similarly, $\mu_2 \sin i_2 = \mu_1 \sin i_3$

$\therefore$ $\frac{\mu_1}{\mu_2} = \frac{\sin i_2}{\sin i_3}$ ... (ii)

From Eqs (i) and (ii) $i_3 = i_1$; i.e. the emergent ray is parallel to the incident ray.

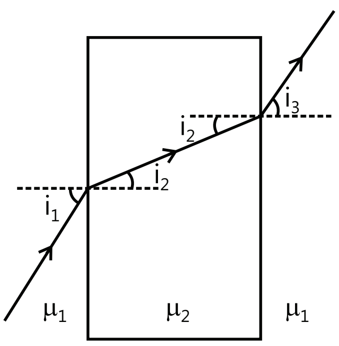


Figure 16.42

Illustration 11: The refractive index of glass with respect to water is (9/8) and the refractive index of glass with respect to air is (3/2). Find the refractive index of water with respect to air. **(JEE MAIN)**

Sol: The refractive index of medium one with respect to medium two is given by ${}_1\mu_2 = \frac{\mu_2}{\mu_1}$.

To find the refractive index of water with respect to air, we need to obtain the ratio between ${}_w\mu_g$ and ${}_a\mu_g$

Given ${}_w\mu_g = 9/8$ and ${}_a\mu_g = 3/2$.

As ${}_a\mu_g \times {}_g\mu_w \times {}_w\mu_a = 1$

$\therefore$ $\frac{1}{{}_w\mu_a} = {}_a\mu_w = {}_a\mu_g \times {}_g\mu_w = \frac{{}_a\mu_g}{{}_w\mu_g}$

$\therefore$ ${}_a\mu_w = \frac{3/2}{9/8} = \frac{4}{3}$

Illustration 12: (i) Find the speed of light of wavelength λ = 780 nm (in air) in a medium of refractive index μ = 1.55.
(ii) What is the wavelength of this light in the given medium? **(JEE MAIN)**

Sol: In a medium of refractive index μ the velocity of wave is given by $v = \frac{c}{\mu}$ and the wavelength of wave is given by $\lambda_{medium} = \frac{\lambda_{air}}{\mu}$

(i) $v = \frac{c}{\mu} = \frac{3.0 x 10^8}{1.55} = 1.94 x 10^8 m/s$

(ii) $\lambda_{medium} = \frac{\lambda_{air}}{\mu} = \frac{780}{1.55} = 503nm$.

5.4 Image Due to Refraction at a Plane Surface

Consider the situation given in the Fig. 16.43. A point object O is placed in a medium of refractive index μ_1. An another medium of refractive index μ_2 has its boundary at PA. Consider two rays OP and OA originating from O. Let OP fall perpendicularly on PA and OA fall on PA at a small angle *i* with the normal. OP enters the second medium without deviating, and OA enters by making an angle *r* with the normal. When produced backward, these rays meet at I that is the virtual image of O. If *i* and *r* are small,

$\sin i = \tan i = \frac{PA}{PO}$;

and $\sin r = \tan r = \frac{PA}{PI}$.

Thus, $\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$

$= \left(\frac{PA}{PO}\right) \cdot \left(\frac{PI}{PA}\right) = \frac{PI}{PO}$. ... (i)

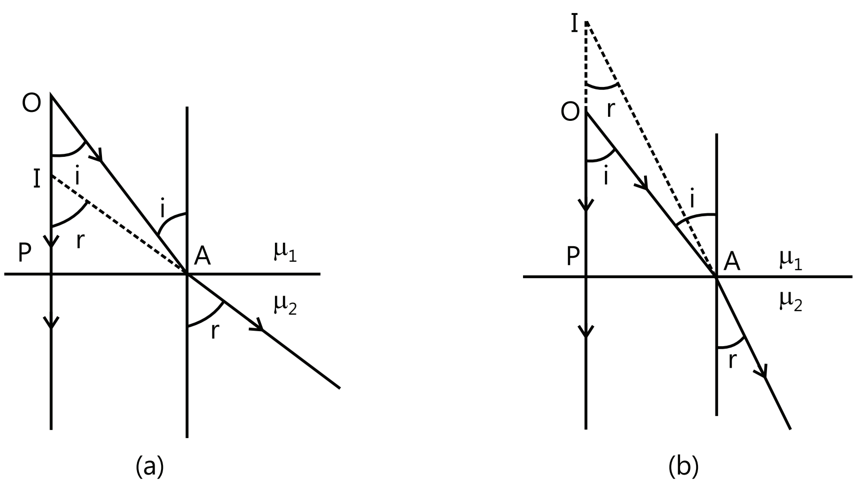


Figure 16.43

Suppose medium 2 is air and an observer looks at the image from this medium. The real depth of the object inside medium 1 is PO, whereas the depth as it appears to the observer is PI. Writing $\mu_2 = 1$ and $\mu_1 = \mu$, Eq. (i) gives,

$$\frac{1}{\mu} = \frac{\text{apparent depth}}{\text{real depth}} \quad \text{or,} \quad \mu = \frac{\text{real depth}}{\text{apparent depth}}.$$

The image shifts closer to the observer's eye by an amount

$OI = PO - PI = \left(\frac{PO - PI}{PO}\right)PO = \left(1 - \frac{PI}{PO}\right)PO$ or, $\Delta t = \left(1 - \frac{1}{\mu}\right)t$, where *t* is the thickness of the medium over the object, and Δt is the apparent shift in its position toward the observer. Note that Δt is positive in Fig16.43 (a) and negative in Fig. 16.43 (b).

Different Scenarios at a Glance:

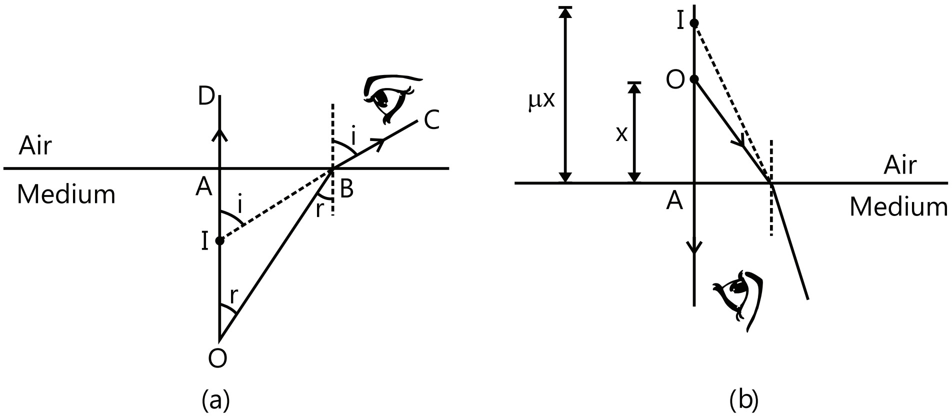


Figure 16.44

Illustration 13: A printed page is kept pressed by a glass cube $(\mu = 1.5)$ of edge 6.0 cm. By what amount will the printed letters appear to be shifted when viewed from the top? **(JEE MAIN)**

Sol: As the glass of thickness t = 6 cm is kept on the page, the image of letter appears to be shifted closer to eye by $\Delta t = \left(1 - \frac{1}{\mu}\right)t$

The thickness of the cube = *t* = 6.0 cm. The shift in its position of the printed letters is

$$\Delta t = \left(1 - \frac{1}{\mu}\right) \times t \quad = \left(1 - \frac{1}{1.5}\right) x 6.0\,cm = 2.0\,cm.$$

5.5 Shift Due to a Glass Slab (Double Refraction from Plane Surfaces)

(a) Normal Shift:

Refer Fig 16.45: An object is placed at O. Plane surface CD forms its image (virtual) at I_1. This image acts as an object for EF which initially forms the image (virtual) at I. Distance OI is called the normal shift, and its value is

$$OI = \left(1 - \frac{1}{\mu}\right)t \Rightarrow \left(\frac{\mu - 1}{\mu}\right)t$$

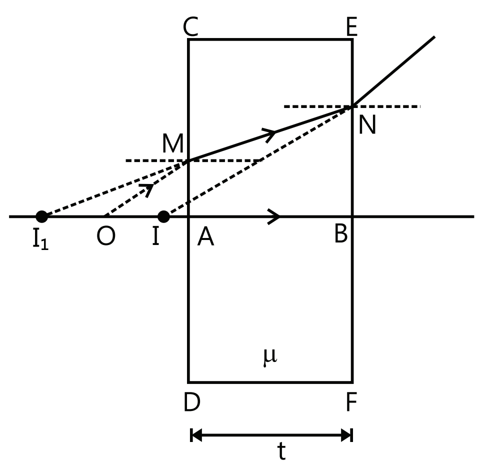


Figure 16.45

Proof: Let $OA = x$

$AI_1 = \mu x$ (Refraction from CD)

$BI_1 = \mu x + t$

$BI = \frac{BI_1}{\mu} = x + \frac{t}{\mu}$ (Refraction from EF)

$\therefore \quad OI = (AB + OA) - BI \quad = (t + x) - \left(x + \frac{t}{\mu}\right) = \left(1 - \frac{1}{\mu}\right)t.$

(b) Lateral Shift: We have already discussed that ray MA is parallel to ray BN, but the emergent ray is displaced laterally by a distance *d*, which depends on μ, *t* and *i*, and its value is given by the relation

$$d = t\left(1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right)\sin i.$$

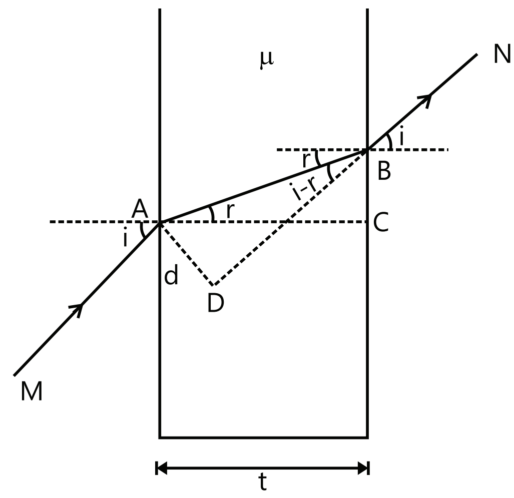


Figure 16.46

Proof:

$$AB = \frac{AC}{\cos r} = \frac{t}{\cos r} \qquad (\text{as } AC = t).$$

Now, $\quad d = AB\sin(i - r) = \frac{t}{\cos r}[\sin i \cos r - \cos i \sin r]$

or $\quad d = t[\sin i - \cos i \tan r] \qquad \ldots (i)$

Further $\quad \mu = \frac{\sin i}{\operatorname{sir} r} \quad$ or $\quad \sin r = \frac{\sin i}{\mu}$

$\therefore \quad \tan r = \frac{\sin i}{\sqrt{\mu^2 - \sin^2 i}}$

Substituting in Eq. (i), we get $d = t\left[1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right]\sin i.$

Illustration 14: A point object O is placed in front of a concave mirror of focal length 10 cm. A glass slab with a refractive index of $\mu = 3/2$ and thickness of 6 cm is inserted between an object and a mirror. Find the position of the final image when the distance *x* shown in the Fig. 16.47 is: (a) 5cm (b) 20 cm **(JEE MAIN)**

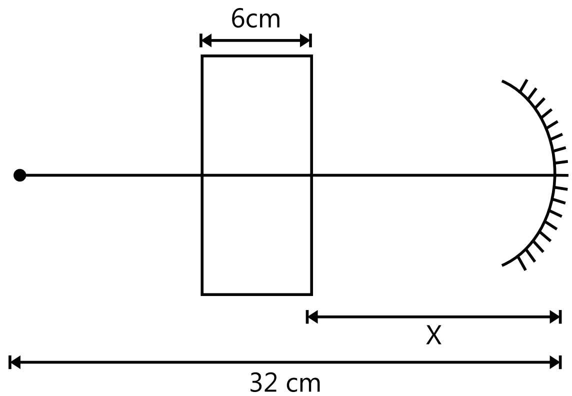


Figure 16.47

Sol: As the glass slab of thickness t = 10 cm is kept in front of the mirror, the image of object from the slab appears to be shifted closer to mirror by $\Delta t = \left(1 - \frac{1}{\mu}\right)t$. The position of the image is given by $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ where u is the distance of object, f is the focal length of the mirror and v is the distance of the image from mirror.

As we know that the normal shift produced by a glass slab is $\Delta x = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{2}{3}\right)(6) = 2\text{cm}$;

i.e. for the mirror, the object is placed at a distance $(32 - \Delta x) = 30\text{cm}$ from it. Now apply the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \quad \frac{1}{v} - \frac{1}{30} = \frac{1}{10} \Rightarrow \; v = -15\text{cm}.$$

(i) **When x = 5cm:** The light falls on the slab on its return path as shown in the Fig. 16.48, but the slab will again shift it by a distance of $\Delta x = 2\text{cm}$. Hence, the final real image is formed at a distance of (15+2) =17 cm from the mirror.

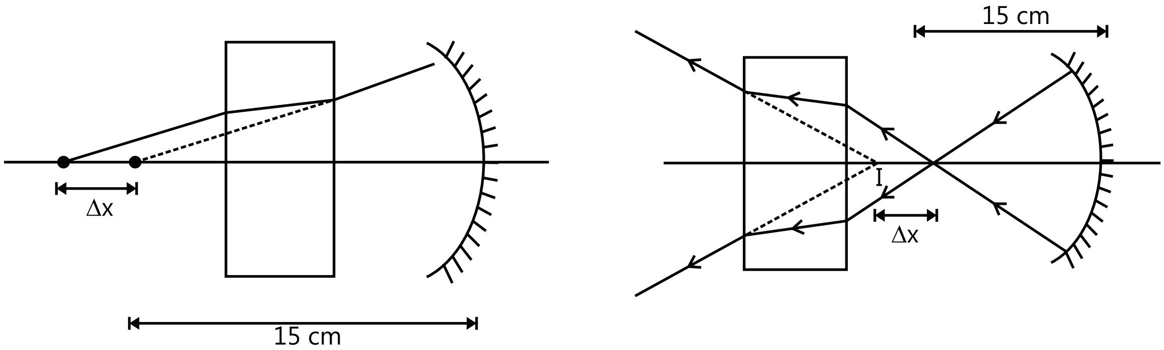


Figure 16.48

(ii) **When *x* = 20 cm:** The final image is at a distance of 17 cm from the mirror in this case also, but it is virtual.

6. CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

When a ray of light passes from an optically denser medium (a medium with larger μ) to an optically rarer medium (a medium with smaller μ), we have $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} < 1$.

When we gradually increase the value of *i*, the corresponding *r* value also increases, and at a certain point, *r* becomes 90°. Let the angle of incidence for this case be θ_c called the critical angle of the given pair of media. If *i* is increased further, there is no *r* that can satisfy the Snell's law. Then, all the light waves are reflected back into the first medium. This is called total internal reflection (TIR). Generally, the critical angle of a medium is quoted for light travelling from the denser to the rarer medium. In this case, $\mu_2 = 1$.

When substituting $\mu_1 = \mu$, then the Snell's law becomes $\frac{\sin\theta_c}{\sin 90^0} = \frac{1}{\mu}$

or $\sin\ \theta_c = (1/\mu)$ or $\theta_c = \sin^{-1}(1/\mu)$.

Illustration 16: A point source of light is placed at the bottom of a tank filled with water up to the level of 80 cm. Find the area of the surface of water through which light from the source emerges out. Assume that the refractive index is equal to 1.33. **(JEE ADVANCED)**

Sol: When light travels from a denser medium (water) to the rarer medium (air) for angle of incidence i greater than critical angle, than by Snell's law we get $\sin C = \frac{1}{\mu}$.

Let the light emerges out of a circular area of radius *r* as shown in the figure.

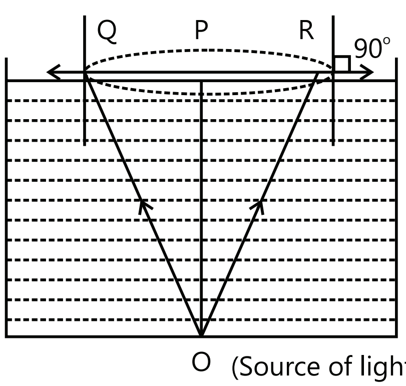


Figure 16.49

Step 1. Using $\sin C = \frac{1}{n}$, We get

$\sin C = \frac{1}{1.33} = 0.7513$

$\therefore$ C = sin⁻¹(0.7519) = 48.75°

Step 2. From ΔPOR,

$\tan\ C = \frac{PR}{OP} = \frac{r}{80}$

$\therefore$ r = 80tanC = 80tan(48.75°) = 80 × 1.14 = 91.2cm

Step 3. $\therefore$ Area through which light emerges out $= \pi r^2 = 3.14 \times (91.2)^2 = 26116.76\,\text{cm}^2 = 2.6\text{m}^2$.

Illustration 17: The critical angle for water is 48.2°. Find its refractive index. **(JEE MAIN)**

Sol: By Snell's law we get $\mu = \frac{1}{\sin C}$.

$\mu = \frac{1}{\sin\theta_c} = \frac{1}{\sin 48.2^\circ} = 1.34$.

Day-to-day life: Due to heating of the earth, the refractive index of air near the surface of the earth is lesser than that above the earth. Light waves from a distant object reach the surface of earth at an angle of $i > \theta_c$, so that the TIR will take place, and the image of an object creates an illusion of water near the object.

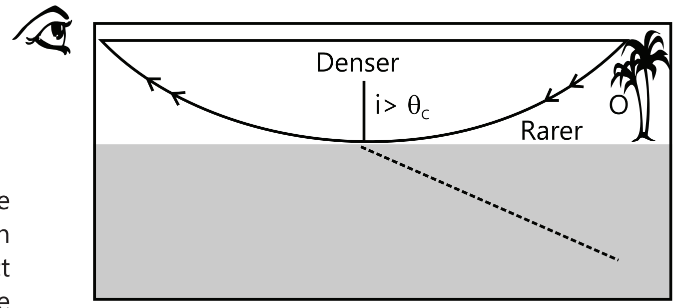


Figure 16.50

7. REFRACTION AT THE SPHERICAL SURFACES

Refraction at a single spherical surface

If the boundary between two transparent media is curved either as a convex or as a concave spherical surface, an object O in a medium of refractive index μ_1 forms an image I in a medium of refractive index μ_2 as shown in the Fig. 16.51. When the ray from O is incident at an angle *i* on the medium of refractive index μ_1, the refracted ray forms an image I after refraction in the medium of refractive index μ_2 at angle *r* according to the Snell's law.

$\mu_1 \sin i = \mu_2 \sin r$.

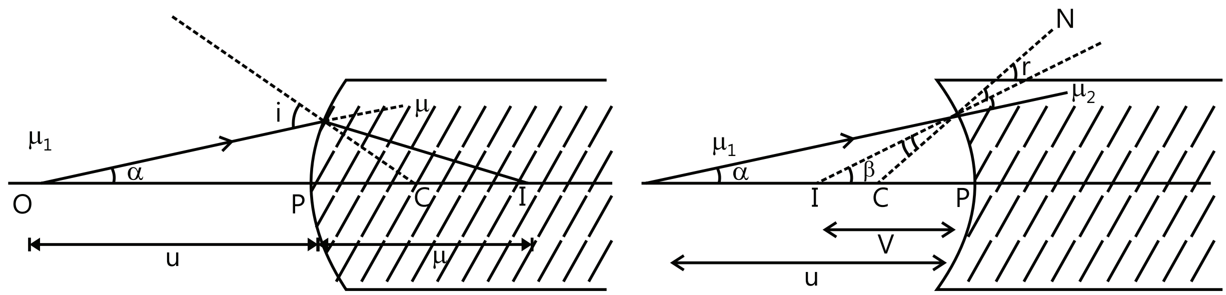


Figure 16.51

The relation between *u*=OP and *v*=IP is given by $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$.

PLANCESS CONCEPTS

The relation is valid for a single spherical surface or plane refracting surfaces, and the sign convention for the spherical mirrors and spherical refracting surfaces are the same.

The refraction formula $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ can also be applied to plane refraction surfaces with $R = \infty$.

Let us derive $d_{app} = \frac{d_{actual}}{\mu}$ using this.

Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ with proper sign and values, we get

PLANCESS CONCEPTS

$$\frac{1}{v}-\frac{\mu}{-d}=\frac{1-\mu}{\infty}=0 \text{ or } v=-\frac{d}{\mu}$$

i.e. the image of object O is formed at a distance $\frac{d}{\mu}$ on the same side.

or $d_{app}=\frac{d_{actual}}{\mu}$.

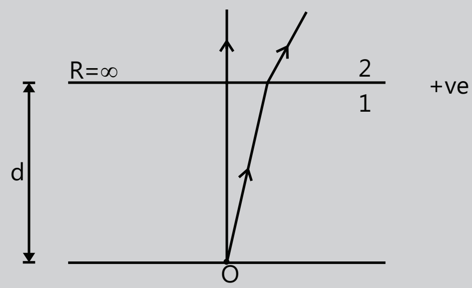


Figure 16.52

Anurag Saraf (JEE 2011 AIR 226)

Illustration 18: A sunshine recorder globe of 30 cm diameter is made of glass of refractive index *n* = 1.5. A ray of light enters the globe parallel to the axis. Find the position from the center of the sphere where the ray crosses the principle axis. **(JEE ADVANCED)**

Sol: As the light enters from air to the globe of refractive index *n* than for the refraction at the surface we have relation $\frac{\mu_1}{u}+\frac{\mu_2}{v}=\frac{\mu_2-\mu_1}{R}$.

First refraction (from the rarer to the denser medium): Here, $u=-\infty, n_2=1.5, R=+15\text{cm}$.

Using the relation, $\frac{n_2}{v}+\frac{n_1}{u}=\frac{n_2-n_1}{R}$

i.e. $$\frac{n_2}{v}=\left(\frac{n_2-n_1}{R}\right)+\frac{n_1}{u}$$

$$\Rightarrow \frac{1.5}{v}=\frac{1.5-1}{15}+\frac{1}{(-\infty)}=\frac{1}{30};$$

$$\Rightarrow v=45\text{cm}.$$

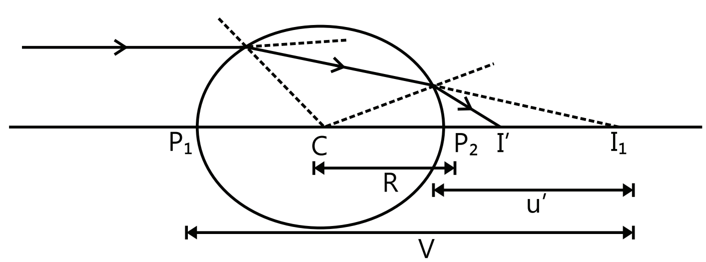


Figure 16.53

Second refraction (from the denser to the rarer medium):

Here, R=-15cm, u'= (45-30) =15cm.

Using the relation, $\frac{n_1}{v'}-\frac{n_2}{u'}=\frac{n_1-n_2}{R}$

i.e. $$\frac{1}{v'}-\frac{1.5}{5}=\frac{1-1.5}{-15}=\frac{1}{30}\Rightarrow \quad v'=\frac{30}{7}=7.5\text{cm}.$$

$\therefore$ Distance at which the image is formed from the center of the globe is (15+7.5) =22.5 cm.

Illustration 19: Locate the image of a point object O in the situation shown in the figure. Point C denotes the center of curvature of the separating surface. **(JEE ADVANCED)**

Sol: According to the sign convention as the object is placed left to the pole, the distance of it is considered to be negative. Using the formula $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ we get the distance of image from the centre of curvature.

Here, u=-15cm, R=30cm, $\mu_1 = 1$ and $\mu_2 = 1.5$ We have

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \quad \frac{1.5}{v} - \frac{1.0}{-15cm} = \frac{1.5-1}{30cm}$$

$$\Rightarrow \quad \frac{1.5}{v} = \frac{0.5}{30\ cm} - \frac{1}{15\ cm}; \qquad \Rightarrow \quad v = -30cm$$

The image is formed 30 cm left to the spherical surface and is virtual.

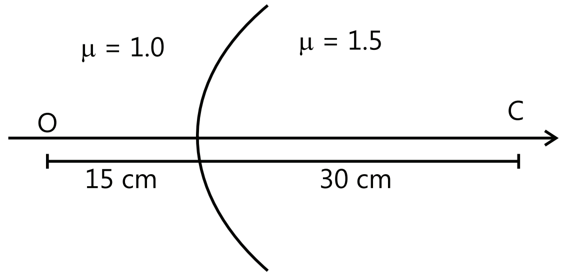


Figure 16.54

Illustration 20: A glass sphere of radius R=10 cm is kept inside water. A point object O is placed at 20 cm from A as shown in the figure. Find the position and nature of the image when seen from other side of the sphere and also draw the ray diagram. Assume μ_g=3/2 and μ_w= 4/3.

(JEE ADVANCED)

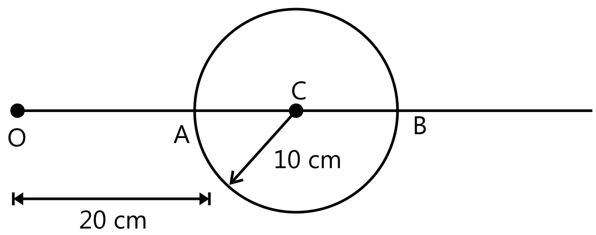


Figure 16.55

Sol: As the light passes through medium of different refractive indices at each refraction the position of image of point object is found by $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$.

A ray of light from object O gets refracted twice. The direction of this light ray is from the left to right. Hence, the distances measured in this direction are positive.

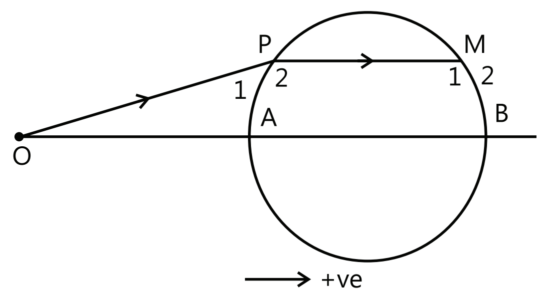


Figure 16.56

When applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ twice with proper signs,

we have $\frac{3/2}{AI_1} - \frac{4/3}{-20} = \frac{3/2 - 4/3}{10}$

or $AI_1 = -30cm$.

Now, the first image I_1 acts an object for the second surface, where

$BI_1 = u = -(30+20) = -50cm$.

$\therefore \frac{4/3}{BI_2} - \frac{3/2}{-50} = \frac{4/3 - 3/2}{-10}$.

$\therefore BI_2 = -100$ cm, i.e. the final image I_2 is virtual and is formed at a distance of 100 cm (toward left) from B. The ray diagram is shown in the figure.

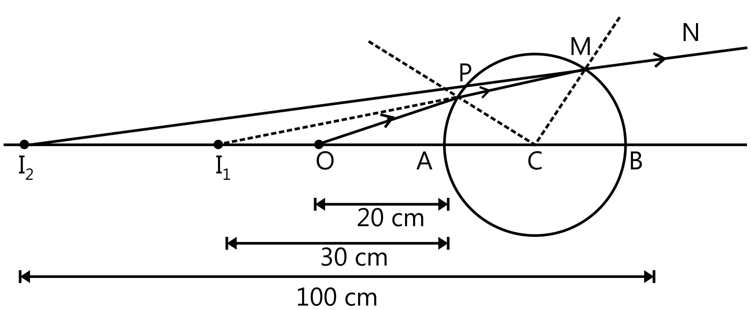


Figure 16.57

The following points should be taken into account while drawing the ray diagram.

(i) At P, the ray travels from a rarer to a denser medium. Hence, it will bend toward normal PC. At M, it travels from a denser to a rarer medium and hence moves away from normal MC.

(ii) PM ray when extended backward meets at I_1, and MN ray when extended meets at I_2.

7.1 Lateral Magnification

The lateral magnification may be obtained with the help of the adjacent Fig. 16.58, where two rays from the tip of an object of height h_0 meet at the corresponding point on an image of height h_1. One ray passes through the center of curvature of the spherical surface so that its direction is unchanged. The path of the second ray is obtained from the Snell's law. With the paraxial approximation,

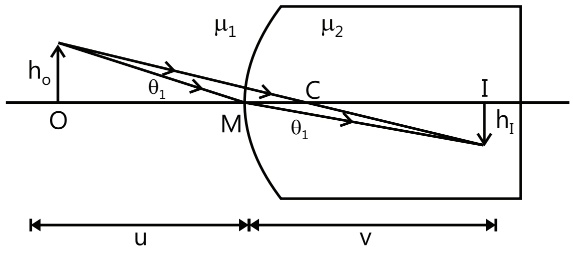


Figure 16.58

$$\sin\theta_1 \approx \frac{h_0}{u} \quad \text{and} \quad \sin\theta_2 \approx \frac{h_i}{v}.$$

Combining these equations with the Snell's law, then,

$$\mu_1\left(\frac{h_0}{u}\right) = \mu_2\left(\frac{h_i}{v}\right) \quad \text{or} \quad \frac{h_i}{h_0} = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right).$$

The lateral magnification *m* is the ratio of the image height to the object height or $\frac{h_i}{h_0}$.

We, therefore, obtain $\left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right)$ i.e. $\frac{h_i}{h_o} = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right)$

PLANCESS CONCEPTS

Here, *v* is +ve, *u* is −ve, h_i is −ve, h_0 is +ve (the distances measured above the axis are positive). So, if we put these sign conventions, in Eq. (vi), we obtain the same result $m = \frac{\mu_1}{\mu_2}\frac{v}{u}$.

Chinmay S Purandare (JEE 2012 AIR 698)

Illustration 21: Find the size of the image formed in the situation shown in the figure. **(JEE MAIN)**

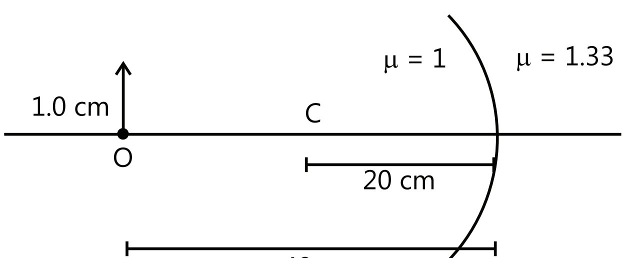


Figure 16.59

Sol: Here as the object is placed left to the pole beyond center of the curvature, then according to sign convention the distance of object is considered to be negative.

The distance of image is found by $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Here, u=−40cm, R=−20cm, $\mu_1 = 1$, $\mu_2 = 1.33$.

We have $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\Rightarrow \frac{1.33}{v} - \frac{1}{-40cm} = \frac{1.33-1}{-20cm} \Rightarrow \frac{1.33}{v} = -\frac{1}{40cm} - \frac{0.33}{20cm}$$

$$\Rightarrow v = -32cm.$$

The magnification is $m = \frac{h_2}{h_1} = \frac{\mu_1 v}{\mu_2 u}$ or $\frac{h_2}{1.0cm} = \frac{-32cm}{1.33x(-40cm)}$ $\Rightarrow h_2 = +6.0$ cm. .

The image is erect.

7.2 Refraction through Lenses

A lens is made of a transparent material with two refracting surfaces such that at least one of these is a curved one. The convex lens is thicker in the middle, and the concave lens is thinner in the middle.

The plano-convex and plano-concave lenses have one plane surface, and the other surfaces are convex and concave, respectively. Different types of typical lenses are shown in the Fig. 16.60.

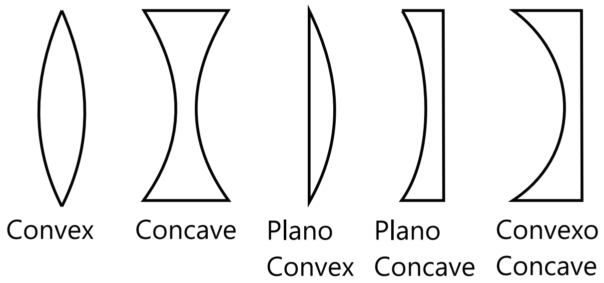


Figure 16.60

7.2.1 Parameters Associated with Lens

The center of curvature C is the center of the sphere, of which the curved surface of the lens is a part. The radius of curvature of either surface of the lens is the radius of the sphere, of which the curved surface is a part.

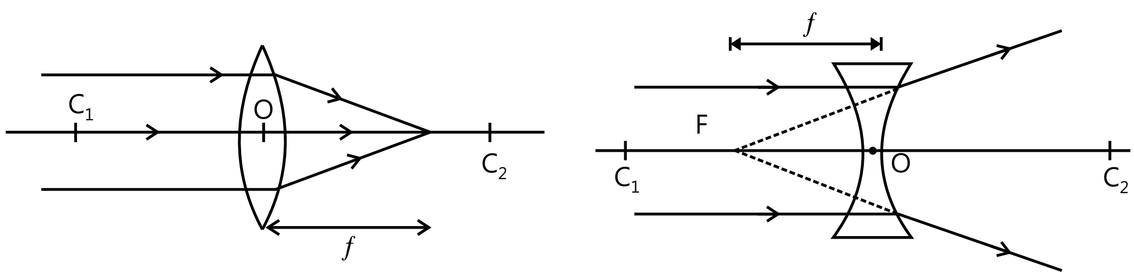


Figure 16.61

The optical center, O, of a lens is a point through which the ray does not get deviated. The principal axis is a line passing through the center (s) of curvature and the optical center. If the object is at infinity, the image is formed at the principal focus and vice versa. The focal length is the distance between the optical center of a lens and a point on which a parallel beam of light converges or appears to converge. The aperture of a lens is the effective diameter of its light-transmitting area. The intensity of the image is directly proportional to the square of the aperture.

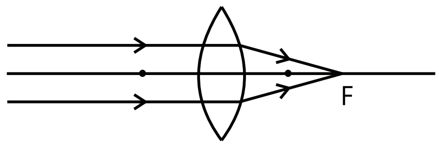
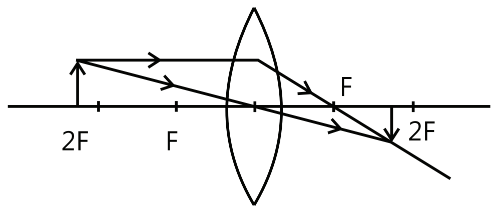
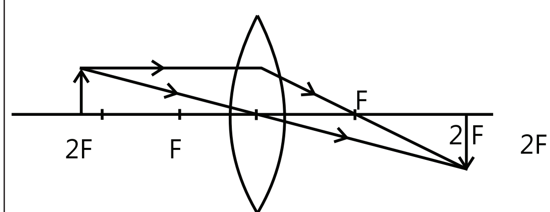
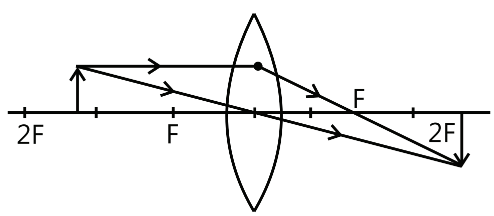
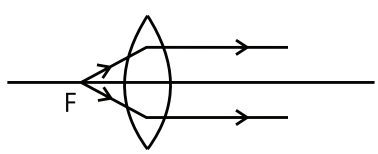
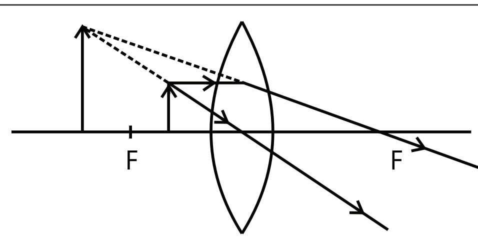
7.2.2 Rules of Image Formation

(a) A ray that passes through the optical center does not get deviated through the lens.

(b) A ray incident parallel to the principal axis after refraction through the lens either passes through the focus or appears to pass through the focus after extrapolation.

(c) A ray that passes through the focus or directed toward it becomes parallel to the principal axis after refraction. The rays of light from a point of object intersect or appear to intersect after refraction through the lens and form an image. If the rays actually intersect, then the image is real, and if the rays appear to intersect, then the image is virtual.

7.3 Image Formation by Convex Lens

S. No	Position of object	Diagram	Position of image	Nature of image
1	Infinity	 Figure 16.62	Principal focus	Real, inverted and diminished
2	Between 2F and ∞	 Figure 16.63	Between F and 2F	Real, inverted and diminished
3	2F	 Figure 16.64	2F	Real, inverted and of same size as the object
4	Between F and 2F	 Figure 16.65	Beyond 2F	Real, inverted and magnified
5	F	 Figure 16.66	Infinity	Real, inverted and highly magnified
6	Between F and optical center	 Figure 16.67	Same side as the optical center	Virtual, erect and magnified

A lens has two foci, which is not a case in a mirror.

First focus (F_1): If an object (real in case of a convex, virtual for concave) is placed at the first focus (F_1), the image of this object is formed at infinity, or we can say through F_1 it becomes parallel to the principal axis after refraction from the lens. The distance from the first focal length is f_1.

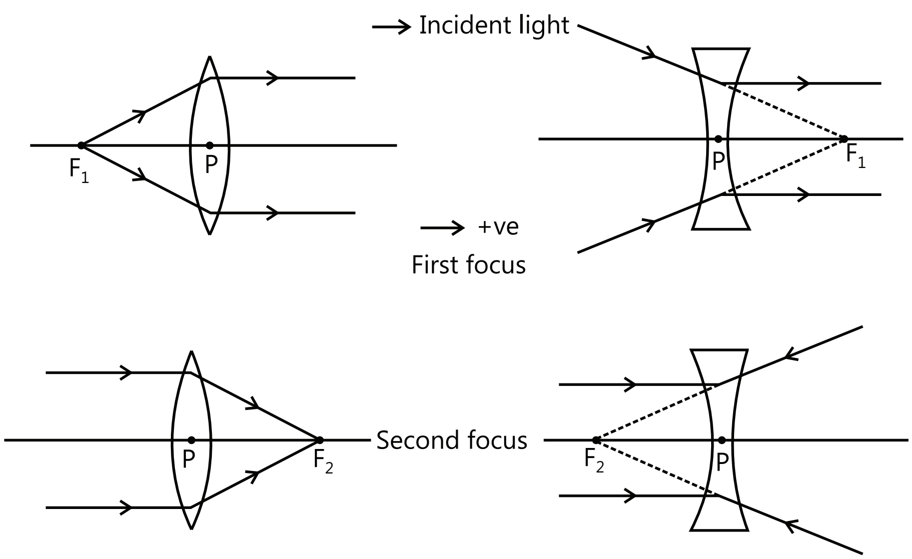


Figure 16.68

Second focus or principal focus (F_2): A narrow beam of light that travels parallel to the principal axis either converge (in case of a convex lens) or diverge (in case of a concave lens) at a refraction (*r*) from the lens. This point F_2 is called the second or principal focus. If the rays converge, the lens is a converging lens, and if the rays diverge, then the lens is a diverging lens. It can be seen from the Fig. 16.68 that f_1 is negative for a convex lens and positive for a concave lens. But f_2 is positive for a convex lens and negative for a concave lens.

$|f_1| = |f_2|$ if the media on the two sides of a thin lens have the same refractive index.

We mainly concern with the second focus f_2. Thus, wherever we write the focal length f, it means the second or principal focal length. Therefore, $f = f_2$ and, hence, f is positive for a convex lens and negative for a concave lens.

7.4 Image Formation by a Concave Lens

An image I formed by a concave lens of a real object O occurred beyond F and the optical center is virtual, erect and diminished. The image is formed at F if the object is at infinity.

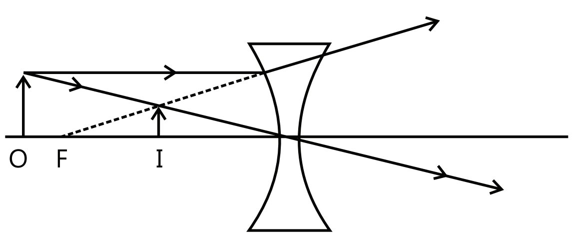


Figure 16.69

7.5 Lens Formulae

If an object is in a medium of refractive index μ_1 at a distance *u* from the optical center of a lens having radii of curvature R_1 and R_2 and of refractive index μ_2, its image is formed at a distance *v* from the optical center, then

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

If is the focal length of the lens, then $$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

- If μ is the refractive index of the material the lens is made of with respect to the surrounding medium air, then $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$.
- If I and O are the lateral or transverse size of the image and object, respectively, the magnification *m* is given by, $m=\frac{I}{O}=\frac{v}{u}$.
- The power *P* of the lens is given by $P=\frac{1}{f(m)}=\frac{100}{f(cm)}$ dioptre.

PLANCESS CONCEPTS

Suppose *m* is positive, it implies *v* and *u* are of same sign, i.e. the object and its image are on the same side (left side), which implies that the image of a real object is virtual. Thus, *m*=+2; it implies that the image is virtual, erect and magnified to two times the actual size, and $|v|=2|u|$. Similarly, $m=-\frac{1}{2}$ implies that the image is real, inverted and diminished, and $|v|=\frac{1}{2}|u|$.

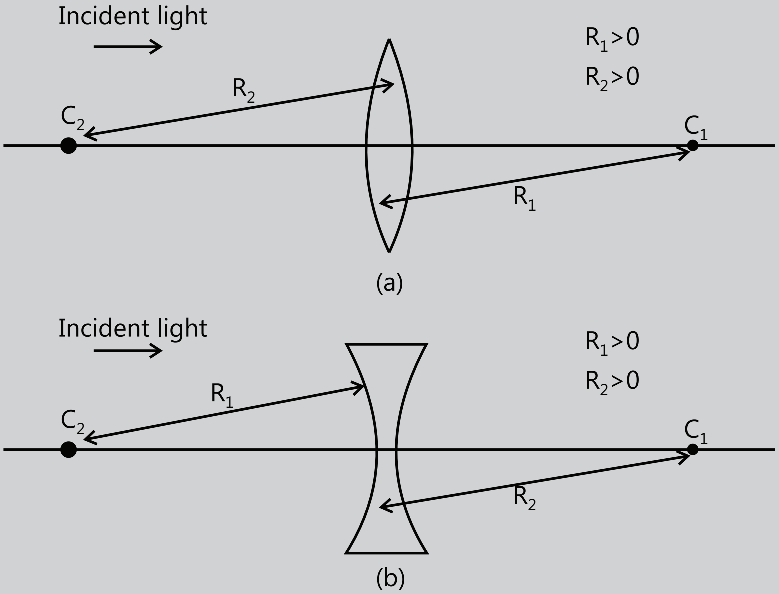


Figure 16.70

For a converging lens, R_1 is positive and R_2 is negative. Therefore, $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ is positive, and if the lens is placed in air, the value of $(\mu-1)$ is also positive. Hence, the focal length f of a converging lens is positive. For a diverging lens, however, R_1 is negative and R_2 is positive at the focal length f.

The focal length of a mirror $\left(f_M=\frac{R}{2}\right)$ depends only on the radius of curvature R, while that of a lens depends on μ_1,μ_2,R_1 and R_2. Thus, when a lens and a mirror both are immersed in a liquid, the focal length lens changes, whereas that of the mirror remains unchanged.

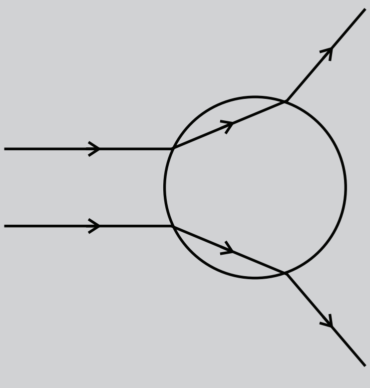


Figure 16.71

Suppose $\mu_2<\mu_1$, i.e. the refractive index of the medium (in which the lens is placed) is more than the refractive index of the material the lens is made of, then $\left(\frac{\mu_2}{\mu_1}-1\right)$ becomes negative, i.e. the lens' behavior changes. A converging lens behaves as a diverging lens and vice versa. An air bubble in water seems to be a convex lens, but behaves as a concave (diverging) lens.

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Illustration 22: The focal length of a convex lens in air is 10 cm. Find its focal length in water. Assume $\mu_g = 3/2$ and $\mu_w = 4/3$ **(JEE MAIN)**

Sol: The focal length of lens is given by $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \left({}_1\mu^2 - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

$$\frac{1}{f_{air}} = \left(\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{... (i)}$$

and $$\frac{1}{f_{water}} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{... (ii)}$$

Dividing Eq. (i) by Eq. (ii), we get $\frac{f_{water}}{f_{air}} = \frac{\left(\mu_g - 1\right)}{\left(\mu_g / \mu_w - 1\right)}$

Substituting the values, we get $f_{water} = \frac{\left(3/2 - 1\right)}{\left(\frac{3/2}{4/3} - 1\right)} f_{air}$; $\quad = 4 f_{air} = 4x10 = 40cm$.

Illustration 23: An image I of point object O is formed by a lens whose optical axis is AB as shown in the Fig. 16.72.

(a) State whether it is a convex or a concave lens?

(b) Draw a ray diagram to locate the lens and its focus.

(JEE ADVANCED)

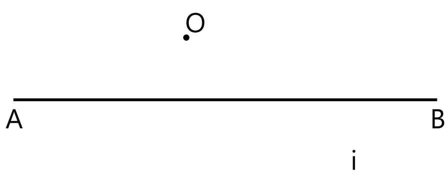


Figure 16.72

Sol: For the convex lens, the image of the object is real, formed on the opposite site and is always inverted while for concave lens the image of an object is always virtual, erect and on the same side of object.

(a) (i) A concave lens always forms an erect image. The given image I is on the other side of the optical axis. Hence, the lens is convex.

(ii) Join O with I. Line OI cuts the optical axis AB at pole P of the lens. The dotted line shows the position of the lens.

Then, draw a line parallel to AB from point O. It cuts the dotted line at M and join M with I. Line MI cuts the optical axis at focus (F) of the lens.

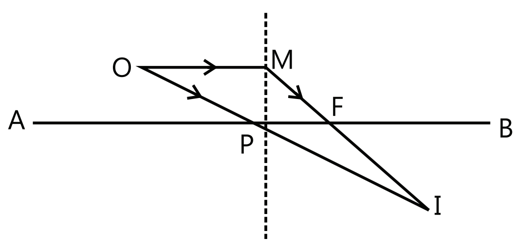


Figure 16.73

Illustration 24: Find the distance of an object from a convex lens if image is magnified two times the actual size. The focal length of the lens is 10 cm. **(JEE MAIN)**

Sol: For the convex lens, the position of the object from the lens is given by $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

A convex lens forms both type of images, i.e. real and virtual. Since the type of the image is not mentioned here, we have to consider both the cases.

When the image is real: In this case, *v* is positive and *u* is negative with $|v| = 2|u|$.

Thus, if $u = -x$ then $v = 2x$ and $f = 10$ cm.

Substituting in $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, We get $\frac{1}{2x}+\frac{1}{x}=\frac{1}{10}$ or $\frac{3}{2x}=\frac{1}{10}$

$\therefore$ x=15cm

x =15 cm; it implies that the object lies between F and 2F.

When the image is virtual: In this case, v and u both are negative. So let, $u=-y$ then $v=-2y$ and $f=10\text{cm}$.

Substituting in $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ We get, $\frac{1}{-2y}+\frac{1}{y}=\frac{1}{10}$

or $\frac{1}{2y}=\frac{1}{10}$; $\therefore$ y=5cm.

y=5 cm; it implies that the object lies between F and P.

7.6 Displacement Method to Determine the Focal Length of a Convex Lens

If the distance d between an object and a screen is greater than four times the focal length of a convex lens, then there are two possible positions of the lens between the object and the screen at which a sharp image of the object is formed on the screen. This method is called displacement method and is used in laboratory to determine the focal length of a convex lens.

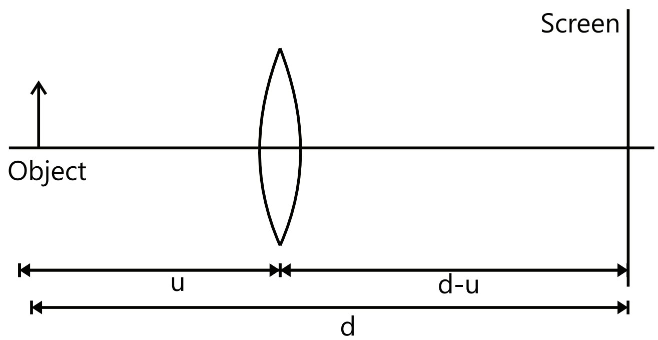


Figure 16.74

To prove this, let us take an object that is placed at a distance u from a convex lens of a focal length f. The distance between the image and the lens v = (d−u).

From the lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$.

We have $\frac{1}{d-u}-\frac{1}{-u}=\frac{1}{f} \Rightarrow u^2-du+df=0$

$$\therefore\ u=\frac{d\pm\sqrt{d(d-4f)}}{2}$$

Now, there are following possibilities:

(i) If $d<4f$, then u is imaginary.

(ii) If $d=4f$, then $u=\frac{d}{2}=2f$. Hence, there is only one possible position and the minimum distance between an object and its real image in case of a convex lens is $4f$.

(iii) If $d>4f$, there are two possible positions of lens at distances $\frac{d+\sqrt{d(d-4f)}}{2}$ and $\frac{d-\sqrt{d(d-4f)}}{2}$, for which an real image is formed on the screen.

(iv) If I_1 is the image length in the first position of the object and I_2 is the image length in the second position, then the object length O is given by $O=\sqrt{I_1I_2}$.

Proof:

$$|u_1|=\frac{d+\sqrt{d(d-4f)}}{2} \qquad \therefore |v_1|=d-|u_1|=\frac{d-\sqrt{d(d-4f)}}{2}$$

$$\left|u_2\right| = \frac{d - \sqrt{d(d-4f)}}{2} \qquad \therefore \left|v_1\right| = d - \left|u_2\right| = \frac{d + \sqrt{d(d-4f)}}{2}$$

Now $\quad \left|m_1 m_2\right| = \frac{I_1}{O} x \frac{I_2}{O} = \frac{\left|v_1\right|}{\left|u_1\right|} x \frac{\left|v_2\right|}{\left|u_2\right|}$

Substituting the values, we get $\quad \frac{I_1 I_2}{O_2} = 1 \qquad$ or $\quad O = \sqrt{I_1 I_2}$

7.7 Lenses in Contact

If two thin lenses of focal lengths f_1 and f_2 are placed in contact, the equivalent focal length *F* of this combination is given by $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$.

If one surface of the lens is coated with silver, the effective focal length *F* of the combination is given by

$\frac{1}{F} = \frac{2}{f_l} + \frac{1}{f_m}$, where f_l is the focal length of the lens, and f_m is the focal length of the silvered surface.

When two lenses of focal lengths f_1 and f_2 are kept distance *d* apart from each other, the focal length of this combination is given by $\frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_2} - \frac{d}{f_l f_2}$

If there is a medium of refractive index μ between the lenses, the equivalent focal length *F* is $\frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_2} - \frac{d/\mu}{f_l f_2}$.

Note: When more than two lenses in contact, the equivalent focal length is given by the formula, $\frac{1}{F} = \sum_{i=1}^{n} \frac{1}{f_i}$

Here, $f_{1,}\ f_2$, ... should be substituted with their respective signs.

Important Observation: To find the position of an image when one face of a lens is coated with silver.

The given system finally behaves as mirror, whose focal length is given by

$1/v + 1/u = 1/f$.

$$\frac{1}{f} = \frac{2(\mu_2/\mu_1)}{R_2} - \frac{2(\mu_2/\mu_1 - 1)}{R_1}.$$

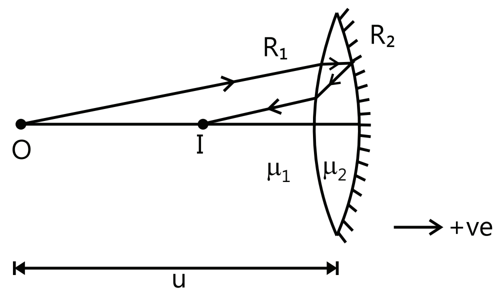


Figure 16.75

PLANCESS CONCEPTS

A lens made of three different materials has three focal lengths. Thus, for a given object, there are three images.

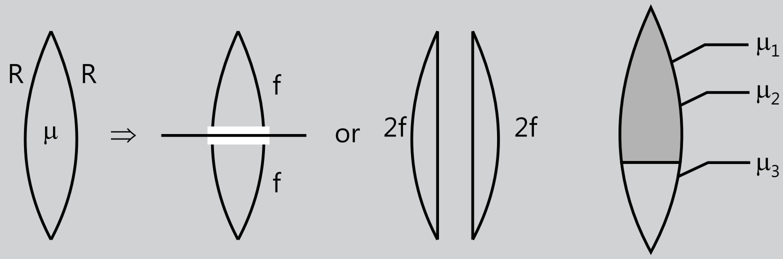


Figure 16.76

PLANCESS CONCEPTS

Memory zone

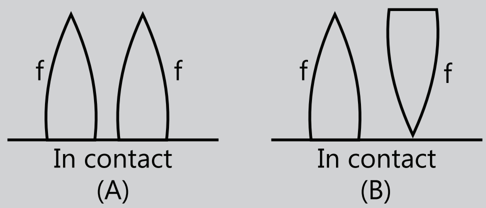


Figure 16.77

(A) The resultant focal length in this case is $\frac{f}{2}$.

(B) The resultant focal length in this case is ∞. This is because the optical axes of both parts have been inverted.

Anurag Saraf (JEE 2011 AIR 226)

Illustration 25: A double concave lens made up of glass of refractive index 1.6 has radii of curvature of 40 cm and 60 cm. Calculate its focal length in air. **(JEE MAIN)**

Sol: For double concave lens, the focal length is given by $\frac{1}{f} = \left({}^{\alpha}\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ where ${}^{\alpha}\mu_g$ = n is refractive index of glass with respect to air.

Using the relation $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. Here $R_1 = -40\,cm$, $R_2 = +60\,cm$ and $x = 1.6$.

We get $\frac{1}{f} = (1.6-1)\left(\frac{1}{-40} - \frac{1}{60}\right) = -0.6\left(\frac{60+40}{60x40}\right) = -\frac{1}{40}$

i.e. $f = -40\,cm$.

Illustration 26: A biconvex lens has a focal length $\frac{2}{3}$ times the radius of curvature of either surface. Calculate refractive index of lens material. **(JEE MAIN)**

Sol: The biconvex or double convex lens, we can find the refractive index of material using $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

Here, $f = \frac{2}{3}R$ $\quad R_1 = R$ and $R_2 = R$.

Using $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$, we get $\frac{3}{2R} = (n-1)\left(\frac{1}{R} + \frac{1}{R}\right) = \frac{2(n-1)}{R}$

$\Rightarrow \quad (n-1) = \frac{3}{4}$ or $n = \frac{3}{4} + 1 = \frac{7}{4} = 1.75$.

Illustration 27: A glass convex lens has a power of 10.0 D. When this lens is fully immersed in a liquid, it acts a concave lens of focal length 50 cm. Calculate the refractive index of the liquid (Assume ${}^{a}\mu_g = 1.5$). **(JEE ADVANCED)**

Sol: Power of convex lens is given by $P = \frac{100}{f}$ where $\frac{1}{f} = \left({}^a\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. When lens is immersed inside the liquid, it now behaves as concave lens thus the focal length of the lens is given as $\frac{1}{f} = \left(\frac{{}^a\mu_g}{{}^a\mu_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$P = \frac{100}{f(\text{in cm})}$ $\qquad \therefore f = \frac{100}{10} = 10\,\text{cm}$.

Now $\frac{1}{f} = \left({}^a\mu_g - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$; $\qquad \frac{1}{10} = 0.5\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\therefore \quad \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{5} \qquad \text{... (i)}$$

When fully immersed in a liquid, $f = -50\,\text{cm}$

$$\therefore \quad -\frac{1}{50} = \left(\frac{{}^a\mu_g}{{}^a\mu_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{{}^a\mu_g}{{}^a\mu_l} - 1\right) x \frac{1}{5}$$

$$\therefore \quad \frac{{}^a\mu_g}{{}^a\mu_l} - 1 = -\frac{1}{10}; \quad \text{or} \quad \frac{{}^a\mu_g}{{}^a\mu_l} = -\frac{1}{10} + 1 = \frac{9}{10} \quad \text{or} \quad {}^a\mu_l = \frac{10}{9} \times {}^a\mu_g = \frac{10}{9} \times 1.5 = 1.67$$

Illustration 28: A thin plano-convex lens of focal length f is split into two halves. One of the halves is shifted along the optical axis as shown in the Fig. 16.78. The separation between the object and image planes is 1.8 m. The magnification of the image formed by one of the half lens is 2. Find the focal length of the lens and the distance between the two halves. Draw the ray diagram of image formation. **(JEE ADVANCED)**

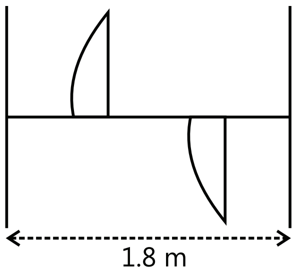


Figure 16.78

Sol: When the lens is cut in two halves along principle axis, the focal length of both the halves is equal as the original len.

For both the halves, the position of the object and the image is same. There is difference only in magnification. Magnification for one of the halves is given as 2 (>1). This is the first one, because $|v| > |u|$. So, for the first half,

$|v/u| = 2 \qquad \text{or} \qquad |v| = 2|u|$

Let $u = -x$, $\qquad$ then $\qquad v = +2x$

and $|u| + |v| = 1.8\,\text{m}$.

i.e. $3x = 1.8\,\text{m}$ $\qquad$ or $\qquad x = 0.6\,\text{m}$

Hence, $u = -0.6\,\text{m}$ $\qquad$ and $\qquad v = +1.2\,\text{m}$

Using $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{1.2} - \frac{1}{-0.6} = \frac{1}{0.4}$ $\qquad f = 0.4\,\text{m}$

For the second half, $\frac{1}{f} = \frac{1}{1.2 - d} - \frac{1}{-(0.6 + d)}$

or $\frac{1}{0.4} = \frac{1}{1.2 - d} + \frac{1}{(0.6 + d)}$.

By solving this, we get $\qquad d = 0.6\,\text{m}$

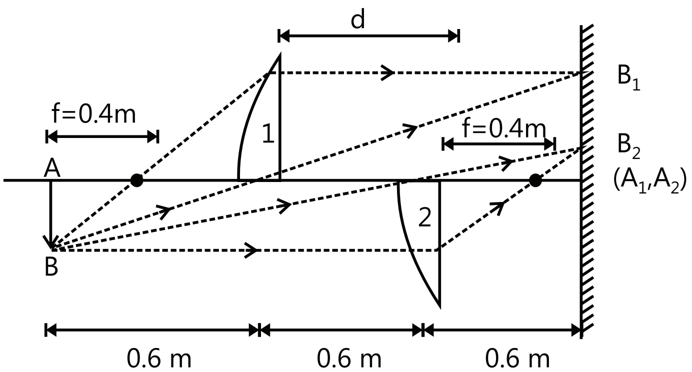


Figure 16.79

Magnification for the second half will be $m_2 = \frac{v}{u} = \frac{0.6}{-(1.2)} = -\frac{1}{2}$,

and for the first half is $m_1 = \frac{v}{u} = \frac{1.2}{-(0.6)} = -2$

The ray diagram is shown in the Fig. 16.79.

Illustration 29: A converging lens of focal length 5.0 cm is placed in contact with a diverging lens of focal length 10.0 cm. Find the combined focal length of the system. **(JEE MAIN)**

Sol: The focal length of the combination of lenses is given by $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

Here, $f_1 = +5.0\,\text{cm}$ and $f_2 = -10.0\,\text{cm}$

Therefore, the combined focal length *F* is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{5.0} - \frac{1}{10.0} = +\frac{1}{10.0}; \qquad \therefore \qquad F = +10.0\,\text{cm}$$

i.e. this combination behaves as a converging lens of focal length 10.0 cm.

7.8 Power of an Optical Instrument

By the optical power of an instrument (whether it is a lens, mirror or a refractive surface), we measure the ability of the instrument to deviate the path of rays passing through it. If the instrument converges the rays parallel to the principal axis, its power is positive, and if it diverges the rays, it is a negative power.

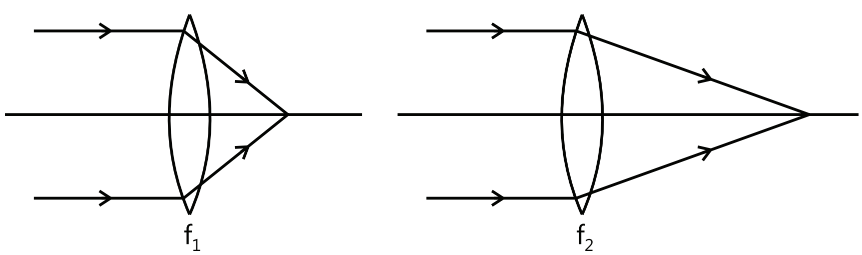
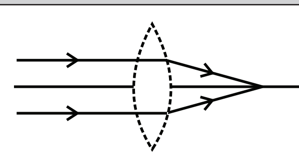


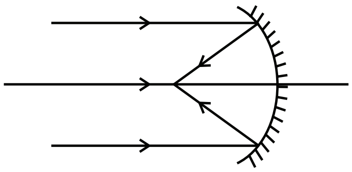
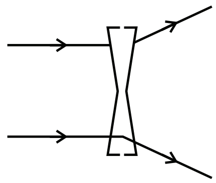
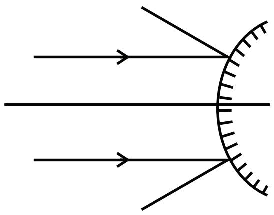
Figure 16.80

The shorter the focal length of a lens (or a mirror), the more it converges or diverges light. As shown in the Fig. 16.80, $f_1 < f_2$, and hence, the power $P_1 > P_2$, because bending of the light ray in case 1 is more than that in case 2. For a lens,

$P\,(\text{in dioptre}) = \frac{1}{f\,(\text{metre})}$, and for a mirror, $P\,(\text{in dioptre}) = \frac{-1}{f\,(\text{metre})}$

At a glance:

Nature of lens/ mirror	**Focal length (f)**	**Power** $P_L = \frac{1}{f}, P_M = -\frac{1}{f}$	**Converging/ diverging**	**Ray diagram**
Convex lens	+ve	+ve	Converging	 **Figure 16.81**

Nature of lens/ mirror	Focal length (f)	Power $P_L = \frac{1}{f}, P_M = -\frac{1}{f}$	Converging/ diverging	Ray diagram
Concave mirror	−ve	+ve	Converging	 Figure 16.82
Concave lens	−ve	−ve	Diverging	 Figure 16.83
Convex mirror	+ve	−ve	diverging	 Figure 16.84

PLANCESS CONCEPTS

Both convex lens and concave mirror have positive power and are converging in nature, whereas both concave lens and convex mirror have negative power and are diverging in nature.

Vaibhav Krishnan (JEE 2009 AIR 22)

Illustration 30: A spherical convex surface separates an object and an image space of refractive index 1.0 and $\frac{4}{3}$. If the radius of curvature of the surface is 10 cm, find its power. **(JEE ADVANCED)**

Sol: For the convex lens power of magnification is given by $P = \frac{1}{f}$

where $\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

We have to find where do the parallel rays converge (or diverge) on the principal axis and call it the focus and the corresponding length is the focal length.

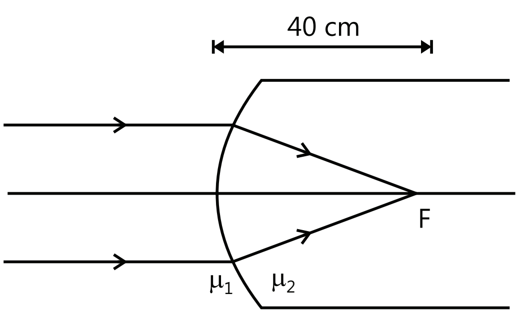


Figure 16.85

Using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$.

With proper values and signs, we have $\frac{4/3}{f} - \frac{1.0}{\infty} = \frac{4/3 - 1.0}{+10}$; or $f = 40\text{cm} = 0.4\text{m}$

Since, the rays are converging, its power should be positive. Hence,

$$P\ (\text{in dioptre}) = \frac{+1}{f\ (\text{metre})} = \frac{1}{0.4}; \qquad \text{or} \quad P = 2.5 \text{ dioptre}$$

Illustration 31: Two lenses of focal length 20 cm and −25cm are placed in contact. Find the total power of this combination. **(JEE MAIN)**

Sol: The power of combination of lenses is given by $P = P_1 + P_2 = \frac{1}{f_1} + \frac{1}{f_2}$.

$$f_1 = 20\text{cm}, \quad f_2 = -25\text{cm}; \qquad \therefore\ P_1 = \frac{100}{f_1} = \frac{100}{20} = 5\text{D}$$

$$P_2 = \frac{100}{-25} = -4\text{D}; \qquad \therefore\ P = P_1 + P_2 = 5 - 4 = 1.0\text{ D}$$

8. PRISM

The figure shows the cross section of a prism. AB and AC represent the refracting surfaces. The angle BAC is the angle of the prism. Assume that the prism is placed in air. A ray PQ, incident on a refracting surface AB, gets refracted along QR. The angle of incidence and the angle of refraction are *i* and *r*, respectively. The ray QR is incident on the surface AC. Here, the light travels from an optically denser medium to an optically rarer medium. If the angle of incidence *r'* is not greater than the critical angle, then the ray is refracted in air along RS. The angle of refraction is i'. The angle i' is also called the angle of emergence. If the prism were not present, the incident ray would have passed un-deviated along PQTU. Due to the presence of the prism, the final ray travels along RS. The angle UTS= δ is called angle of deviation. From triangle TQR,

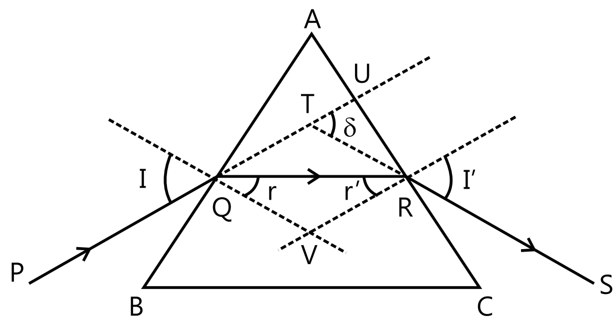


Figure 16.86

$$\angle UTS = \angle TQR + \angle TRQ$$

or $$\delta = (\angle TQV - \angle RQV) + (\angle TRV - \angle QRV)$$

$$= (i - r) + (i' - r') \qquad = (i + i') - (r + r') \qquad \text{.... (i)}$$

Now, the sum of four angles of the quadrangle AQVR is 360°. The angles AQV and ARV both are 90°. Thus,

$$A + \angle QVR = 180°.$$

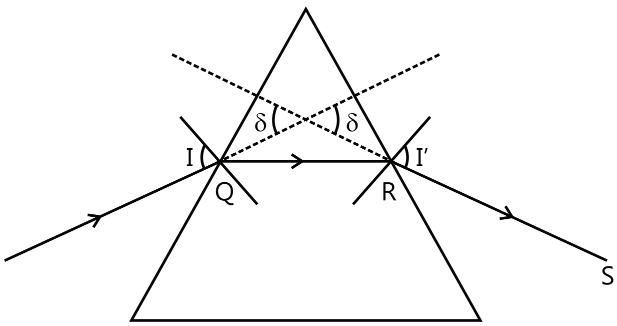
Also, from the triangle QRV, $r + r' + \angle QVR = 180°$

Hence $r + r' = A$

Substituting in Eq. (i) $\delta = i + i' - A$

8.1 Angle of Minimum Deviation

The angle *i'* is determined by the angle of incidence *i*. Thus, the angle of deviation δ is also determined by *i*. For a particular value of *i*, δ is minimum. In this case, the ray passes symmetrically through the prism, so that *i=i'*.



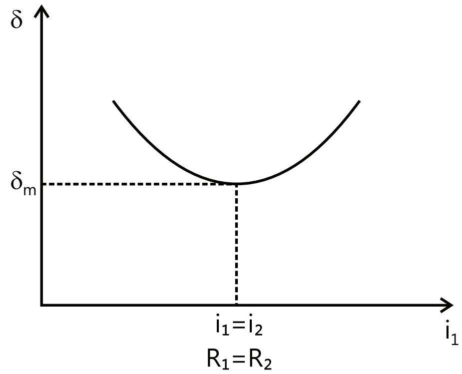


Figure 16.87

8.2 Relation between the Refractive index and the Angle of Minimum Deviation

Let the angle of minimum deviation be δ_m. For a minimum deviation, i=i' and r=r'.

We have $\delta_m = i + i' - A \quad = 2i - A$

or $i = \dfrac{A+\delta_m}{2}$ ... (i)

Also $r + r' = A$; or $r = A/2$ ... (ii)

The refractive index is $\mu = \dfrac{\sin i}{\sin r}$.

Using (i) and (ii) $\mu = \dfrac{\sin\dfrac{A+\delta_m}{2}}{\sin\dfrac{A}{2}}$.

If the angle of prism A is small, δ_m is also small. Then, the equation becomes

$$\mu = \frac{\dfrac{A+\delta_m}{2}}{\dfrac{A}{2}} \quad \Rightarrow \delta_m = (\mu - 1)A.$$

Illustration 32: The angle of minimum deviation from a prism is 37°. If the angle of prism is 53°, find the refractive index of the material of the prism. **(JEE MAIN)**

Sol: For prism, the refractive angle is given by $\mu = \dfrac{\sin\dfrac{A+\delta_m}{2}}{\sin\dfrac{A}{2}}$

$$\mu = \frac{\sin\dfrac{A+\delta_m}{2}}{\sin\dfrac{A}{2}} = \frac{\sin\dfrac{53^0+37^0}{2}}{\sin\dfrac{53^0}{2}} = \frac{\sin 45^0}{\sin 26.5^0} = 1.58.$$

Illustration 33: A ray of light passes through a glass prism such that the angle of incidence is equal to the angle of emergence. If the angle of emergence is ¾ times the angle of the prism, then calculate the angle of deviation when the angle of prism is 30°. **(JEE MAIN)**

Sol: For prism, we have relation $i+e=A+\delta$, where i and e are angle of incidence and emergence respectively. A is angle of prism and δ is angle of minimum deviation

Given that $i=e$, $e=\left(\frac{3}{4}\right)A$ and $A=30°$.

Using the relation $i+e=A+\delta$, we get $\delta=i+e-A=e+e-A=2e-A$

$=2 \times \left(\frac{3}{4}\right)A-A=0.5A=0.5\times 30^0=15^0$.

8.3 Condition of No Emergence

In this section, we want to find the condition such that a ray of light entering the face AB does not come out of the face AC for any angle i_I, i.e. TIR takes place on AC

$r_1+r_2=A$; $\therefore$ $r_2=A-r_1$

or $(r_2)_{min}=A-(r_1)_{max}$. ... (x)

Now, r_1 will be maximum when i_I is maximum, and the maximum value of i_I is 90°

Hence, $\mu=\frac{\sin(i_I)_{max}}{\sin(r_I)_{max}}=\frac{\sin 90^0}{\sin(r_1)_{max}}$

$$\operatorname{sir}(r_I)_{max}=\frac{1}{\mu}=\sin\theta_c\ ;\ \therefore \qquad (r_I)_{max}=\theta_c.$$

From Eq. (x) $(r_2)_{min}=A-\theta_c$. ... (xi)

Now, if the minimum value of r_2 is greater than θ_c, then obviously all the values of r_2 will be greater than θ_c and TIR will take place under all the conditions. Thus, the condition of no emergence is,

$(r_2)_{min}>\theta_c$ or $A-\theta_c>\theta_c$ .... (xii)

or $A>2\theta_c$.

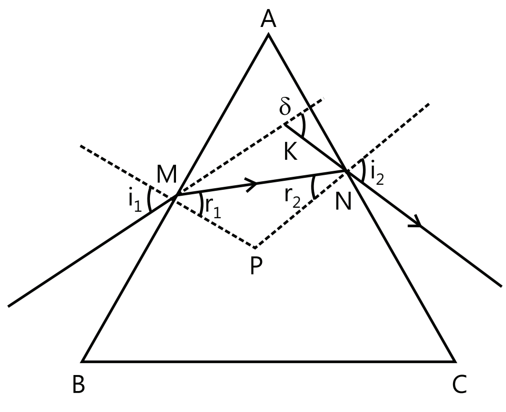


Figure 16.88

PLANCESS CONCEPTS

Equation $r_1 + r_2 = A$ can be applied at any of the three vertices. For example in the Fig. 16.89, $r_1 + r_2 = B$.

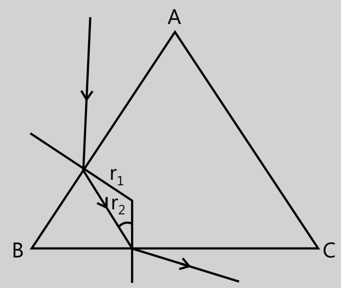


Figure 16.89

Anand K (JEE 2011 AIR 47)

9. DISPERSION OF LIGHT

When a beam of white light that consists of spectrum of various wavelengths ranging from long wavelengths in red color to short wavelengths in violet color passes through a prism, it is split into its constituent colors. This phenomenon is called dispersion. The dispersion of light takes place because the refractive index μ of a medium depends upon the wavelength λ of light according to the Cauchy's relation $\mu \simeq A + \frac{B}{\lambda^2}$, where μ is maximum for violet and minimum for red color.

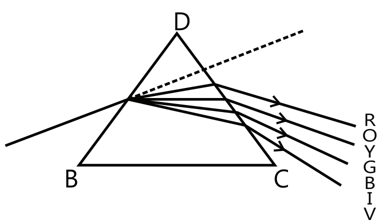


Figure 16.90

9.1 Angle of Dispersion

It is defined as an angle between emerging violet and red color rays, i.e. angle of dispersion is given by $\alpha = \delta v - \delta_R \; ; = A\left(\mu_V - \mu_R\right)$.

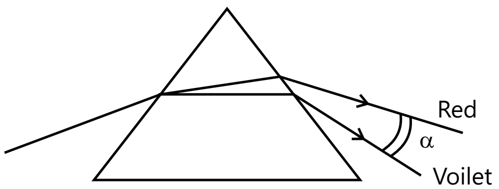


Figure 16.91

9.2 Dispersive Powers

The ratio of the angle of dispersion to the angle of deviation of the mean yellow color, δ, of the ray produced by any prism is called the dispersive power ω of the material of the prism

Dispersive power $= \omega = \dfrac{\delta_v - \delta_R}{\delta}$.

Also, $= \omega = \dfrac{\mu_v - \mu_R}{\mu - 1}$, where μ_v, μ_R and μ are the refractive indices of violet, red and mean yellow colors respectively.

9.3 Deviation without Dispersion

If two suitable prisms of small angle made up of two different transparent materials are combined and placed opposite to each other, then the net dispersion is equal to zero, and these prisms are called an achromatic pair of prisms that produce a deviation of ray of light, but without any dispersion.

For the two prisms of angle *A* and *A'* and refractive indices for violet and red colors μ_v, μR and $\mu_v{}'$ $\mu R'$, respectively, the net dispersion $= \left(\mu_v - \mu_R\right)A + \left(\mu_v{}' - \mu_R{}'\right)A' = 0$

$$A' = -\frac{(\mu_v - \mu_R)A}{(\mu'_v - \mu'_R)}$$

If ω and ω′ are dispersive powers of these prisms, and δ and δ' are their mean deviations, then

$$\frac{(\mu_v - \mu_R)}{(\mu - 1)}(\mu - 1)A + \frac{\mu'_v - \mu'_R}{\mu' - 1}(\mu' - 1)A' = 0$$

$\omega\delta + \omega'\delta' = 0$.

9.4 Dispersion without Deviation

If the deviation produced by the first prism for the mean ray is equal and opposite to that produced by the second prism, the combination of such two prisms produces dispersion without any deviation. For zone net deviation, $\delta + \delta' = 0$.

$(\mu - 1)A + (\mu' - 1)A' = 0$; $A' = -\frac{(\mu - 1)A}{(\mu' - 1)}$, (− sign denotes that the second prism is inverted),

where μ and μ′ are refractive indices for the mean color for prisms of angles *A* and *A*′, respectively.

Note: Most of the problems of prisms can be easily solved by drawing proper ray diagram and then applying laws of geometry with the basic knowledge of prism formulae.

Illustration 34: The angle of a prism is *A*, and its one surface is coated with silver. A light ray falling at an angle of incidence 2*A* on the first surface returns back through the same path after suffering reflection at the second silvered surface. Find the refractive index of material. **(JEE MAIN)**

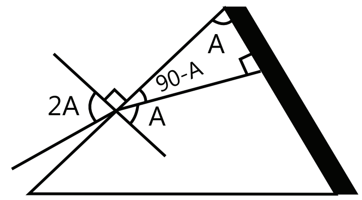


Figure 16.92

Sol: According to Snell's law we have $\mu = \frac{\sin i}{\sin r}$. And the angle of refraction r =90-i

Given i = 2A. From the figure, it can be obtained that *r* = *A*.

$$\therefore \mu = \frac{\sin i}{\sin r} = \frac{\sin 2A}{\sin A} = \frac{2\sin A\cos A}{\sin A} = 2\cos A.$$

Illustration 35: A crown glass prism of angle 5° is to be combined with a glass prism in such a way that the mean ray passes undeviated. Find (i) the angle of the flint glass prism needed and (ii) the angular dispersion produced by the combination when white light passes through it. Refractive indices for red, yellow and violet color light are 1.514, 1.523, respectively, for crown glass and 1.613, 1.620 and 1.632, respectively, for flint glass. **(JEE ADVANCED)**

Sol: For the angle of minimum deviation, we have the relation $\delta = (\mu - 1)A$. As prism of two different materials are joined together, the ratio $\frac{\delta'}{\delta}$ gives the angle of minimum deviation for flint glass is obtained.

The deviation produced by the crown prism is $\delta = (\mu - 1)A$, and by the flint prism is $\delta' = (\mu' - 1)A'$.

The prisms are placed at their inverted angles with respect to each other. The deviations are also in opposite directions. Thus, the net deviation is

$$D = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A' \quad \text{... (i)}$$

(i) If the net deviation for the mean ray is zero, $(\mu - 1)A = (\mu' - 1)A'$

or $A' = \frac{(\mu - 1)}{(\mu' - 1)}A = \frac{1.517 - 1}{1.620 - 1} x 5^0 = 4.2^0$.

(ii) The angular dispersion produced by the crown prism is $\delta_v - \delta_r = (\mu_v - \mu_r)A$, and that by the flint prism is $\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A'$.

The net angular dispersion is

$$\delta = (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A' = (1.523 - 1.514)x5^0 - (1.632 - 1.613)x4.2^0 = -0.0348^0.$$

The angular dispersion is 0.0348^0.

Illustration 36: An isosceles glass prism has one of its faces coated with silver. A ray of light is incident normally on the other face (which has an equal size to the silvered face). The ray of light is reflected twice on the same-sized faces and then emerges through the base of the prism perpendicularly. Find the angles of prism.

(JEE ADVANCED)

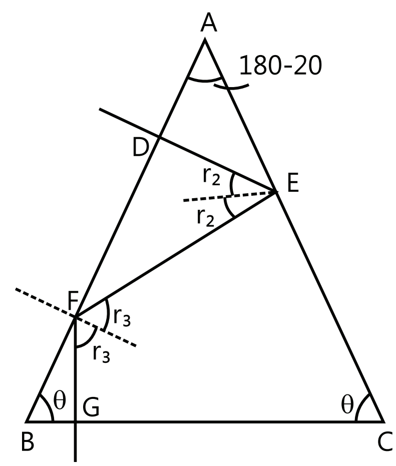


Figure 16.93

Sol: The angles of prism add up to 180°.

$r_1 = 0 \quad \therefore r_2 = A = 180^0 - 2\theta$ ... (i)

$\angle DFE = 180^0 - 90^0 - 2r_2 = 180^0 - 90^0 - 360^0 + 4\theta = 4\theta - 270^0$ ... (ii)

$\therefore \quad r_3 = 90^0 - \angle DFE = 360^0 - 4\theta$ ... (iii)

$\angle BFG = 90^0 - \theta = 90^0 - r_3$; or $r_3 = \theta$ ... (iv)

From Eqs (iii) and (iv),

$5\theta = 360^0$; $\quad \therefore \theta = 72^0$ and $180^0 - 2\theta = 36^0$.

$\therefore$ Angles of prism are 72°, 72° and 36°.

10. DEFECTS OF IMAGES

The relations developed regarding the formation of images by lenses and mirrors are approximate, which do not produce focused, perfect image of objects. The imperfections or defects in such images are called aberrations. If the defect in image is due to such approximations, these can produce spherical aberration apart from coma, distortion, astigmatism and so on. If the defect in image is due to the dispersion of white light into constituent colors similar to that in prism, it is called chromatic aberration producing blurred colored image of the object. The spherical and chromatic aberrations are briefly described in the following subsections:

10.1 Spherical Aberration

This aberration is produced due to the spherical nature of a lens or a mirror. The rays of light from point object when incident near the principal axis called paraxial rays are focused at a larger distance at point I_M, and the rays incident near the periphery or margin of the lens are focused near the lens at I_P as shown in the Fig. 16.94. It gives a rise to a defocused long image of a point object. The spherical aberration can be reduced by any one of the following methods:

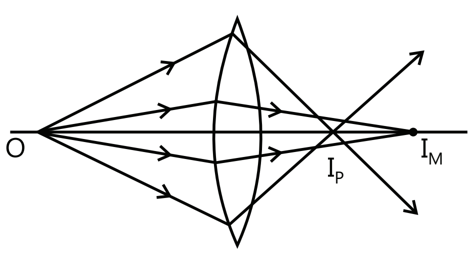


Figure 16.94

(a) Using two thin convex lenses of focal lengths f_1 and f_2 separated at a distance $d = f_1 - f_2$.

(b) Using a plano-convex lens with its convex surface facing the incident or emergent light, whichever is more parallel.

(c) Either using a lens of large focal length or using a specially designed aplanatic lens, crossed lens or parabolic reflectors.

10.2 Chromatic Aberration

As the refractive index for different wavelengths or colors comprising white or any composite light is different, the image of an object illuminated by white light is colored due to the chromatic aberration. Such a defect can by removed by using an achromatic doublet comprising a convex and a concave lens of focal lengths f_1 and f_2 and dispersive powers ω_1 and ω_2 such that the ratio of their focal lengths is equal to the ratio of their dispersive powers.

$$\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0 \qquad \text{or} \qquad \frac{\omega_1}{\omega_2}=-\frac{f_1}{f_2},$$

where dispersive power $\omega=\frac{\mu_v-\mu_r}{\mu-1}$ and μ_v, μ_r and μ are respective indices for wavelengths of violet, red and mean yellow colors of white light.

11. OPTICAL INSTRUMENTS

Optical instruments are used to assist the eye in viewing an object. Let us first discuss about the human eye and the mechanism through which we see.

11.1 The Eye

The eye has a nearly spherical shape of diameter 1 inch each. Following are some of the terms related to the eye.

(a) **Cornea** – The front portion of the eye is more sharply curved and is covered by a transparent protective membrane called the cornea.

(b) **Aqueous humor** – Behind the cornea, there is a space filled with a liquid called aqueous humor.

(c) **Crystalline lens** – The part just behind aqueous humor is called crystalline lens.

(d) **Iris** – It is the muscular diaphragm between the aqueous humor and lens and is the colored part that we see in the eye.

(e) **Pupil** – The small hole in the iris is called the pupil. Varying aperture of the pupil controls the amount of light entering into the eye with the help of iris.

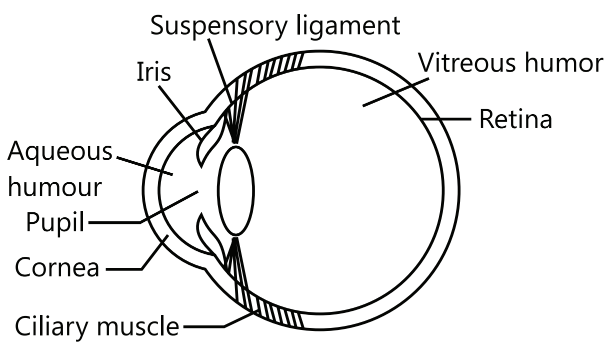


Figure 16.95

(f) **Retina** – This is a screen-like structure on which the eye forms an image. The retina contains rods and cones that receive light signal.

(g) **Accommodation –** When the eye is focused on a distant object, the ciliary muscles are relaxed so that the focal length of the eye lens has its maximum value that is equal to its distance from the retina. The parallel rays that enter into the eyes are focused on the retina, and we see the object clearly. When the eye is focused on a closer object, the ciliary muscles are strained and the focal length of the eye lens decreases. The ciliary muscles adjust the focal length in such a way that the image is again formed on the retina and we see the object clearly. This process of adjusting focal length is called accommodation. However, the muscles cannot be strained beyond a limit, and hence, if the object is brought too close to the eye, the focal length cannot be adjusted to form the image on the retina. Thus, there is a minimum distance for the clear vision of an object.

The nearest point at which the image can be formed on the retina is called the near point of the eye. The distance of the near point from the eye is called the least distance for clear vision. This varies from person to person and with age. At a young age (say below 10 years), the muscles are strong and flexible and can bear more strain. The near point may be as close as 7−8 cm at this age. In old age, the muscles cannot bear more strain and the near point shifts to large values, say 1 to 2 m or even more. We shall discuss about these defects of vision and use of glasses in a later section. The average value of the least distance for clear vision for a normal eye is generally 25 cm.

11.2 Apparent Size

The size of an object is related to the size of the image formed on the retina. A larger image on the retina activates larger number of rods and cones attached to it, and the object looks larger. As it is clear from the Fig. 16.96, if an object is taken away from the eye, the size of the image on the retina decreases, and hence, the same object looks smaller. Furthermore the size of the image on the retina is roughly proportional to the angle subtended by the object on the eye. This angle is called the visual angle, and optical instruments are used to increase this artificially in order to improve the clarity.

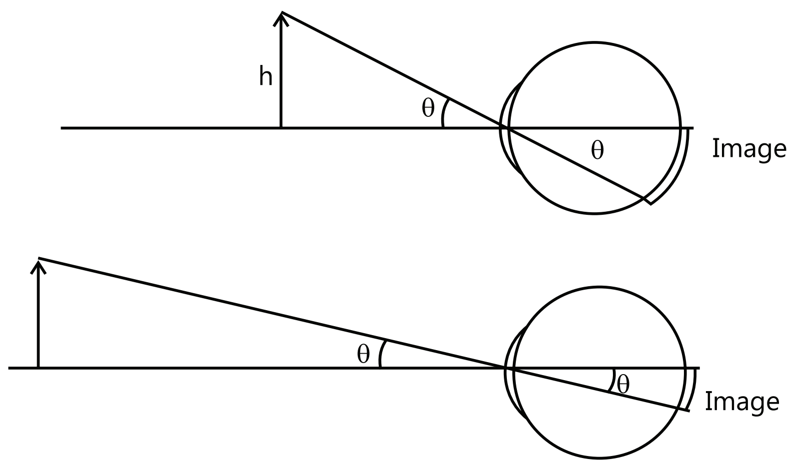


Figure 16.96

Illustration 37: Two boys, the one is 52 inches tall and the other 55 inches tall, are standing at distances 4.0 m and 5.0 m, respectively, from an human eye. Which boy will appear taller? **(JEE MAIN)**

Sol: The angle subtended by any object is given by $\alpha = \dfrac{\text{height of object}}{\text{Distance of object from observer}}$.

The boy which subtends the larger angle will appear taller.

The angle subtended by the first boy on the eye is $\alpha_1 = \dfrac{52\text{inch}}{4.0\text{m}} = 13\text{inch}/\text{m}$.

And the angle subtended by the second boy is $\alpha_2 = \dfrac{52\text{inch}}{5.0\text{m}} = 11\text{inch}/\text{m}$.

As $\alpha_1 > \alpha_2$, the first boy will look taller when seen through the eye.

12. SIMPLE MICROSCOPE

When we view an object with naked eyes, the object must be placed somewhere between infinity and the near point. The angle subtended on the eye is maximum when the object is placed at the near point. This angle is

$$\theta_0 = \frac{h}{D}, \qquad \text{... (i)}$$

where *h* is the size of the object, and *D* is the least distance for clear vision.

This angle can further be increased if a converging lens of short focal length that is called a simple microscope or a magnifier is placed just in front of the eye.

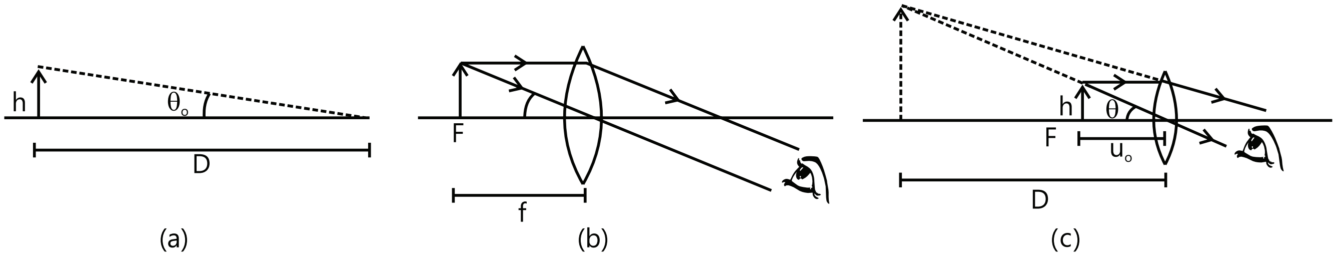


Figure 16.97

Suppose, the lens has a focal length f that is lesser than *D* and let us move the object to the first focal point F. The eye receives the rays that come from infinity. The actual size of the image is infinite, but the angle subtended on the lens (and hence on the eye) is

$$\theta = \frac{h}{f} \qquad \text{... (ii)}$$

As f < D, Eqs (i) and (ii) show that $\theta > \theta_0$. Hence, the eye perceives a larger image than it could have had without the microscope. Because the image is occurred at infinity, the ciliary muscles are least strained to focus the final image on the retina. This is called normal adjustment. The magnifying power of a microscope is θ / θ_0, where θ is the angle subtended by the image on the eye when the microscope is used, and θ_0 is the angle subtended on the naked eye when the object is placed at the near point. This is also known as the angular magnification. Thus, the magnifying power is a factor by which the image on the retina can be enlarged by using the microscope.

In the normal adjustment, the magnifying power of a simple microscope is by Eqs. (i) and (ii),

$$m = \frac{\theta}{\theta_0} = \frac{h/f}{h/D} \quad \text{or} \quad m = \frac{D}{f}$$

If $f < D$, the magnifying power is greater than 1.

The magnifying power can further be increased by moving the object more closer to the lens. Suppose we move the object to a distance u_0 from the lens such that the virtual, erect image is formed at the near point, although the eye is strained, it can still see the image clearly. The distance u_0 is calculated using the lens formula,

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}.$$

Here, $v = -D$ and $u = -u_0$, so that

$$\frac{1}{-u_0} = -\frac{1}{D} - \frac{1}{f}; \qquad \text{or,} \quad \frac{D}{u_0} = 1 + \frac{D}{f} \qquad \text{... (iii)}$$

The angle subtended by the image on the lens (and hence on the eye) is $\theta' = \frac{h}{u_0}$

In this case, the angular magnification or magnifying power is

$$m = \frac{\theta'}{\theta_0} = \frac{h/u_0}{h/D} = \frac{D}{u_0} = 1 + \frac{D}{f}$$

The above equations show that the magnification is high when the focal length f is small. However, due to several other aberrations, the image becomes too defective at a large magnification with simple microscope. Approximately, a magnification up to 4 is trouble-free.

The magnifying power is measured in a unit X; therefore, if a magnifier produces an angular magnification of 10, it is called as 10 X magnifier.

13. COMPOUND MICROSCOPE

The Fig. 16.98 shows a simplified version of a compound microscope and the ray diagram of image formation. It consists of two converging lenses set coaxially. The one that faces the object is called the objective, and the one that is close to the eye is called the eyepiece or ocular. The objective has a smaller aperture and smaller focal length than the eyepiece. The distance between the objective and the eyepiece can be varied by appropriate screws fixed on the panel of the microscope.

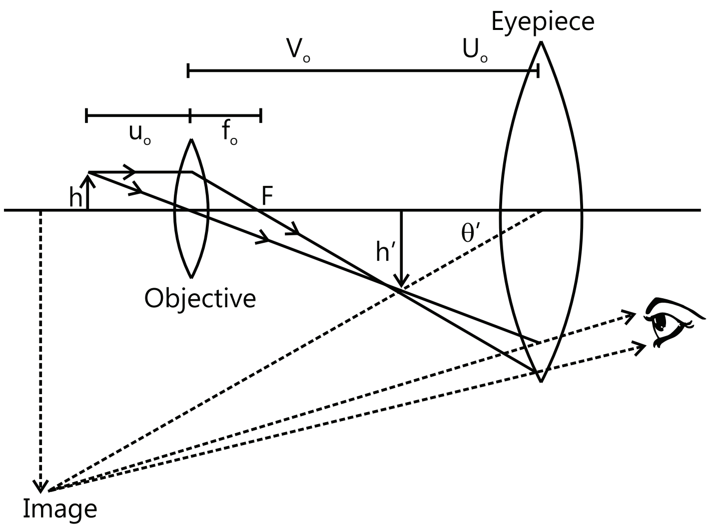


Figure 16.98

The object is placed at a distance u_0 from the objective which is slightly greater than its focal length f_0. A real image and an inverted image are formed at a distance v_0 on the other side of the objective. This image becomes the object for the eyepiece. For normal adjustment, the position of the eyepiece is adjusted such that the image formed by the objective falls in the focal plane of the eyepiece. Then, the final image is formed at infinity. It is erect with respect to the first image and, hence, inverted with respect to the object. The eye is least strained in this adjustment as it has to focus the parallel rays coming toward it. The position of the eyepiece can also be adjusted in such a way that the final virtual image is formed at the near point. The angular magnification is increased in this case. The ray diagram in the Fig. 16.98 refers to this case.

The eyepiece acts as a simple microscope effectively used to view the first image. Thus, the magnification by a compound microscope is a two-step process. In the first step, the objective produces a magnified image of the given object. In the second step, the eyepiece produces an angular magnification. The overall angular magnification is the product of the two.

Magnifying power

Refer to the figure, if an object of height *h* is seen by the naked eye and placed at the near point, the largest image is formed on the retina. The angle formed by the object on the eye in this situation is $\theta_0 = \frac{h}{D}$. ... (i)

When a compound microscope is used, the final image subtends an angle θ′ on the eyepiece (and hence on the eye) given by $\theta' = \frac{h'}{u_e'}$ (ii)

Where *h′* is the height of the first image, and u_e is the distance between the first image and the eyepiece.

The magnifying power of the compound microscope is, therefore,

$$m = \frac{\theta'}{\theta_0} = \frac{h'}{u_e} x \frac{D}{h} = \left(\frac{h'}{h}\right)\left(\frac{D}{u_e}\right) \quad \text{... (iii)}$$

Also from the figure $\frac{h'}{h}=-\frac{v_0}{u_0}=\frac{v}{u}$ ... (iv)

Now, D/u_e is the magnifying power of the eyepiece that acts as a simple microscope. Using the equations given above, in normal adjustment, this value becomes D/f_e when the image is formed at infinity and $1+D/f_e$ when the image is formed at the least distance for clear vision, i.e. at *D*. Thus, for the normal adjustment, the magnifying power of the compound microscope is, by Eq. (iii), $m=\frac{v}{u}\left(\frac{D}{f_e}\right)$ when the image is formed at infinity and is $m=\frac{v}{u}\left(1+\frac{D}{f_e}\right)$ when the final image is formed at the least distance for clear vision.

Using lens equation for the objective,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow \quad 1-\frac{v}{u}=\frac{v}{f_0} \qquad \Rightarrow \quad \frac{v}{u}=1-\frac{v}{f_0}$$

In general, the focal length of the objective is very small, so that $\frac{v}{f_0}>>1$. Furthermore, the first image is close to the eyepiece, so that $v\approx l$, where *l* is the tube length (distance between the objective and the eyepiece). Thus, $\frac{v}{u}=1-\frac{v}{f_0}\approx-\frac{v}{f_0}\approx-\frac{l}{f_0}$.

If these conditions are satisfied, for the normal adjustment, the magnifying power of the compound microscope is $m=-\frac{l}{f_0}\frac{D}{f_0}$ when the image is formed at infinity and is $m=-\frac{l}{f_0}\left(1+\frac{D}{f_0}\right)$ when the final image is formed at the least distance for clear vision.

In an actual compound microscope, all the objectives and the eyepieces consist of a combination of several lenses instead of a single lens assumed in the simplified version.

Illustration 38: A compound microscope has an objective of focal length 1 cm and an eyepiece of focal length 2.5 cm. An object has to be placed at a distance of 1.2 cm away from the objective for normal adjustment.

(i) Find the angular magnification.

(ii) Find the length of the microscope tube. **(JEE ADVANCED)**

Sol: As the objective lens of microscope is convex lens, the focal length is obtained as $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ and magnification is $m=\frac{v}{u}$. The length of microscopic tube is given as $L=v+f_e$ where v is the distance of image formed by objective.

(i) If the first image is formed at a distance *v* from the objective, we get

$$\frac{1}{v}-\frac{1}{(-1.2\,cm)}=\frac{1}{1\,cm} \qquad \text{or,} \quad v=6\,cm.$$

The angular magnification in normal adjustment is $m=\frac{v}{u}\frac{D}{f_e}=-\frac{6\,cm}{1.2\,cm}.\frac{25\,cm}{2.5\,cm}=-50$.

(ii) For normal adjustment, the first image must be in the focal plane of the eyepiece. The length of the tube is, therefore, $L=v+f_e$ =6 cm + 2.5 cm = 8.5 cm.

14. TELESCOPES

A microscope is used to view the object placed close to it, i.e. within few centimeters. To look at the distant objects such as stars, planets and a distant tree, we use telescope. There are three types of telescopes that are used.

(A) Astronomical Telescope

The Fig. 16.99 shows the construction and working principle of a simplified version of an astronomical telescope.

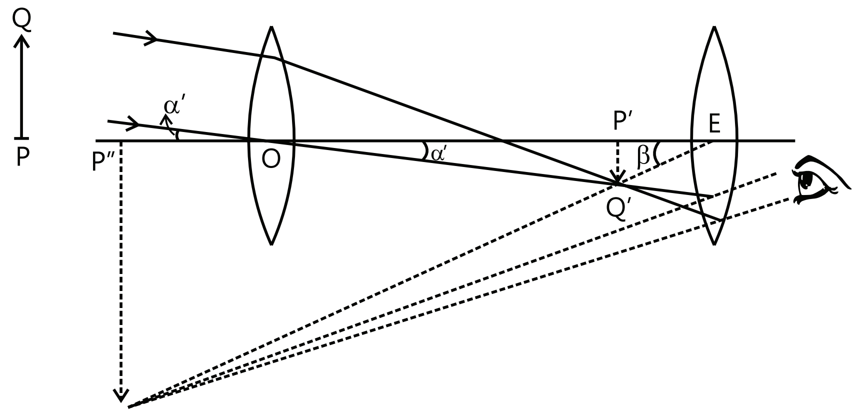


Figure 16.99

The telescope consists of two converging lenses placed coaxially. The one that faces the distant object is called the objective, and it has larger aperture and focal length. The other is called the eyepiece, as it is placed closer to the eye and has smaller aperture and focal length. The lenses are fixed in tubes. The eyepiece tube can slide within the objective tube so that the distance between the objective and the eyepiece can be changed.

When the telescope is directed toward a distant object PQ, the objective forms a real image of that object in its focal plane. If the point P is on the principal axis, the image point P′ is at the second focus of the objective. The rays from Q are focused at Q′. The eyepiece forms a magnified virtual image P″Q″ of P′Q′. This image is finally seen by the eye. In normal adjustment, the position is adjusted such that the final image is formed at infinity. In such a case, the first image P′Q′ is formed in the first focal plane of the eyepiece. The eye is least strained to focus this final image. The image can be brought closer by pushing the eyepiece closer to the first image. A maximum angular magnification is produced when the final image is formed at the near point.

Magnifying Power

Let the objective and the eyepiece have focal lengths f_0 and f_e, respectively, and the object is placed at a large distance u_0 from the objective. The object PQ in the Fig. 16.99 subtends an angle α on the objective. Since the object is at infinity, the angle it would subtend on the eye, if there were no telescope, is α′.

As u_0 is very large, the first image P′Q′ is formed in the focal plane of the objective.

From the figure $$|\alpha| = |\alpha'| \approx |\tan\alpha'| = \frac{P'Q'}{OP'} = \frac{P'Q'}{f_0} \qquad \text{... (i)}$$

The final image P″Q″ subtends an angle β on the eyepiece (and hence on the eye). From the triangle P′Q′E,,

$$|\beta| \approx |\tan\beta| = \frac{P'Q'}{EP'} \Rightarrow \left|\frac{\beta}{\alpha}\right| = \frac{f_0}{EP'} \qquad \text{... (ii)}$$

If the telescope is adjusted for normal adjustment so that the final image is formed at infinity, the first image P′Q′ must be in the focal plane of the eyepiece.

Then $EP' = f_e$.

Thus, Eq. (ii) becomes $$\left|\frac{\beta}{\alpha}\right| = \frac{f_0}{f_e}. \qquad \text{... (iii)}$$

The angular magnification or the magnifying power of the telescope is

$$m = \frac{\text{Angle subtended by the final image on the eye}}{\text{Angle subtended by the object on the unaided eye}}.$$

The angles β and α are formed on the opposite sides of the axis. Hence, the signs of these angles are opposite, and β/α is negative. Hence, $m = \frac{\beta}{\alpha} = -\left|\frac{\beta}{\alpha}\right|$.

Using Eq. (iii), $m = -\frac{f_0}{f_e}$.

If the telescope is adjusted so that the final image is formed at the near point of the eye, the angular magnification is further increased. Let us apply the lens equation to the eyepiece in this case.

Here, $u = -EP'$ and $v = -EP'' = -D$.

The lens equation is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{-D} - \frac{1}{-EP'} = \frac{1}{f_e} \quad \Rightarrow \quad \frac{1}{-EP'} = \frac{1}{f_e} + \frac{1}{D} = \frac{f_e + D}{f_e D} \qquad \text{... (iv)}$$

By Eq. (ii), $\left|\frac{\beta}{\alpha}\right| = \frac{f_0(f_e + D)}{f_e D}$

The magnification is $m = \frac{\beta}{\alpha} = -\left|\frac{\beta}{\alpha}\right| = -\frac{f_0(f_e + D)}{f_e D} = -\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$

Length of the Telescope

From the Fig. 16.106, we see that the length of the telescope is $L = OP' + P'E' = f_0 + P'E$.

For normal adjustment, $P'E = f_e$ so that $L = f_0 + f_e$. For adjustment for the near-point vision, we get, by Eq. (iv),

$P'E = \frac{f_e}{f_e + D}$, so that the length is $L = f_0 + \frac{f_e D}{f_e + D}$.

14.1 Resolving Power of a Telescope

The resolving power of a microscope is defined as the reciprocal of the distance between two objects, which can be resolved when seen through the microscope. It depends on the wavelength λ of the light, the refractive index μ of the medium between the object and the objective of the microscope and the angle θ subtended by a radius of the objective on one of the object. It is given by

$$R = \frac{1}{\Delta d} = \frac{2\mu \sin\theta}{\lambda}$$

To increase the resolving power, the objective and the object are kept immersed in oil. It increases μ and hence R.

The resolving power of a telescope is defined as the reciprocal of the angular separation between two distant objects which are just resolved when viewed through a telescope. It is given by

$$R = \frac{1}{\Delta\theta} = \frac{a}{1.22\,\lambda},$$

where *a* is the diameter of the objective of the telescope. The telescopes with larger objective aperture (1 m or more) are used in astronomical studies.

15. DEFECTS OF VISION

As described earlier, the ciliary muscles control the curvature of the lens in the eye and hence can change the effective focal length of the system. When the muscles are fully relaxed, the focal length is maximum. When the muscles are strained, the curvature of the lens increases and the focal length decreases. For clear vision, the image must be formed on the retina. The image distance is, therefore, fixed for clear vision, and it equals the distance of the retina from the eye lens. It is about 2.5 cm for a grown-up person. If we apply the lens formula to the eye, the magnitudes of the object distance, the image distance and the effective focal length satisfy

$$\frac{1}{v_0}+\frac{1}{u_0}=\frac{1}{f} \quad \text{or} \quad \frac{1}{u_0}=\frac{1}{f}-\frac{1}{v_0} \qquad \text{... (i)}$$

Here, v_0 is fixed, and hence by changing f, the eye is focused on the objects placed at different values of u_0. We see from Eq. (i) that when f increases, u_0 increases, and when f decreases, u_0 decreases. The maximum distance one can see is

$$\frac{1}{u_{max}}=\frac{1}{f_{max}}-\frac{1}{v_0}, \qquad \text{... (ii)}$$

where f_{max} is the maximum focal length possible for the eye lens.

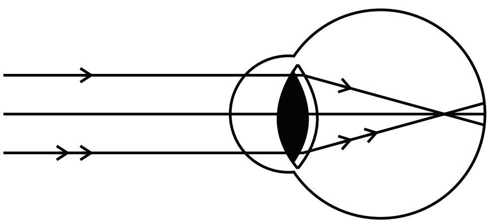
The focal length is maximum when the ciliary muscles are fully relaxed. In a normal eye, this focal length equals the distance v_0 from the lens to the retina. Thus,

$v_0 = f_{max}$ by (ii), $u_{max} = \infty$.

Theoretically, a person can have clear vision of the objects placed at any large distance from the eye. For the closer objects, u is smaller, and hence, f should be smaller. The smallest distance at which a person can have a clear vision is related to the minimum possible focal length f. The ciliary muscles are most strained in this position. By Eq. (ii), the closest distance for clear vision is given by

$$\frac{1}{u_{min}}=\frac{1}{f_{min}}-\frac{1}{v_0}. \qquad \text{... (iii)}$$

For an average grown-up person, u_{min} should be around 25 cm or less. This is a convenient distance at which one can hold an object in his/her hand and can see. Thus, a normal eye can clearly see objects placed in the range from about 25 cm from the eye to a large distance of the order of several kilometers. The nearest point and the farthest point up to which an eye can clearly see are called the near point and the far point. For a normal eye, the distance of the near point should be around 25 cm or less, and the far point should be at infinity. We now describe some common defects of vision.



(A) Defective-eye

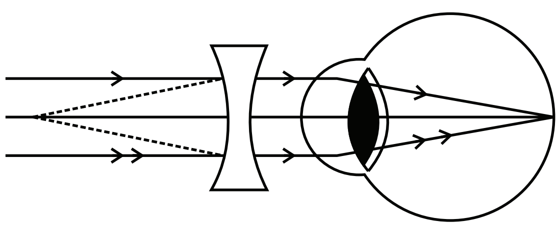
Figure 16.100

By the eye lens, real, inverted and diminished image is formed at retina.

The common defects of vision are as follows:

(a) Myopia or short sightedness: The distant objects are not clearly visible in this defect. The image of a distant object is formed before the retina.

The defect can be remedied by using a concave lens.



(B) Defective-eye

Figure 16.101

Illustration 39: A nearsighted man can clearly see the objects up to a distance of 1.5 m. Calculate the power of the lens of the spectacles necessary for the remedy of this defect. **(JEE MAIN)**

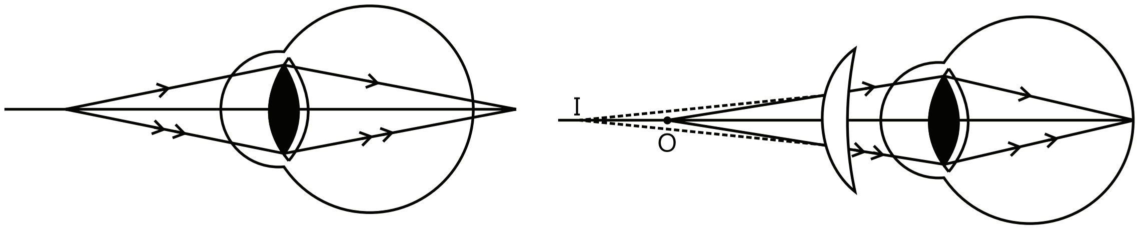
Sol: As the man has near sighted vision, he need to wear concave lens which can form virtual and erect images.

The power of magnification of lens is $P=\frac{1}{f}$.

The lens should form a virtual image of a distant object at 1.5 m from the lens. Thus, it should be a divergent lens, and its focal length should be −1.5 m. Hence,

$$f = -1.5\text{m} \quad \Rightarrow \quad P = \frac{1}{f} = -\frac{1}{1.5}\text{m}^{-1} = -0.67\text{D}.$$

(b) Hypermetropia or far sightedness: The near objects are not clearly visible in this defect. The image of a near object is formed behind the retina.



(A) Deflected-eye (B) Corrected-eye

Figure 16.102

This defect is remedied by using a convex lens.

(c) Presbyopia: In this defect, both near and far objects are not clearly visible. This is remedied either by using two separate lenses or by using a single spectacle having bifocal lenses.

(d) Astigmatism: In this defect, the eye cannot see objects in two orthogonal (perpendicular) directions clearly simultaneously. This defect is remedied by using a cylindrical lens.

PLANCESS CONCEPTS

While testing your eye by reading a chart, if doctor finds it to 6/12, it implies that you can read a letter from 6 m which the normal eye can read from 12 m. Thus, 6/6 is the normal eye sight.

Worth Knowing: The persistence of vision is $\frac{1}{10}$ s, i.e. if the time interval between two consecutive light rays is less than 0.1 s, the eye cannot distinguish them separately. Hence, the fps (frames per second) of a video should be more than 10.

Anurag Saraf (JEE 2011 AIR 226)

PROBLEM-SOLVING TACTICS

1. Of *u*, *v* and *f*, any two values will be known to us and we will be asked to find the third. In such type of problems, two cases are possible.

Case 1: When signs of all the three will be known to us from the given information, substitute all the three with the known sign; then, we can get only the numerical value of the unknown (i.e. the third quantity) without sign.

Case 2: When the sign of the third unknown quantity is not known to us, substitute only the known quantities with sign. Then, the numerical value of the unknown with its respective sign can be obtained.

2. The experiments show that if the boundaries of the media are parallel, the emergent ray CD, although laterally displaced, is parallel to the incident ray AB if $\mu_1 = \mu_5$. We can also directly apply the Snell's law $\left(\mu \sin i = \text{constant}\right)$ in media 1 and 5, i.e. $\mu_1 \sin i_1 = \mu_5 \sin i_5$ $\mu_1 \sin i_1 = \mu_2 \sin i_2 = \mu_3 \sin i_3 = ... = \mu_i \sin i_i$

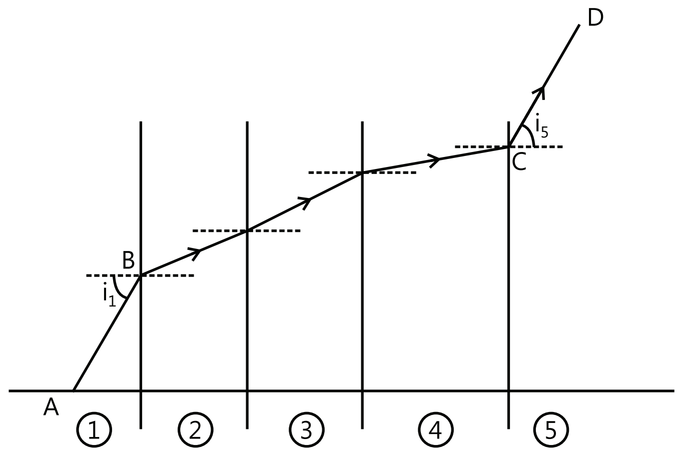


Figure 16.103

Notice that an apparent depth is multiple of either μ or $1/\mu$. It can be find out by knowing whether the medium through which light is entering is a denser or rarer medium.

3. Sometimes only a part of a prism will be given. To solve such problems, first complete the prism and then solve the problems.

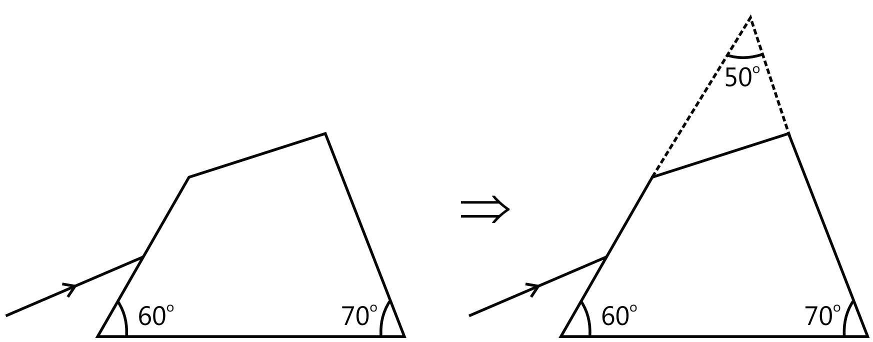


Figure 16.104

FORMULAE SHEET

<table>
<tr><th>S. No</th><th>KEY CONCEPTS</th><th>DESCRIPTIONS</th></tr>
<tr><td>1</td><td>Law of reflection</td><td>(i) The incident ray (AB), the reflected ray (BC) and normal (NN′) to the surface (SS′) of reflection at the point of incidence (B) lie in the same plane. This plane is called the plane of incidence (also the plane of reflection).

(ii) The angle of incidence (the angle between the normal and the incident ray) and reflection angle (the angle between the reflected ray and the normal) are equal.

$\angle i = \angle r$.

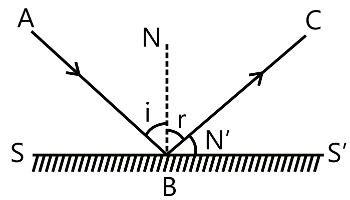


Figure 16.105</td></tr>
<tr><td>2</td><td>Object</td><td>(a) Real: Point from which rays actually diverge.

(b) Virtual: Point toward which rays appear to converge.</td></tr>
<tr><td>3</td><td>Image</td><td>The image is decided by the reflected or refracted rays only. The point image for a mirror is that point

(i) Toward which the rays reflected from the mirror actually converge (real image), OR

(ii) From which the reflected rays appear to diverge (virtual image).</td></tr>
<tr><td>4</td><td>Characteristics of reflection by a plane mirror</td><td>(a) The size of the image is the same as that of the object.

(b) For a real object, the image is virtual, and for a virtual object, the image is real.

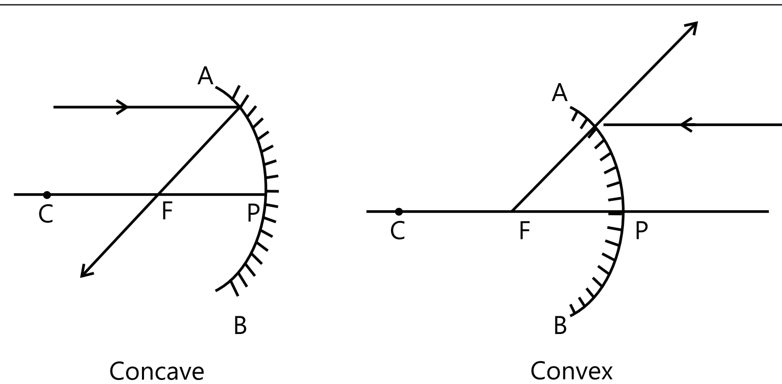
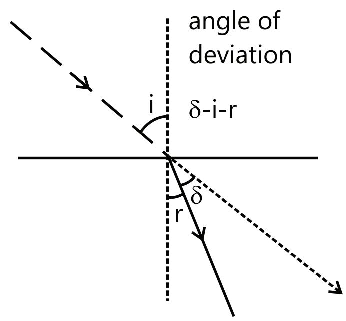
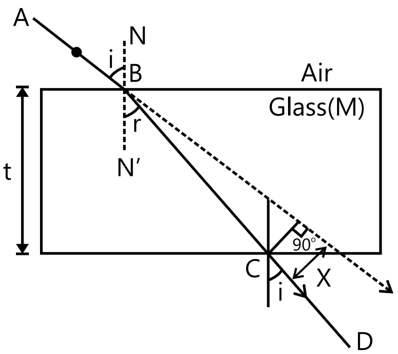
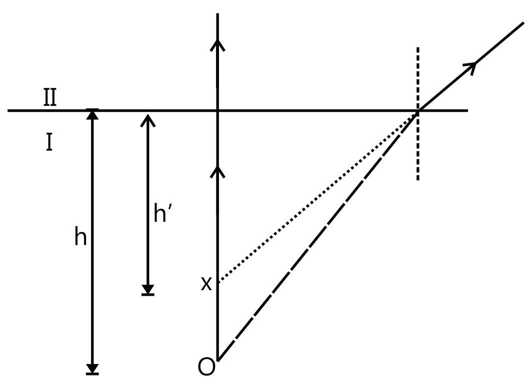
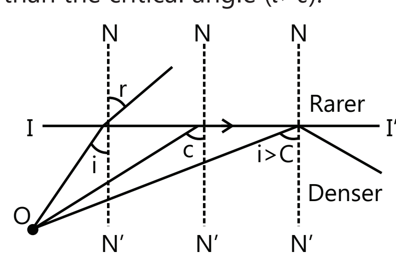
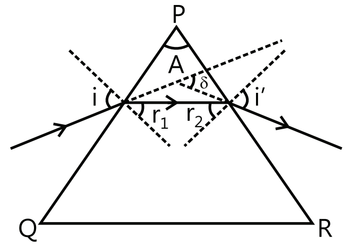
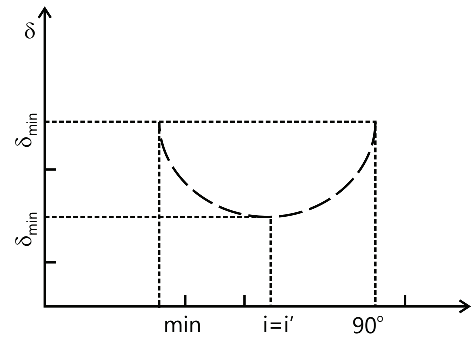
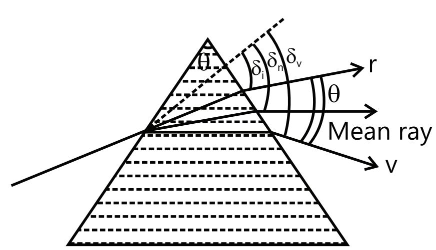
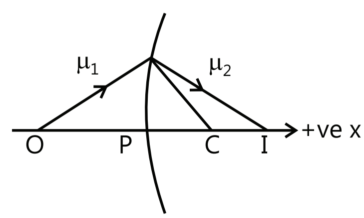
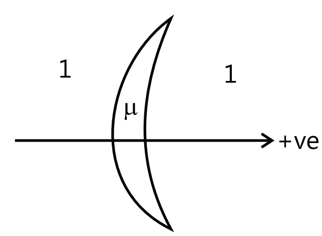
(c) For a fixed incident light ray, if the mirror is rotated through an angle θ, the reflected ray turns through an angle of 2θ.</td></tr>
<tr><td>5</td><td>Spherical mirrors</td><td>

Figure 16.106</td></tr>
<tr><td>6</td><td>Paraxial rays</td><td>Rays that form very small angle with principal axis are called paraxial rays.</td></tr>
</table>

S. No	KEY CONCEPTS	DESCRIPTIONS
7	**Sign convention**	We follow the Cartesian coordinate system convention according to which (a) The pole of the mirror is the origin. (b) The direction of the incident rays is a positive x-axis. (c) Vertically up is positive y-axis. **Note:** According to this, a convention radius of curvature and focus of concave mirror are negative and of convex mirror are positive.
8	**Mirror formula**	*f*=x-coordinate of focus; $\boxed{\frac{1}{f}=\frac{1}{v}+\frac{1}{u}}$, where *v*=x-coordinate of the image; *u*=x-coordinate of the object. **Note: Valid only for paraxial rays.**
9	**Transverse magnification**	$h_2 = y$ -coordinate of the image $\boxed{m=\frac{h_2}{h_1}=-\frac{v}{u}}$. $h_1 = y$ -coordinate of the object. (both are perpendicular to the principle axis of the mirror)
10	**Optical power**	Optical power of a mirror (in dioptres) = $-\frac{1}{f}$, where *f* is the focal length (in m) with a respective sign.
REFRACTION – PLANE SURFACE		
1	**Laws of refraction (at any refracting surface)**	**(i)** The incident ray (AB), the normal (NN') to the refracting surface (II') at the point of incidence (B) and the refracted ray (BC) all lie in the same plane called the plane of incidence or the plane of refraction. **(ii)** $\boxed{\frac{\sin i}{\sin r}=\text{Constant}:}$ for any two given media and light of a given wavelength. This is the Snell's law. $\boxed{\frac{\sin i}{\sin r}={}_1n_2=\frac{n_2}{n_1}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}}$ **Note:** The frequency of light does not change during refraction.
2	**Deviation of a ray due to refraction**	 **Figure 16.107**

S. No	KEY CONCEPTS	DESCRIPTIONS
3	**Refraction through a parallel slab**	**(i)** Emerged ray is parallel to the incident ray, if medium is same on both sides. **(ii)** Lateral shift $\boxed{x = \dfrac{t\sin(i-r)}{\cos r}}$, where t= thickness of the slab.  **Figure 16.108** **Note:** Emerged ray is not parallel to the incident ray if the media on both the sides are different.
4	**Apparent depth of a submerged object**	At near normal incidence, $\boxed{h' = \dfrac{\mu_2}{\mu_1}h}$  **Figure 16.109** **Note:** *h* and *h*′ are always measured from the surface.
5	**Critical angle & total internal reflection (TIR.)**	**(i)** Ray travels from a denser to a rarer medium. **(ii)** The angle of incidence should be greater than the critical angle (i>c). Critical angle $\boxed{C = \sin^{-1}\dfrac{n_r}{n_i}}$  **Figure 16.110**

S. No	KEY CONCEPTS	DESCRIPTIONS
6	**Refraction through prism**	**1.** $\delta = (i + i') - (r + r')$. **2.** $r + r' = A$. **3.** Variation in δ versus (shown in diagram). **4.** There is one and only one angle of incidence, for which the angle of deviation is minimum.  Figure 16.111 When $\delta = \delta_m$ then $i = i'$ & $r = r'$, the ray passes symmetrically through the prism, and then (where *n*=absolute RI of glass), $\boxed{n = \frac{\sin\left[\frac{A+\delta m}{2}\right]}{\sin\left[\frac{A}{2}\right]}}$.  Figure 16.112 **Note:** When the prism is dipped in a medium, then (where *n*=RI of glass w.r.t. medium). **5.** For a thin prism, $\left(A < 10^0\right)$; $\delta = (n-1)A$. **6. Dispersion of light:** The angular splitting of a ray of white light into a number of components when it is refracted in a medium other than air is called dispersion of light. **7. Angle of dispersion:** An angle between the rays of the extreme colors in the refracted (dispersed) light is called angle of dispersion. $\theta = \delta_v - \delta_r$. **8. Dispersive power** (ω) of the medium of the material of prism. $(\omega) = \frac{\text{Angular dispersion}}{\text{Derivation of mean ray (yellow)}}$ For a small-angled prism, (A<10°) $\boxed{\omega = \frac{\delta_v - \delta_R}{\delta_y} = \frac{n_v - n_R}{n-1}; n = \frac{n_v + n_R}{2}}$,  Figure 16.113 where n_v, n_R and *n* are RI of the material for violet, red and yellow colors, respectively.

S. No	KEY CONCEPTS	DESCRIPTIONS
		9. Combination of two prisms: **(i)** Achromatic combination: It is used for deviation without dispersion. Condition for this is $\left(n_v - n\right)A = \left(n'_v - n'_r\right)A'$. Net mean Deviation $= \left[\dfrac{n_v + n_R}{2} - 1\right] A - \left[\dfrac{n'_v + n'_R}{2} - 1\right] A'$. Or $\omega\delta + \omega'\delta' = 0$ where ω, ω' are dispersive powers for the two prisms and δ, δ' are the mean deviations. **(ii)** Direct vision combination: It is used to produce dispersion without deviation; condition for this is $\left[\dfrac{n_v + n_R}{2} - 1\right] A = \left[\dfrac{n'_v + n'_R}{2} - 1\right] A'$. Net angle of dispersion $\left(n_v - n\right)A - \left(n'_v - n'_r\right)A'$.
REFRACTION AT A SPERICAL SURFACE		
1		**(a)** $\boxed{\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}}$; *v*, *u* a *R* are kept with sign. As *v* = PI *u* = −PO *R* = PC (**Note radius is with sign).** **(b)** $\boxed{m = \dfrac{\mu_1 v}{\mu_2 u}}$  **Figure 16.114**
2	**Lens formula**	**(a)** $\boxed{\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}}$ **(b)** $\boxed{\dfrac{1}{f} = \left(\mu - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)}$ **(c)** $\boxed{m = \dfrac{v}{u}}$  **Figure 16.115**

Solved Examples

JEE Main/Boards

Example 1: A person sees the point A on the rim at the bottom of a cylindrical vessel when the vessel is empty through the telescope T. When the vessel is completely filled with a liquid of refractive index 1.5, he observes a mark at the center B, of the bottom, without moving the telescope or the vessel. What is the height of the vessel if the diameter of its cross section is 10 cm?

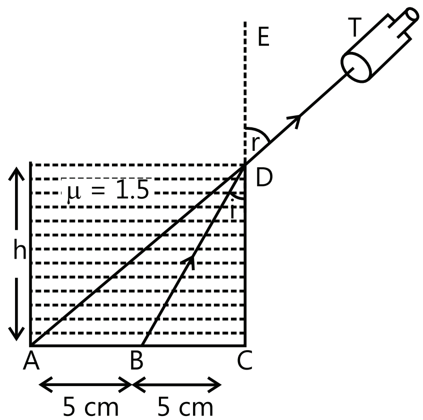
Sol: The vessel is filled with a liquid of refractive index μ when the ray from B reaches the telescope.

$\therefore \mu \sin i = 1.0 \times \sin r$, ... (i)

where $\angle r = \angle ADC$, $\angle i = \angle BDC$

$$\sin i = \frac{R}{\sqrt{R^2+h^2}}, \ \sin r = \frac{2R}{\sqrt{h^2+(2R)^2}}$$

where *h* is the height of the liquid, and *R* is the radius of the vessel.



Substituting these values in Eq. (i),

$$\mu \times \frac{R}{\sqrt{R^2+h^2}} = \frac{2R}{\sqrt{h^2+(2R)^2}} \Rightarrow \mu = \frac{2\sqrt{R^2+h^2}}{\sqrt{h^2+(2R)^2}}.$$

$$4R^2\left(\mu^2-1\right) = h^2\left(4-\mu^2\right) \Rightarrow h = 2R\sqrt{\left[\frac{\mu^2-1}{4-\mu^2}\right]}$$

$$h = 10\sqrt{\frac{(1.5)^2-1}{4-(1.5)^2}} = 10\sqrt{\frac{5}{7}} = 8.45\,\text{cm}.$$

Example 2: A parallel beam of light rays is incident on a transparent sphere of radius *R* and a refractive index μ in the direction of one of the diameters. At what distance from the center of the sphere, will the rays be focused? Assume that μ < 2.

Sol: For refraction of light ray from surface of sphere, the distance of image is obtained by $\frac{\mu_1}{v} - \frac{\mu_2}{r} = \frac{\mu_1-\mu_2}{R}$.

The ray refracted at one surface becomes object for the opposite surface.

For the first refraction surface:

$\mu_1 = 1$, $\mu_2 = \mu$, $u = -\infty$, $v = ?$, $r = +R$

$$\therefore \quad \frac{\mu}{v} - \frac{1}{-\infty} = \frac{\mu-1}{R} \Rightarrow v = \frac{\mu R}{\mu-1}$$

For refraction at the second surface:

I′ serves as an object for the second surface. Now, $\mu_1 = \mu$ (since light travels from sphere to air) and $\mu_2 = 1$,

$$u = BI' = +\left(\frac{\mu R}{\mu-1} - 2R\right) = \frac{R(2-\mu)}{\mu-1}$$

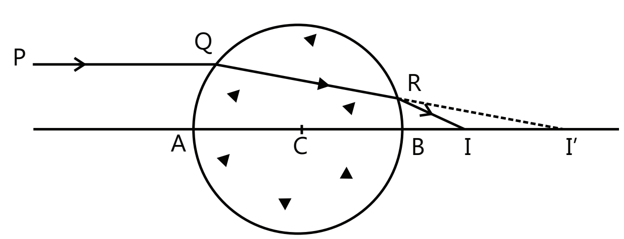
(It is positive because light travels from the left to the right, and the distance is also in the same direction.) $v = ?$, $r = -R$.

$$\therefore \quad \frac{1}{v} - \frac{\mu(\mu-1)}{R(2-\mu)} = \frac{1-\mu}{-R} \ \text{ or } \ v = \frac{R(2-\mu)}{2(\mu-1)}$$

This distance is positive and is referred from the second surface.

$\therefore$ Distance of the focal point from the center

$$= \frac{R(2-\mu)}{2(\mu-1)} + R = \frac{\mu R}{2(\mu-1)}.$$



Example 3: A rectangular glass block of thickness 10 cm and refractive index 1.5 is placed over a small coin. A beaker is filled with water of refractive index 4/3 to a height of 10 cm and is placed over the block.

(i) Find the apparent position of the object when it is viewed from normal incidence.

(ii) Draw a neat ray diagram.

(iii) If the eye is slowly moved away from the normal, at a certain position, the object is found to disappear due to the TIR. At which surface, does this happen and why?

Sol: The image of coin formed at upper surface of block, becomes object for beaker containing water. The image thus formed at distance v is given by $\frac{\mu_1}{v}-\frac{\mu_2}{r}=\frac{\mu_1-\mu_2}{R}$.

For the first surface: $\mu_2=\frac{4}{3}, \quad \mu_1=1.5,$

$u=-10\,cm,\ R=\infty,\ v_1=?$

$$\frac{\frac{4}{3}}{v_1}-\frac{1.5}{-10}=\frac{\frac{4}{3}-1.5}{\infty}\Rightarrow v_1=-\frac{80}{9}cm$$

I' serves as an object for the second surface. For the second surface:

Note: This is an alternative to the apparent depth relation.

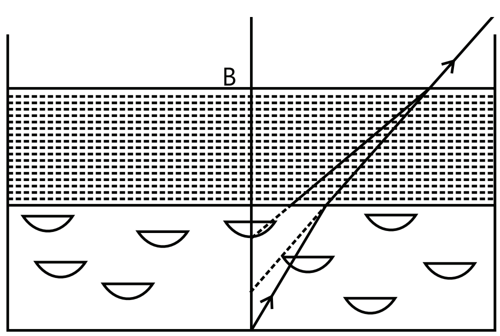
$$u=-BI'=-(BA+AI')=-\left(10+\frac{80}{9}\right)=-\frac{170}{9}cm$$

$\mu_1=\frac{4}{3},\ \mu_2=1, R=\infty,\ v_2=?$

$$\therefore \frac{1}{v_2}-\frac{\frac{4}{3}}{-\frac{170}{9}}=\frac{1-\frac{4}{3}}{\infty} \quad\Rightarrow\quad v_2=14.2cm$$

θ_c: critical angle for glass–water interface

$$=\sin^{-1}\frac{1}{{}^{w}\mu_g} \quad\Rightarrow\quad \theta_c=\sin^{-1}\frac{1}{{}^{w}\mu_a\times{}^{a}\mu_g}$$



$$=\sin^{-1}.\frac{1}{\frac{3}{4}x\frac{3}{2}}=\sin^{-1}\frac{8}{9}=62.7^0$$

The critical angle for water–air interface

$$=\sin^{-1}\frac{1}{{}^{a}\mu_w}=\sin^{-1}\frac{3}{4}=48^0.$$

Obviously, therefore, TIR takes place earlier at the water–air interface.

Example 4: How long will the light take in travelling a distance of 500 m in water? Given that μ for water is 4/3 and the velocity of light in vacuum is $3x10^{10}$ cm/s. Also calculate the equivalent path.

Sol: The velocity of light in a medium is given as $v=\frac{c}{\mu}$.

The optical path travelled by light is $\ell=\mu\times d$ where d is the distance travelled in water.

We know that

$$\mu=\frac{\text{Velocity of light in vaccum}}{\text{Velocity of light in water}}.$$

$$\frac{4}{3}=\frac{3x10^{10}}{\text{Velocity of light in water}}.$$

Velocity of light in water = 2.25 x 10^{10}cm/s.

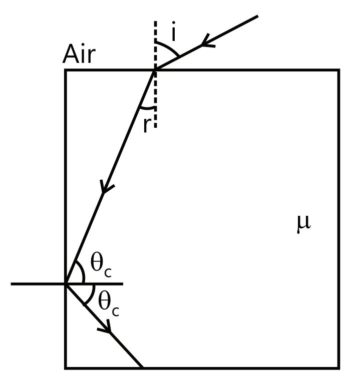
$$\text{Time taken}=\frac{500x100}{2.25x10^{10}}=2.22x10^{-6}\text{ sec}.$$

Equivalent optical path

$=\ \mu$ x distance travelled in water

$$=\frac{4}{3}x500=666.64\,m.$$

Example 5: In the figure shown, for an angle of incidence *i* at the top surface, what is the minimum refractive index needed for TIR at the vertical face?



Sol: Total internal reflection of light occurs inside body when the angle of incidence is greater than critical angle θ_C and according to Snell's law $\theta_C=\sin^{-1}\left(\frac{1}{\mu}\right)$

Applying the Snell's law at the top surface,

$\mu \sin r = \sin i$. ... (i)

For TIR, the vertical face

$\mu \sin \theta_c = 1$

Using geometry, $\theta_c = 90° - r$

$\mu \sin(90 - r) = 1 \quad \Rightarrow \quad \mu \cos r = 1$ ... (ii)

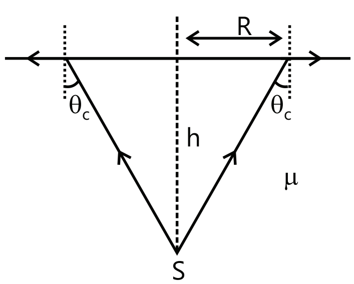
On squaring and adding Eqs (i) and (ii), we get

$\therefore \; \mu^2 \sin^2 r + \mu^2 \cos^2 r = 1 + \sin^2 i$

$\Rightarrow \; \mu = \sqrt{1 + \sin^2 i}.$

Example 6: A point source of light is placed at the bottom of a tank containing a liquid (refractive index = μ) up to a depth *h*. A bright circular spot is seen on the surface of the liquid. Find the radius of this bright spot.

Sol: The light waves incident on the surface of the water at an incident angle greater than critical angle will get reflected in side water. This light waves forms cone in the volume of the tank. Thus we get relation of critical angle as $\tan\theta_C = \frac{R}{h}$

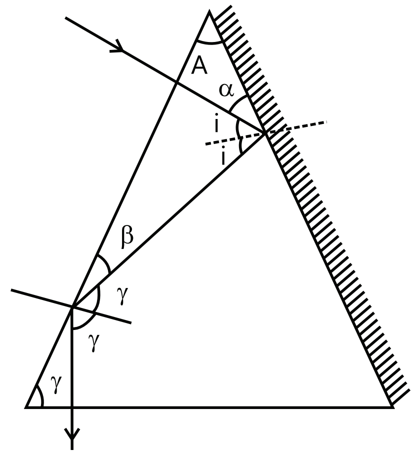


Rays coming out of the source and incident at an angle greater than θ_c will be reflected back into the liquid; therefore, the corresponding region on the surface will appear dark. As it is obvious from the figure, the radius of the bright spot is given by

$R = h \tan \theta_c = \frac{h \sin\theta_c}{\cos\theta_c} \quad \Rightarrow \quad R = \frac{h\sin\theta}{\sqrt{1 - \sin^2 \theta_c}}$

Since $\sin\theta_c = \frac{1}{\mu}$; $\quad \therefore \; R = \frac{h}{\sqrt{\mu^2 - 1}}$.

Example 7: The cross section of the glass prism has the form of an isosceles triangle. One of the equal faces is coated with silver. A ray of light incident normally on the other equal face and after getting reflected twice emerges through the base of prism along the normal. Find the angle of the prism.



Sol: The angles of prism add up to 180°.

From the figure, $\alpha = 90^0 - A$

$i = 90^0 - \alpha$; $\quad i = 90^0 - \alpha = A$ ... (i)

Also, $\quad \beta = 90^0 - 2i = 90^0 - 2A$

and $\quad \gamma = 90^0 - \beta = 2A$

Thus, $\quad \gamma = r = 2A$

From geometry,

$A + \gamma + \gamma = 180^0. \qquad \text{or} \quad A = \frac{180}{5} = 36^0.$

Example 8: A lens has a power of +5 dioptre in air. What will be its power if completely immersed in water?

Given $\mu_g = \frac{3}{2}$; $\mu_w = \frac{4}{3}$.

Sol: According to the lens maker's formula the focal length 1/f is $\frac{1}{f} = \left(\frac{\mu_1}{\mu_2} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

Let f_a and f_w be the focal lengths of the lens in air and water, respectively, then,

$P_a = \frac{1}{f_a} \quad$ and $\quad P_w = \frac{\mu_w}{f_w}$.

$f_a = 0.2\text{m} = 20\text{cm}$.

Using the lens maker's formula,

$P_a = \frac{1}{f_a} = (\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ ... (i)

$\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$$P_w = \frac{1}{f_w} = \left(\frac{\mu_g - \mu_w}{\mu_w}\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{...(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get,

$$\frac{P_w}{P_a} = \frac{(\mu_g - \mu_w)}{\mu_w(\mu_g - 1)} = \frac{1}{3}.\frac{1}{\mu_w}$$

$$\Rightarrow \quad P_w = P_a\left(\frac{1}{3}\right)\left(\frac{3}{4}\right) = \frac{+5}{4}\text{D}$$

Example 9: The distance between two point sources of light is 24 cm. Find out where you would place a converging lens of focal length 9 cm, so that the images of both the sources are formed at the same point.

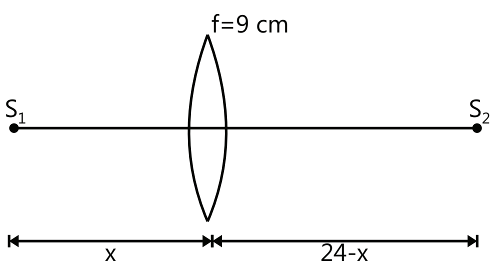
Sol: For lens the distance of the image formed from the lens is given by $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ where u, v and f are distance of object, distance of image and focal length respectively.

$$\text{For } S_1 : \frac{1}{v_1} - \frac{1}{-x} = \frac{1}{9}$$

$$\therefore \frac{1}{v_1} = \frac{1}{9} - \frac{1}{x} \quad \text{... (i)}$$

$$\text{For } S_2 : \frac{1}{v_2} - \frac{1}{-(24-x)} = \frac{1}{9} \quad \text{... (ii)}$$

$$\therefore \quad \frac{1}{v_2} = \frac{1}{9} - \frac{1}{24-x}.$$



Since, the sign convention for S_1 and S_2 is just opposite. Hence,

$$v_1 = -v_2.$$

$$\Rightarrow \quad \frac{1}{v_1} = -\frac{1}{v_2}$$

$$\therefore \quad \frac{1}{9} - \frac{1}{x} = \frac{1}{24-x} - \frac{1}{9}$$

Solving this equation, we get *x* = 6 cm. Therefore, the lens should be kept at a distance of 6 cm from either of the object.

Example 10: Two equiconvex lenses of focal lengths 30 cm and 70 cm, made of material of refractive index = 1.5, are held in contact coaxially by a rubber band round their edges. A liquid of refractive index 1.3 is introduced in the space between the lenses filling it completely. Find the position of the image of a luminous point object placed on the axis of the combination lens at a distance of 90 cm from it.

Sol: This system is combination of three lenses. Two lenses of glass one lens of liquid. Add the powers to get total power.

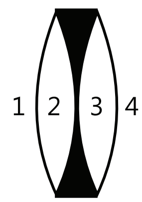
$|R_1| = |R_2| = f_1 = 30\text{cm}$ (As $\mu = 1.5$).

Similarly, $|R_3| = |R_4| = f_2 = 70\text{cm}$.

The focal length of the liquid lens (in air),

$$\frac{1}{f_3} = (\mu - 1)\left(\frac{1}{R_2} - \frac{1}{R_3}\right)$$

$$= (1.3-1)\left(\frac{1}{-30} - \frac{1}{70}\right) = -\frac{1}{70}$$



(b) $m = \frac{v}{u}$

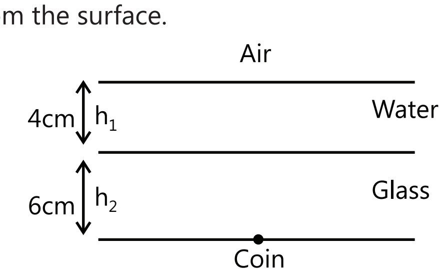
$$\therefore m_1 = \frac{(5.0-4.0)}{(-4.0)} = -0.25,$$

$$\text{and } m_2 = \frac{(5.0-1.0)}{(-1.0)} = -4.00.$$

Hence, both the images are real and inverted, the first is magnification −0.25, and the second is −4.00.

JEE Advanced/Boards

Example 1: A 4-cm-thick layer of water covers a 6-cm-thick glass slab. A coin is placed at the bottom of the slab and is being observed from the air side along the normal to the surface. Find the apparent position of the coin from the surface.



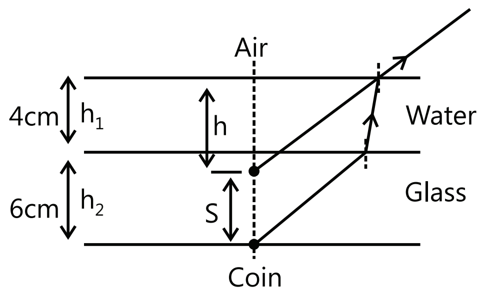
Sol: As the thick layer of water is placed over the glass slab. The coin placed beneath the glass slab will appear to shift upwards due to both glass and water by distance s. This apparent shift is thus given by

$$s = s_1 + s_2 = \left(1 - \frac{1}{\mu_1}\right) \times h_1 + \left(1 - \frac{1}{\mu_2}\right) \times h_2$$

The total apparent shift is

$$s = h_1\left(1 - \frac{1}{\mu_1}\right) + h_2\left(1 - \frac{1}{\mu_2}\right)$$

$$s = 4\left(1 - \frac{1}{4/3}\right) + 6\left(1 - \frac{1}{3/2}\right) = 3 \text{ cm.}$$



Thus, $h = h_1 + h_2 - s = 4 + 6 - 3$.

= 7.0 cm.

Example 2: An achromatic lens of focal length 100 cm is made up of crown and flint glass lenses. Find the focal length of each lens. Given that for crown glass $\mu_v = 1.5245$, $\mu_r = 1.5155$ and for flint glass $\mu_v' = 1.659$ and $\mu_r' = 1.641$.

Sol: The focal length of the combination of lens is given by $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$.

If μ and μ' are mean refractive indices for crown and flint glasses, respectively, then,

$$\mu = \frac{\mu_v + \mu_r}{2} = \frac{1.5245 + 1.5155}{2} = 1.52 .$$

$$\mu' = \frac{\mu_v' + \mu_r'}{2} = \frac{1.659 + 1.641}{2} = 1.65 .$$

The dispersive powers ω and ω' for crown and flint glass, respectively, are

$$\omega = \frac{\mu_v - \mu_r}{\mu - 1} = \frac{1.5245 - 1.5155}{1.52 - 1} = \frac{0.009}{0.52} = \frac{9}{520}$$

$$\omega' = \frac{\mu_v' - \mu_r'}{\mu' - 1} = \frac{1.659 - 1.641}{1.65 - 1} = \frac{0.018}{0.65} = \frac{9}{325}$$

To have an achromatic combination,

$$\frac{\text{focal length for crown glass}}{\text{focal length for flint glass}} = \frac{f}{f'} = -\frac{\omega}{\omega'}$$

$$= -\frac{9}{520} \times \frac{325}{9} = -\frac{5}{8}; \qquad \frac{1}{f'} = -\frac{5}{8f} \qquad \text{... (i)}$$

As the focal length of the combination is 100 cm,

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f'} = \frac{1}{f} - \frac{5}{8f} = \frac{3}{8f} = \frac{1}{100} .$$

$$f = \frac{3}{8} \times 100 = 37.5 \text{ cm} .$$

$$f' = \frac{-8}{5} \times f = \frac{-8}{5} \times \frac{75}{2} = -60 \text{ cm} .$$

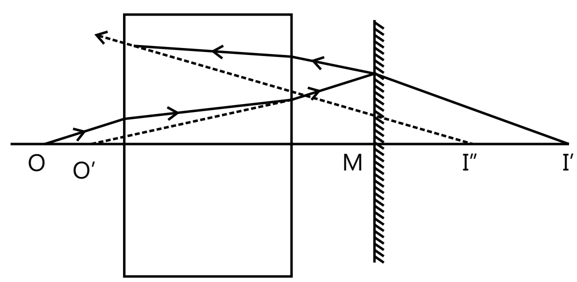
The achromatic doublet requires a convex lens of focal length 37.5 cm made of crown glass and a concave lens of focal length 60 cm made of flint glass.

Example 3: A 20-cm-thick slab of glass of refractive index 1.5 is placed in front of a plane mirror and a pin is placed in front of it in the air at a distance of 40 cm from the mirror. Find the position of the image.

Sol: The slab of thickness t forms the image of the object O at the point O′.

The slab shifts the image of object by $(t - t/\mu)$. O′ serves as an object for the plane mirror.

Object distance for the mirror is MO.



$$= 40 - \frac{20}{3} = \frac{100}{3} \text{ cm} .$$

Since in a plane mirror, object distance = image distance.

$$MI' = MO' = \frac{100}{3} \text{ cm} .$$

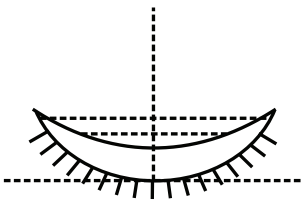
Now, I' serves as an object for the slab again.

I' is shifted to I'' by $20 - \frac{20}{1.5} = \frac{20}{3}$ cm.

$\therefore$ Distance of the final image (I'') from the mirror

$$= \frac{100}{3} - \frac{20}{3} = \frac{80}{3} \text{ cm} .$$

Example 4: The convex surface of a thin concavo–convex lens of glass of refractive index 1.5 has a radius of curvature 20 cm. The concave surface has a radius of curvature of 60 cm. The convex side is coated with silver and placed at a horizontal surface as shown in the figure.



(a) Where a pin should be placed on the optical axis such that its image is formed at the same place?

(b) If the concave part is filled with water of refractive index 4/3, find the distance through which the pin should be moved, so that the image of the pin again coincides with pin.

Sol: When the convex side of the concavo–convex lens is coated with silver, the combination becomes a mirror. This combination consist two lenses and one mirror placed close to each other. The powers of all the three will be added. When the water is filled on concave side, we get plano-convex water lens whose focal length is found by lens makers formula. This combination consists of four lenses (two lenses of glass and two lenses of water) and one mirror.

(a) The refraction takes place from the first surface, reflection from the lower surface and finally refraction from the first surface of focal lengths f_g, f_m and f_g, respectively. The combined focal length F is given by

$$\frac{1}{F}=\frac{1}{f_g}+\frac{1}{f_m}+\frac{1}{f_g}=\frac{2}{f_g}+\frac{1}{f_m} \quad \text{... (i)}$$

$$f_m = R_2/2 = 20/2 = 10\,\text{cm}..$$

The value of f_g can be obtained by using the formula

$$\frac{1}{f_g}=\left({}^{a}\mu_g-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=\left(1.5-1\right)\left(\frac{1}{20}-\frac{1}{60}\right)$$

$$f_g = 60\,\text{cm}.$$

Substituting these values in Eq. (i),

$$\frac{1}{F}=\frac{2}{60}+\frac{1}{10}=\frac{2}{15}$$

$$F=\frac{15}{2}=7.5\,\text{cm}.$$

For the image to be informed at the same point as the object

$$u = 2F = 2\times 7.5 = 15\,\text{cm}.$$

The object should be placed at a distance of 15 cm from the lens on the optical axis.

(b) If f_w is the focal length of lens in water, the focal length F' of this combination is given by

$$\frac{1}{F'}=\frac{1}{f_w}+\frac{1}{f_g}+\frac{1}{f_m}+\frac{1}{f_g}+\frac{1}{f_w}$$

$$\frac{1}{F'}=\frac{2}{f_w}+\frac{2}{f_g}+\frac{1}{f_m} \quad \text{... (ii)}$$

$f_g = 60$ cm, $f_m = 10$ cm.

The value of f_w is calculated by using the relation,

$$\frac{1}{f_w}=\left({}^{a}\mu_w-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f_w}=\left(\frac{4}{3}-1\right)\left(\frac{1}{60}\right)=\frac{1}{180}$$

f_w = 180 cm.

Substituting these values in Eq. (ii), we get

$$\frac{1}{F'}=\frac{2}{180}+\frac{2}{60}+\frac{1}{10};\ F=\frac{90}{13}$$

$$u' = 2F = \frac{2\times 90}{13}=\frac{180}{13}\ \text{cm}.$$

Displacement of the pin

$$=u-u'=15-\frac{180}{13}=\frac{15}{13}=1.14\ \text{cm}.$$

Example 5: The radius of curvature of the convex face of plano-convex lens is 12 cm, and its $\mu = 1.5$.

(a) Find the focal length of the lens.

The plane surface of the lens is coated with silver.

(b) At what distance from the lens, will the parallel rays incident on the convex surface converge?

(c) Sketch the ray diagram to locate the image, when a point object is placed on the axis at a distance of 20 cm from the lens.

(d) Calculate the image distance when the object is placed as in (c).

Sol: Use the lens maker's formula to find the focal length of the plane-convex lens. When the plane side of the lens is coated with silver, the combination becomes a mirror. This combination consist two lenses and one mirror placed close to each other. The powers of all the three will be added.

(a) The focal length *f* of lens of refractive index μ is given by the formula.

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right).$$

In case of plano-convex lens,

$R_1=\infty,\ R_2=-12\,cm,\ \mu=1.5$

$$\therefore\quad \frac{1}{f}=(1.5-1)\left(\frac{1}{12}\right)$$

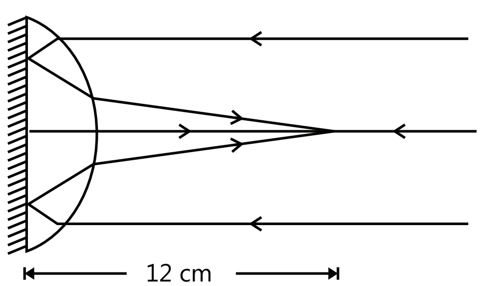
$f=24\,cm.$

(b) The focal length, $f_{m,}$ of plane-silvered surface is infinity. The focal length *F* of the plano-convex lens, when the plane surface is coated with silver is given by

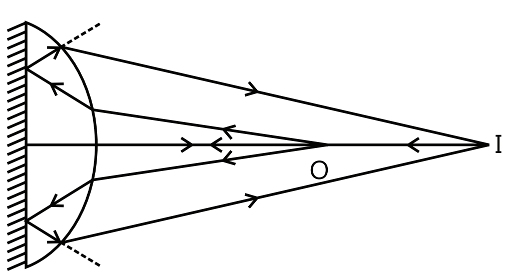
$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f_m}+\frac{1}{f}=\frac{2}{f}+\frac{1}{\infty}$$

$$\therefore\quad F=\frac{f}{2}=\frac{24}{2}=12\,cm.$$

Such a lens behaves like a concave mirror of focal length 12 cm. The parallel rays converge at a distance of 12 cm from the silvered surface.



(c) The figure shows the ray diagram of the image formed by this lens when the object is placed at a distance of 20 cm from the lens. The light is incident from the right to the left.



(d) For a lens,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{F}.$$

For *F* = 12 cm, $u=+20cm$ and $v=?$

$$\frac{1}{v}=\frac{1}{12}-\frac{1}{20}=\frac{1}{30}$$

$v=30\,cm.$

A real image is formed.

Example 6: An object O is placed at a distance of 15 cm from a convex lens A of focal length 10 cm and its image I_1 is formed on a screen on the other side of the lens. A concave lens B is now placed midway between A and I_1, then the screen is moved back 10 cm to receive a clear image I_2.

Find (a) The focal length of the concave lens and

(b) Linear magnification of the final image.

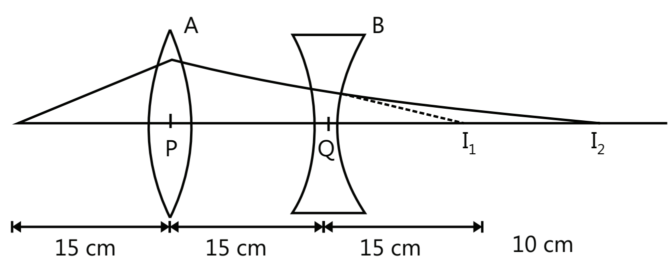
Sol: The convex lens forms the image of the object at point I_1. This image acts as the object for concave lens and the final image is formed at I_1'.

For refraction through convex lens,

$u_1=OP=-15\,cm.$

$f=+10\,cm, v_1=PI_1.$

Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$



$$\frac{1}{v_1}=\frac{1}{f}+\frac{1}{u}=\frac{1}{10}+\left(\frac{1}{-15}\right)=\frac{3-2}{30}=\frac{1}{30}$$

$\Rightarrow v_1=30\ cm.$

I_1 serves as a virtual object for lens B. For refraction through lens B,

$u_2=QI_1=PI_1-15=30-15=+15\,cm,$

$v_2=15+10=+25\,cm.$

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{25}-\frac{1}{15}=\frac{3-5}{75}=-\frac{2}{75}$$

$$f=-\frac{75}{2}=-37.5\,cm,$$

The negative sign implies it is a concave lens.

Linear magnification = $\frac{v_1}{u_1} \times \frac{v_2}{u_2}$

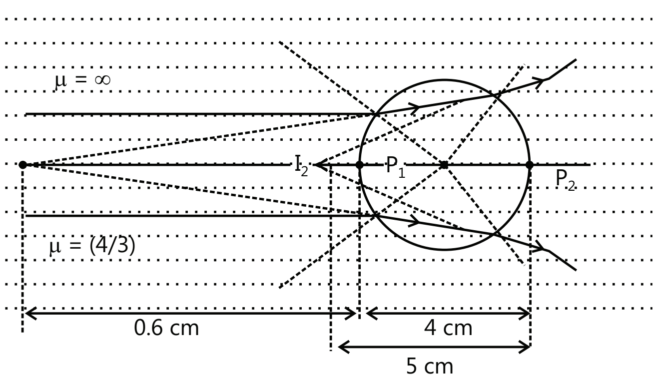
$= \frac{30}{15} \times \frac{5}{3} = -\frac{10}{3} = -3.33$

The negative sign shows that the final image is inverted.

Example 7: A parallel beam of light travelling in water $(\mu = 4/3)$ is refracted by a spherical air bubble of radius 2 mm placed in water. Assume that the light rays to be paraxial.

(a) Find the position of the image due to refraction at the first surface and the position of the final image.

(b) Draw a ray diagram showing the positions of both images.



Sol: The image of the object formed by the first refraction by the water-glass surface acts as the object for the second refraction at glass-water surface.

(a) For the refraction from a single spherical surface, we have

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}.$$

Let P_1 be pole at the first surface and P_2 to be pole at the second surface. At P,

$\mu_1 = (4/3);\ \mu_2 = 1;\ R = +0.2\text{cm},\ u_1 = \infty.$

So $\frac{1}{v_1} - \frac{(4/3)}{\infty} = \frac{1-(4/3)}{+0.2}.$

$\therefore \quad v_1 = -0.6\,\text{cm}.$

The first surface will form a virtual image I_1 at a distance 0.6 cm to the left of P_1 as shown in the figure.

This image acts as an object for the second surface. So for the second surface at P_2,

$\mu_1 = 1, \quad \mu_2 = (4/3), \quad R = -0.2\,\text{cm}$

and $\mu_2 = -(0.6 + 0.4) = -1.0\,\text{cm}$

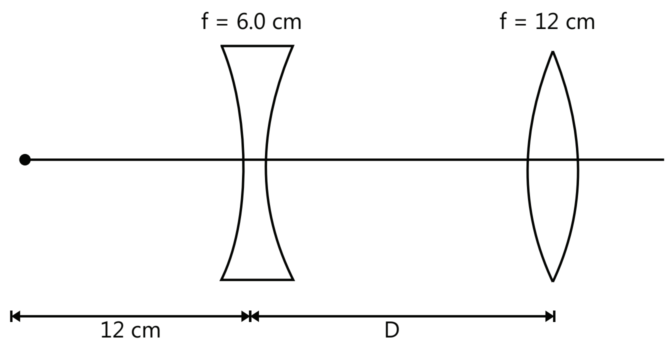
$$\frac{(4/3)}{v_2} - \frac{1}{(-1)} = \frac{(4/3)-1}{-0.2}$$

$v_2 = -0.5$ cm

The final image I_2 is formed at a distance of 0.5 cm to the left of the second surface P_2. The final image is at a distance of $0.5 - 0.4 = 0.1\,\text{cm}$ to the left of the first surface as shown in figure.

(b) The ray diagram is already shown in the figure drawn.

Example 8: An object is placed at a distance of 12 cm to the left of a diverging lens of focal length −6.0 cm. A converging lens with a focal length of 12.0 cm is placed at a distance *d* to the right of the diverging lens. Find the distance *d* that corresponds to a final image at infinity.



Sol: The concave lens forms the image of the object at point say I_1. This image acts as the object for convex lens and the final image is formed at say I_1'.

Applying lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ twice, we have

$$\frac{1}{v_1} - \frac{1}{-12} = \frac{1}{-6} \quad \text{... (i)}$$

$$\frac{1}{\infty} - \frac{1}{v_1 - d} = \frac{1}{12} \quad \text{... (ii)}$$

Solving Eqs (i) and (ii), we have

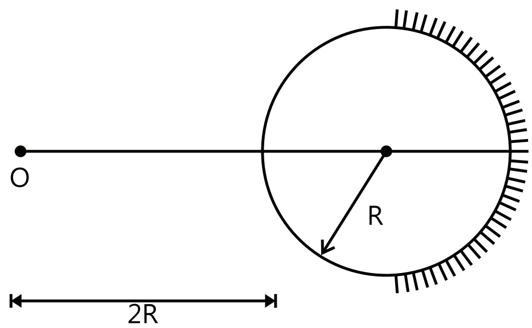
$v_1 = -4\text{cm}.$

And $d = 8\text{cm}.$

Example 9: A solid glass sphere with a radius *R* and a refractive index of 1.5 is coated with silver over a hemisphere. A small object is located on the axis of the sphere at a distance 2*R* to the left of the vertex of the un-silvered hemisphere. Find the position of the final image after all refractions and reflections have taken place.

Sol: The image formed by first refraction at air-glass spherical surface acts as the object for the concave mirror. The image formed by the mirror acts as the object for spherical glass-air spherical surface.

The ray of light first gets refracted, then reflected and then again refracted. For the first refraction and then reflection, the ray of light travels from the left to the right, while for the last refraction it travels from the right to the left. Hence, the sign convention will change accordingly.



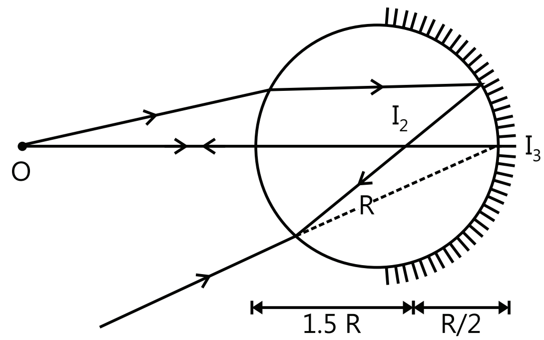
First refraction: Using $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ with proper sign conventions, we have

$$\frac{1.5}{v_1}-\frac{1.0}{-2R}=\frac{1.5-1.0}{+R} \qquad \therefore v_1=\infty.$$

Second reflection: Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{R}$ with proper sign conventions, we have

$$\frac{1}{v_2}+\frac{1}{\infty}=-\frac{2}{R} \quad \therefore v_2=-\frac{R}{2}$$

Third refraction: Again using $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ with reversed sign convention, we have



$$\frac{1.0}{v_3}-\frac{1.5}{-1.5R}=\frac{1.0-1.5}{-R}$$

$\Rightarrow \quad v_3=-2R$;

i.e. final image is formed on the vertex of the silvered face.

JEE Main/Boards

Exercise 1

Q.1 A ray of light incident on an equilateral glass prism shows a minimum deviation of $30°$. Calculate the speed of light through the glass prism.

Q.2 Where an object should be placed from a converging lens of focal length 20 cm so as to obtain a real image of magnification 2?

Q.3 What changes in the focal length of (i) a concave mirror and (ii) a convex lens occur, when the incident violet light on them is replaced with red light?

Q.4 State the conditions for TIR of light to take place. Calculate the speed of light in a medium, whose critical angle is 45^o.

Q.5 An object is placed at the focus of concave lens. Where will its image be formed?

Q.6 Draw a graph to show the variation of the angle of deviation '*D*' with that of the angle of incidence '*I*' for a monochromatic ray of light passing through a glass prism refracting angle '*A*'. Hence, deduce the relation.

Q.7 Draw a ray diagram of an astronomical telescope in the normal adjustment position. Write down the expression for its magnifying power.

Q.8 A spherical surface of radius of curvature *R* separates a rarer and a denser medium. Complete the path of the incident ray of light, showing the formation of the real image. Hence, derive the relation connecting an object distance '*u*', image distance '*v*', radius of curvature *R* and the refractive indices n_1 and n_2 of the two media. Briefly explain how the focal length of a convex lens changes, with an increase in wavelength of incident light.

Q.9 (a) Draw a labelled ray diagram to show the formation of the image by a compound microscope. Write the expression for its magnifying power.

(b) How does the resolving power of a compound microscope change and when?

(i) Refractive index of the medium between the object and the objective lens increases and

(ii) Wavelength of the radiation used is increased.

Q.10 Draw a labeled ray diagram to show the image formation I of a refractive-type astronomical telescope. Why should the diameter of the objective of a telescope be large?

Q.11 A beam of light converges to a point P. A lens is placed in the path of the convergent beam at a distance of 12 cm from P. At what point, does the beam converge if the lens is

(i) A convex lens of focal length 20 cm.

(ii) A concave lens of focal length 16cm?

Do the required calculations.

Q.12 A double convex lens of glass of refractive index 1.6 has its both surfaces of equal radii of curvature of 30 cm each. An object of height 5 cm is placed at a distance of 12.5 cm from the lens. Calculate the size of the image found.

Q.13 Why does the bluish color predominate in a clear sky?

Q.14 How does the angle of minimum deviation of a glass prism of a refractive index 1.5 change, if it is immersed in a liquid of refractive index 1.3?

Q.15 Draw a labeled ray diagram, showing the image formation of an astronomical telescope in the normal adjustment position. Write the expression for its magnifying power.

Q.16 Derive the lens formula, $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ for a concave lens, using the necessary ray diagram.

Two lenses of power 10 D and −5 D are placed in contact.

(i) Calculate the power of the new lens.

(ii) Where should an object be held from the lens so as to obtain a virtual image of magnification 2?

Q.17 Two thin lenses of power +6 D and −2 D are in contact. What is the focal length of the combination?

Q.18 Define the refractive index of a transparent medium. A ray of light passes through a triangular prism. Plot a graph showing the variation of the angle of deviation with an angle of incidence.

Q.19 (a) (i) Draw a labeled ray diagram to show the formation of image in an astronomical telescope for a distant object.

(ii) Write three distinct advantages of a reflecting-type telescope over a refracting-type telescope.

(b) A convex lens of focal length 10 cm is placed coaxially 5 cm away from a concave lens of focal length 10 cm. If an object is placed 30 cm in front of the convex lens, find the position of the final image formed by the combined system.

Q.20 A converging lens is kept coaxially in contact with a diverging lens – both the lenses being of equal focal lengths. What is the focal length of the combination?

Q.21 (a) (i) Draw a neat labeled ray diagram of an astronomical telescope in the normal adjustment. Explain briefly its working.

(ii) An astronomical telescope uses two lenses of power 10 D and 1 D. What is its magnifying power in the normal adjustment?

(b) (i) Draw a neat labeled ray diagram of a compound microscope. Explain briefly its working.

(ii) Why must both the objective and the eyepiece of a compound microscope have short focal lengths?

Q.22 Draw a labeled ray diagram of a reflecting telescope. Mention its two advantages over the refracting telescope.

Q.23 (i) An object is placed between two plane mirrors inclined at 60^o to each other. How many images do you expect to see?

(ii) An object is placed between two plane parallel mirrors. Why do the distant images get fainter and fainter?

Q.24 The magnifying power of a compound microscope is 20. The focal length of its eye piece is 3 cm. Calculate the magnification produced by the objective lens.

Q.25 An astronomical telescope having a magnifying power of 8 consists of two thin lenses 45 cm apart. Find the focal length of the lenses.

Exercise 2

Single Correct Choice Type

Q.1 Two plane mirrors are inclined at 70^o. A ray incident on one mirror at an angle of θ after reflection falls on the second mirror and is reflected from there parallel to the first mirror, θ is:

(A) 50^o (B) 45^o

(C) 30^o (D) 55^o

Q.2 There are two plane mirror with reflecting surfaces facing each other. Both the mirrors are moving with the speed of v away from each other. A point object is placed between the mirrors. The velocity of the image formed due to the n^{th} reflection will be

(A) nv (B) $2nv$

(C) $3nv$ (D) $4nv$

Q.3 A man of height 'h' is walking away from a street lamp with a constant speed 'v'. The height of the street lamp is $3h$. The rate at which the length of the man's shadow is increasing when he is at a distance of $10h$ from the base of the street lamp is:

(A) $v/2$ (B) $v/3$

(C) $2v$ (D) $v/6$

Q.4 A boy of height 1.5 m with his eye level at 1.4 m stands before a plane mirror of length 0.75 m fixed on the wall. The height of the lower edge of the mirror above the floor is 0.8 m. Then,

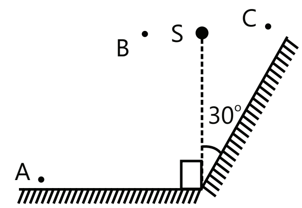
(A) The boy will see his full image.

(B) The boy cannot see his hair.

(C) The boy cannot see his feet.

(D) The boy cannot see neither his hair nor his feet.

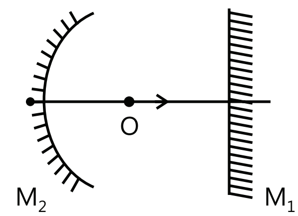
Q.5 A point source of light S is placed in front of two large mirrors as shown. Which of the following observers will see only one image of S?



(A) Only A (B) Only C

(C) Both A and C (B) Both B and C

Q.6 In the figure shown if the object 'O' moves toward the plane mirror, then the image I (which is formed after successive reflections from M_1 & M_2, respectively) will move:



(A) Toward right

(B) Toward left

(C) With zero velocity

(D) Cannot be determined

Q.7 A point source of light is 60 cm away from a screen and is kept at the focus of a concave mirror that reflects light on the screen. The focal length of the mirror is 20 cm. The ratio of average intensities of the illumination on the screen when the mirror is present and when the mirror is removed is:

(A) 36:1 (B) 37:1

(C) 49:1 (D) 10:1

Q.8 A concave mirror is placed on a horizontal table, with its axis directed vertically upward. Let O be the pole of the mirror and C its center of curvature. A point object is placed at C. It has a real image, also located at C (a condition called auto-collimation). If the mirror is now filled with water, the image will be:

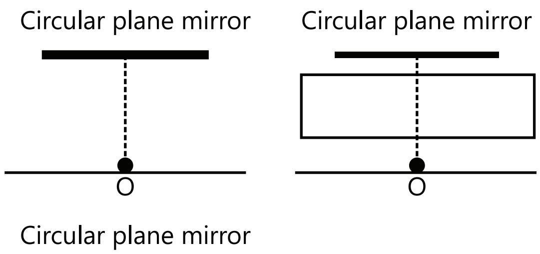
(A) Real and will remain at C

(B) Real and located at a point between C and ∞

(C) Virtual and located at a point between C and O

(D) Real and located at a point between C and O

Q.9 In the diagram shown below, a point source O is placed vertically below the center of a circular plane mirror. The light rays starting from the source are reflected from the mirror such that a circular area A on the ground receives light. Now, a glass slab is placed between the mirror and the source O. What will be the magnitude of the new area on the ground receiving light?



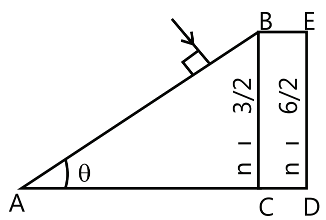
(A) A

(B) Greater than A

(C) Less than A

(D) Cannot say, as the information given is insufficient

Q.10 In the figure ABC is the cross section of a right-angled prism, and BCDE is the cross section of a glass slab. The value of θ, so that light incident normally on the face AB does not cross the face BC, is (given $\sin^{-1}(3/5) = 37^o$)



(A) $\theta \le 37^o$ (B) $\theta > 37^o$

(C) $\theta \le 53^o$ (D) $\theta < 53^o$

Q.11 A small source of light is 4 m below the surface of a liquid of refractive index 5/3. In order to cut off all the light coming out of liquid surface, the minimum diameter of the disc placed on the surface of liquid is:

(A) 3m (B) 4m (C) 6m (D) ∞

Q.12 A point source of light is placed at a distance *h* below the surface of a large deep lake. What is the percentage of light energy that escapes directly from the water surface? μ of the water=4/3? (Neglect partial reflection)

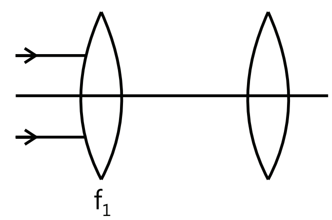
(A) 50% (B) 25% (C) 20% (D) 17%

Q.13 When the object is at distances u_1 and u_2, the images formed by the same lens are real and virtual, respectively, and of same size. Then, the focal length of the lens is:

(A) $\frac{1}{2}\sqrt{u_1u_2}$ (B) $\frac{1}{2}(u_1+u_2)$

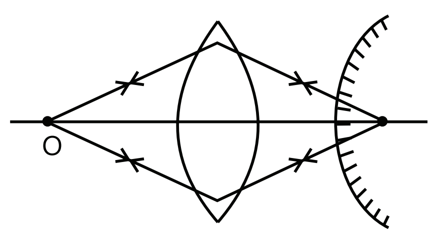
(C) $\sqrt{u_1u_2}$ (D) $2(u_1+u_2)$

Q.14 Parallel beam of light is incident on a system of two convex lenses of focal lengths f_1 = 20 cm and f_2 = 10 cm. What should be the distance between the two lenses so that rays after refraction from both lenses get un-deviated?



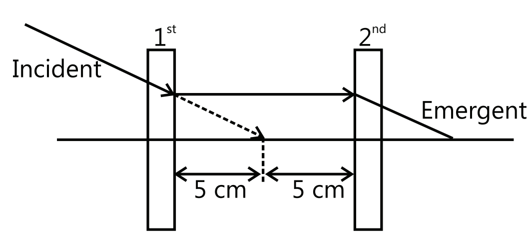
(A) 60 cm (B) 30 cm (C) 90 cm (D) 40 cm

Q.15 An object is placed at a distance of 15 cm from a convex lens of a focal length 10 cm. On the other side of the lens, a convex mirror is placed at its focus such that the image formed by the combination coincides with the object itself. The focal length of the convex mirror is



(A) 20 cm (B) 10 cm (C) 15 cm (D) 30 cm

Q.16 Look at the ray diagram shown, what will be the focal length of the 1st and the 2nd lens, if the incident light ray is parallel to emergent ray.



(A) −5cm and −10cm (B) +5cm and +10cm

(C) −5cm and +5cm (D) +5cm and +5cm

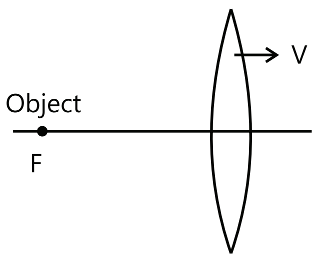
Q.17 A point object is kept at the first focus of a convex lens. If the lens starts moving toward right with a constant velocity, the image will

(A) Always move toward right

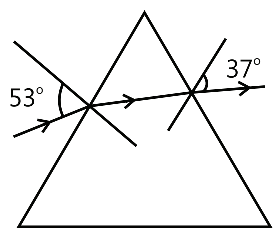
(B) Always move toward left

(C) First move toward right and then toward left.

(D) First move toward left and then toward right.



Q.18 A ray incident at an angle 53° on a prism emerges at an angle of 37° as shown. If the angle of incidence is 50°, which of the following is a possible value of the angle of emergence



(A) 35° (B) 42° (C) 40° (D) 38°

Q.19 A prism has a refractive index $\sqrt{\frac{3}{2}}$ and a refracting angle 90°. Find the minimum deviation produced by the prism.

(A) 40° (B) 45° (C) 30° (D) 49°

Q.20 A certain prism is found to produce a minimum deviation of 38°. It produces a deviation of 44° when the angle of incidence is either 42° or 62°. What is the angle of incidence when it is undergoing a minimum deviation?

(A) 45° (B) 49° (C) 40° (D) 55°

Q.21 A thin prism of angle 5° is placed at a distance of 10 cm from the object. What is the distance of the image from the object? (Given μ of prism = 1.5)

(A) $\frac{\pi}{8}$cm (B) $\frac{\pi}{12}$cm

(C) $\frac{5\pi}{36}$cm (D) $\frac{\pi}{7}$cm

Q.22 Light ray is incident on a prism of angle $A = 60^\circ$ and refractive index $\mu = \sqrt{2}$. The angle of incidence at which the emergent ray grazes the surface is given by

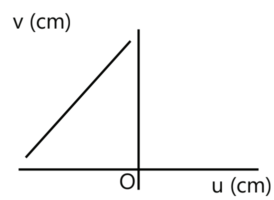
(A) $\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$ (B) $\sin^{-1}\left(\frac{1-\sqrt{3}}{2}\right)$

(C) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (D) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$

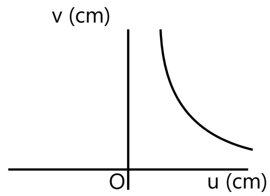
Previous Years' Questions

Q.1 A student measures the focal length of convex lens by putting an object pin at a distance '*u*' from the lens and measuring the distance '*v*' of the image pin. The graph between '*u*' and '*v*' plotted by the student should look like ***(2002)***

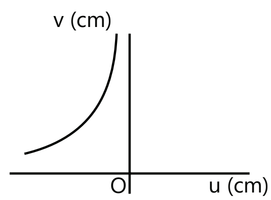
(A)



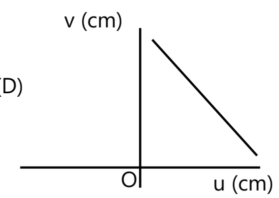
(B)



(C)



(D)



Q.2 An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment, distances are measured by ***(2003)***

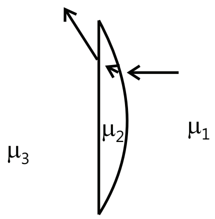
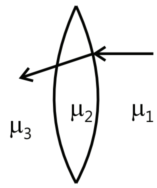
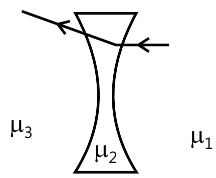
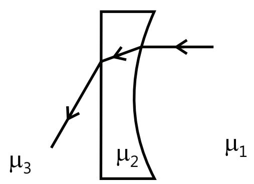
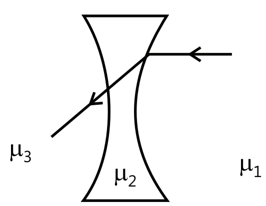
(A) A vernier scale provided on the microscope

(B) A standard laboratory scale

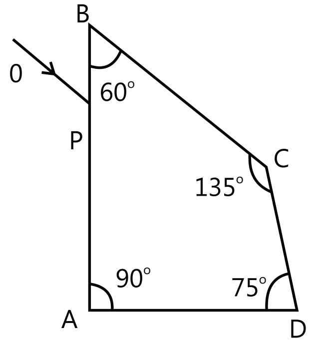
(C) A meter scale provided on the microscope

(D) A screw gauge provided on the microscope

Q.3 Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in the figures in Column II. A ray traversing these media is also shown in the figures. In Column I, the different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagram shown in Column II. ***(2007)***

Column I	Column II
(A) $\mu_1 < \mu_2$	(p) 
(B) $\mu_1 > \mu_2$	(q) 
(C) $\mu_2 = \mu_3$	(r) 
(D) μ	(s) 
	(t) 

Q.4 A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60° (see figure) If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct? ***(2009)***



(A) The ray gets totally internally reflected at face CD.

(B) The ray comes out through face AD.

(C) The angle between the incident ray and the emergent ray is 90°.

(D) The angle between the incident ray and the emergent ray is 120°.

Q.5 The focal length of a thin biconvex lens is 20cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is ***(2005)***

Q.6 An object at distance 2.4 m in front of a lens forms a sharp image on a film at distance 12 cm behind the lens. A 1-cm-thick glass plate of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens), should the object be shifted to be in sharp focus on film? ***(2009)***

(A) 7.2 m (B) 2.4 m (C) 3.2 m (D) 5.6m

Q.7 A spectrometer gives the following reading when used to measure the angle of prism.

Main scale reading: 58.5°

Vernier scale reading: 09 divisions

Given that 1 division on the main scale corresponds to 0.5°. Total divisions on the vernier scale are 30 and match with 29 divisions of the main scale. The angle of the prism from the above data ***(2011)***

(A) 58.59° (B) 58.77°

(C) 58.65° (D) 59°

Q.8 An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants, and I is the intensity of the light beam. The intensity of the beam is decreasing with an increasing radius.

As the beam enters the medium, it will ***(2013)***

(A) Diverge

(B) Converge

(C) Diverge near the axis and converge near the periphery

(D) Travel as a cylindrical beam 4

Q.9 The initial shape of the wave in front of the beam is ***(2000)***

(A) Convex

(B) Concave

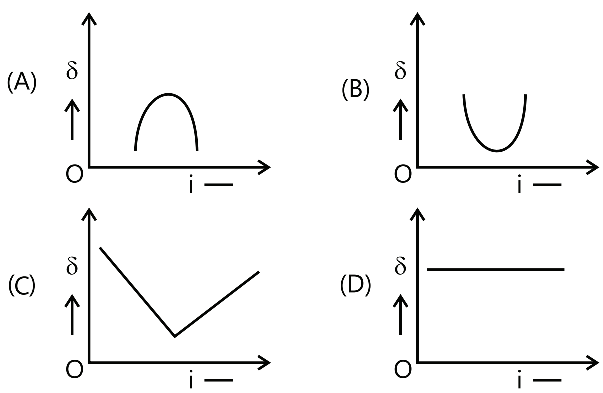
(C) Convex near the axis and concave near the periphery

(D) Planar

Q.10 An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object be shifted to be in sharp focus on film? ***(2012)***

(A) 7.2 m (B) 2.4 m (C) 3.2 m (D) 5.6 m

Q.11 The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by: ***(2013)***



Q.12 Diameter of plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m / s, the focal length of the lens is: ***(2013)***

(A) 20 cm (B) 30 cm (C) 10 cm (D) 15 cm

Q.13 A thin convex lens made from crown glass $\left(\mu=\frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is: ***(2014)***

(A) $f_2 > f$ and f_1 becomes negative

(B) f_1 and f_2 both become negative

(C) $f_1 = f_2 < f$

(D) $f_1 > f$ and f_2 becomes negative

Q.14 A green light is incident from the water to the air – water interface at the critical angle (θ). Select the correct statement ***(2014)***

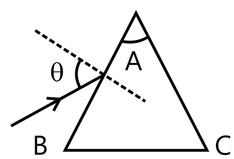
(A) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.

(B) The entire spectrum of visible light will come out of the water at various angles to the normal.

(C) The entire spectrum of visible light will come out of the water at an angle of $90°$ to the normal.

(D) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

Q.15 Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided: ***(2015)***



(A) $\theta - \sin^{-1}\left[\mu \sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(B) $\theta > \cos^{-1}\left[\mu \sin\left(A+\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(C) $\theta < \cos^{-1}\left[\mu \sin\left(A+\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(D) $\theta > \sin^{-1}\left[\mu \sin\left(A-\sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

Q.16 Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is: ***(2015)***

(A) 30 μm (B) 100 μm

(C) 300 μm (D) 1 μm

Q.17 In an experiment for determination of refractive index of glass of a prism by $i-\delta$, plot, it was found that a ray incident at angle $35°$, suffers a deviation of $40°$ and that it emerges at angle $79°$. In that case which of the following is closest to the maximum possible value of the refractive index ? ***(2016)***

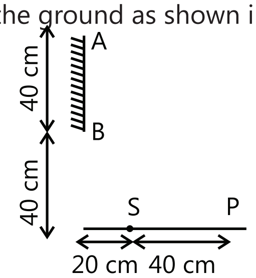
(A) 1.6 (B) 1.7 (C) 1.8 (D) 1.5

JEE Advanced/Boards

Exercise 1

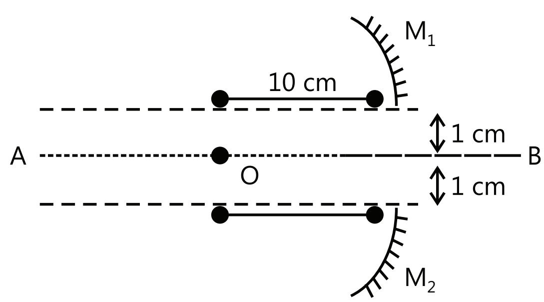
Q.1 Two flat mirrors have their reflecting surfaces facing each other, with an edge of one mirror in contact with an edge of the other so that the angle between the mirrors is $60°$. Find all the angular positions of the image with respect to x-axis. Take the case when a point object is between the mirrors at (1, 1). Point of intersections is (0, 0) and 1st mirror is along the x-axis.

Q.2 In the figure shown, AB is a plane mirror of length 40 cm placed at a height 40 cm from ground. There is a light source S at a point on the ground. Find the minimum and maximum height of a man (eye height) required to see the image of the source if he is standing at a point P on the ground as shown in the Fig.



Q.3 A plane mirror of circular shape with radius r=20 cm is fixed to the ceiling. A bulb is placed on the axis of the mirror. A circular area of radius R=1 on the floor is illuminated after the reflection of light from the mirror. The height of the room is 3 m. What is the maximum distance from the center of the mirror and the bulb so that the required area is illuminated?

Q.4 A concave mirror of focal length 20 cm is cut into two parts from the middle, and the two parts are moved perpendicularly by a distance 1 cm from the previous principal axis AB. Find the distance between the images formed by the two parts?



Q.5 A balloon is rising up along the axis of a concave mirror of radius of curvature 20 m. A ball is dropped from the balloon at a height 15 m from the mirror when the balloon has velocity 20 m/s. Find the speed of the image of the ball formed by a concave mirror after 4 s? [Take: $g = 10\ m/s^2$]

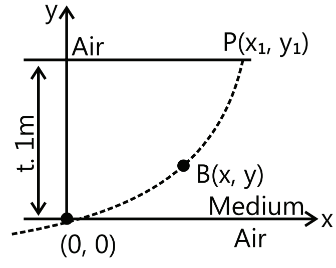
Q.6 An observer whose least distance of distinct vision is '*d*' views his own face in a convex mirror of radius or curvature '*r*'. Prove that the magnification produced cannot exceed $\dfrac{r}{d+\sqrt{d^2+r^2}}$.

Q.7 A surveyor on one bank of canal observes the images of the 4-inch mark and 17-ft mark on a vertical staff, which is partially immersed in the water and held against the bank directly opposite to him. He see that the reflected and refracted rays come from the same point which is the center of the canal. If the 17-ft mark and the surveyor's eye are both 6 ft above the water level, estimate the width of the canal, assuming that the refractive index of the water is 4/3. Zero mark is at the bottom of the canal.

Q.8 A ray of light travelling in air is incident at a grazing angle (incident angle = $90°$) on a long rectangular slab of a transparent medium of thickness t−1.0 (see figure). The point of incidence is the origin A (0, 0). The medium has a variable index of refraction $n(y)$ given by

$$n(y) = \left[ky^{3/2} + 1\right]^{1/2}, \text{ where } k = 1.0\ m^{-3/2},$$

the refractive index of air is 1.0



(i) Obtain a relation between the slope (dy/dx) of the trajectory of the ray at a point B (x, y) in the medium and the incident angle at that point.

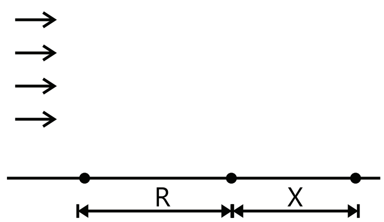
(ii) Find the value of $n \sin i$.

(iii) Obtain an equation for the trajectory $y(x)$ of the ray in the medium.

(iv) Determine the coordinate $(x_1 y_1)$ of point P, where the ray intersects the upper surface of the slab–air boundary.

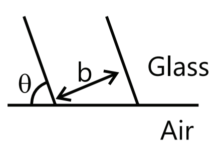
(v) Indicate the path of the ray subsequently.

Q.9 A uniform, horizontal beam of light is incident upon a quarter cylinder of radius R = 5 cm and has a refractive index $2/\sqrt{3}$. A patch on the table at a distance 'x' from the cylinder is unilluminated. Find the value of 'x'?



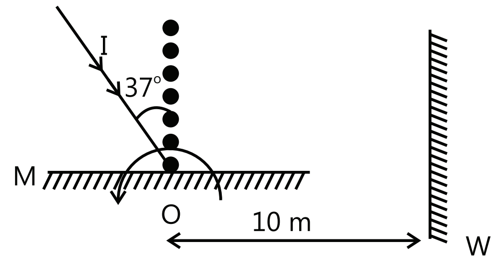
Q.10 An opaque cylindrical tank with an open top has diameter of 3.00 m and is fully filled with water. When the sunlight reaches at an angle of 37° above the horizon, sunlight ceases to illuminate any part of the bottom of the tank. How deep the tank is?

Q.11 A beam of parallel rays of width b propagates in a glass at an angle θ to its plane face. The beam width after it enters into air through this face is _______ if the refractive index of glass is μ.



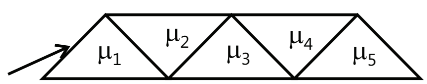
Q.12 A parallel beam of light falls normally on the first face of a prism of a small angle. At the second face, it is partly transmitted and partly reflected, and the reflected beam strikes at the first face again and emerges from it in a direction by making an angle $6°30'$ with the reversed direction of the incident beam. The refracted beam has undergone a deviation of $1°15'$ from the original direction. Find the refractive index of the glass and the angle of the prism.

Q.13 A light ray I is incident on a plane mirror M. The mirror is rotated in the direction as shown in the figure by an arrow at a frequency $9/\pi \text{rev}/\text{sec}$. The light reflected by the mirror is received on the wall W at a distance 10 m from the axis of rotation. When the angle of incidence becomes 37°, find the speed of the spot (a point) on the wall?

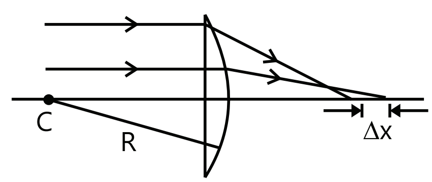


Q.14 The diagram shows five isosceles right-angled prisms. A light ray incident at 90° at the first face emerges at the same angle with the normal from the last face. Find the relation between the refractive indices?

Q.15 Two rays travelling parallel to the principal axis

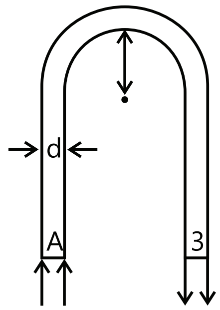


strike a large plano-convex lens of a refractive index of 1.60. If the convex face is spherical, a ray near the edge does not pass through the focal point (spherical aberration occurs). If this face has a radius of curvature of 20.0 cm and the two rays are at $h_1 = 0.50$ cm and $h_2 = 12.0$ cm from the principal axis, then find the difference in the positions where they cross the principal axis.



Q.16 A room contains air in which the speed of sound is 340m/s. The walls of the room are made of concrete, in which the speed of sound is 1700m/s. (a) Find the critical angle for the TIR of sound at the concrete–air boundary. (b) In which medium must the sound be undergone the TIR?

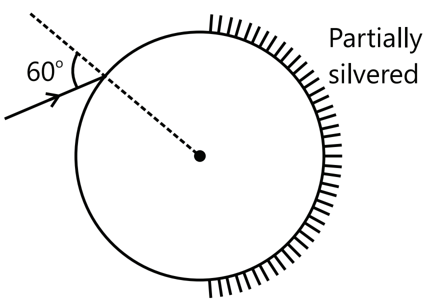
Q.17 A rod made of glass $(\mu = 1.5)$ and of square cross section is bent as shown in the figure. A parallel beam of light falls perpendicularly on the plane flat surface A. Referring to the diagram, d is the width of a side, and R is the radius of inner semicircle. Find the maximum value of ratio $\frac{d}{R}$ so that all the light rays entering the glass through surface A emerge from the glass through surface B.



Q.18 A prism of refractive index $\sqrt{2}$ has a refracting angle of $30°$. One of the refracting surfaces of the prism is polished. For the beam of monochromatic light to retrace its path, find the angle of incidence on the refracting surface.

Q.19 An equilateral prism deviates a ray by $23°$ for two angles of incidence differing by $23°$. Find μ of the prism?

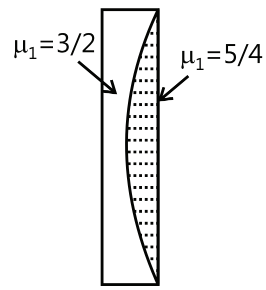
Q.20 A ray is incident on a glass sphere as shown in the figure The opposite surface of the sphere is partially coated with silver. If the net deviation of the ray transmitted at the partially silvered surface is 1/3rd of the net deviation suffered by the ray reflected at the partially silvered surface (after emerging out of the sphere), find the refractive index of the sphere.



Q.21 Two thin similar watch glass pieces are joined together front to front, with rear convex portion is coated with silver, and the combination of glass pieces is placed at a distance $\alpha = 60$ cm from a screen. A small object is placed normal to the optical axis of the combination such that its two times magnified image is formed on the screen. If air between the glass pieces is replaced by water $(\mu = 4/3)$, calculate the distance through which the object must be displaced so that a sharp image is again formed on the screen.

Q.22 A spherical light bulb with a diameter of 3.0 cm radiates light equally in all directions, with a power of 4.5π W. (a) Find the light intensity at the surface of the bulb. (b) Find the light intensity 7.50 m from the center of the bulb. (c) At this 7.50-m distance, a convex lens is set up with its axis pointing toward the bulb. The lens has a circular face with a diameter of 15.0 cm and a focal length of 30.0 cm. Find the diameter of the image of the bulb formed on a screen kept at the location of the image. (d) Find the light intensity at the image.

Q.23 A thin plano-convex lens fits exactly into a plano-concave lens with their plane surface parallel to each other as shown in the figure. The radius of curvature of the curved surface R = 30 cm. The lenses are made of different material having a refractive index $\mu_1 = 3/2$ and $\mu_2 = 5/4$ as shown in the Fig.

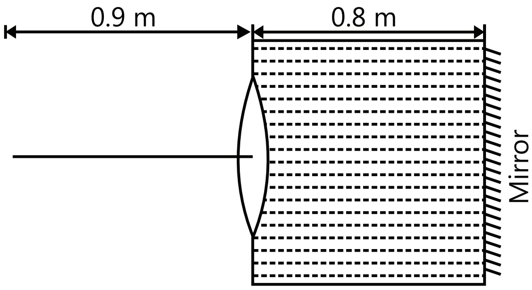


(i) If plane surface of the plano-convex lens is coated with silver, then calculate the equivalent focal length of this system and also the nature of this equivalent mirror.

(ii) An object having a transverse length of 5 cm in placed on the axis of equivalent mirror (in par 1) at a distance 15 cm from the equivalent mirror along the principal axis. Find the transverse magnification produced by the equivalent mirror.

Q.24 Two identical convex lenses L_1 and L_2 are placed at a distance of 20 cm from each other on the common principal axis. The focal length of each lens is 15 cm, and lens L_2 is placed right to lens L_1. A point object is placed at a distance of 20 cm left to lens L_1 on the common principal axis of two lenses. Find where a convex mirror of radius of curvature 5 cm should be placed to the right of L_2 so that the final image coincides with the object?

Q.25 A thin equiconvex lens of glass of a refractive index $\mu = 3/2$ and a focal length 0.3 m in air is sealed into an opening at one end of a tank filled with water $(\mu = 4/3)$. On the opposite side of the lens, a mirror is placed inside the tank on the tank wall perpendicular to the lens axis, as shown in the figure. The distance between the lens and the mirror is 0.8 m. A small object is placed outside the tank in front of the lens at a distance of 0.9 m from the lens along its axis. Find the position (relative to the lens) of the image of the object formed by the system.



Exercise 2

Single Correct Choice Type

Q.1 An object is placed in front of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cm, there is no gap between the images formed by the two mirrors. The radius of the convex mirror is:

(A) 12.5 cm (B) 25 cm

(C) 50 cm (D) 100 cm

Q.2 An infinitely long rod lies along the axis of a concave mirror of a focal length *f*. the near end of the rod is at a distance *u*>*f* from the mirror. Its image will have a length.

(A) $f^2/(u-f)$ (B) $uf/(u-f)$

(C) $f^2/(u+f)$ (D) $uf/(u+f)$

Q.3 A luminous point object is moving along the principal axis of a concave mirror of a focal length 12 cm toward it. When its distance from mirror is 20 cm, its velocity is 4 cm/s. The velocity of the image in cm/s at that instant is:

(A) 6 toward the mirror

(B) 6 away from the mirror

(C) 9 away from the mirror

(D) 9 toward the mirror

Q.4 A thin lens has a focal length *f*, and its aperture has a diameter *d*. It forms an image of intensity *I*. Now the central part of the aperture up to diameter (*d*/2) is blocked by an opaque paper. The focal length and image intensity would change to

(A) *f*/2, *I*/2 (B) *f*, *I*/4

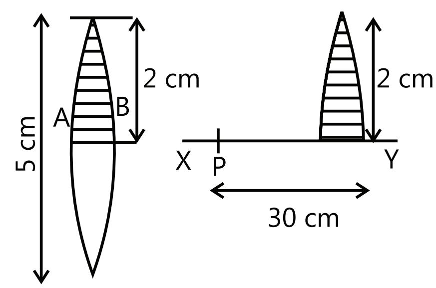
(C) 3*f*/4, *I*/2 (D) *f*, 3*I*/4

Q.5 An object is placed in front of a thin convex lens of focal length 30 cm, and a plane mirror is placed 15 cm behind the lens. If the final image of the object coincides with the object, the distance of the object from the lens is

(A) 60 cm (B) 30 cm

(C) 15 cm (D) 25 cm

Q.6 A converging lens of a focal length 20 cm and diameter 5 cm is cut along line AB. The part of the lens shaded in the diagram is used to form an image of a point P placed 30 cm away from it on line XY, which is perpendicular to the plane of the lens. The image of P will be formed.



(A) 0.5 cm above XY (B) 1 cm below XY

(C) On XY (D) 1.5 cm below XY

Q.7 A screen is placed 90 cm away from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. The focal length of the lens is

(A) 18 cm (B) 21.4 cm

(C) 60 cm (D) 85.6 cm

Q.8 In the above problem, if the sizes of the images formed on the screen are 6 cm and 3 cm, then the height of the object is nearly:

(A) 4.2 cm (B) 4.5 cm

(C) 5 cm (D) 9 cm

Q.9 A concave mirror cannot form

(A) A virtual image of a virtual object

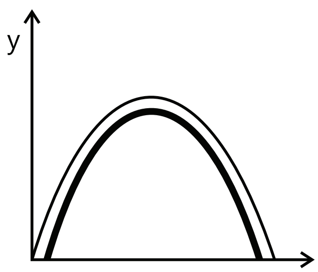
(B) A virtual image of a real object

(C) A real image of a real object

(D) A real image of a virtual object

Multiple Correct Choice Type

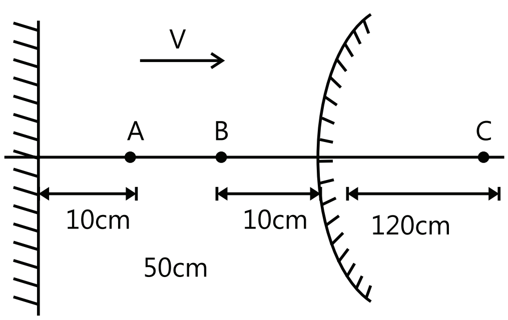
Q.10 A reflecting surface is represented by the equation $y=\frac{2L}{\pi}\sin\left(\frac{\pi x}{L}\right), 0 \le x \le L$. A ray travelling horizontally becomes vertical after reflection. The coordinates of the point(s) where this ray is incident is



(A) $\left(\frac{L}{4}, \frac{\sqrt{2}L}{\pi}\right)$ (B) $\left(\frac{L}{3}, \frac{\sqrt{3}L}{\pi}\right)$

(C) $\left(\frac{3L}{4}, \frac{\sqrt{2}L}{\pi}\right)$ (D) $\left(\frac{2L}{3}, \frac{\sqrt{3}L}{\pi}\right)$

Q.11 In the figure shown, consider the first reflection at the plane mirror and second at the convex mirror. AB is object.



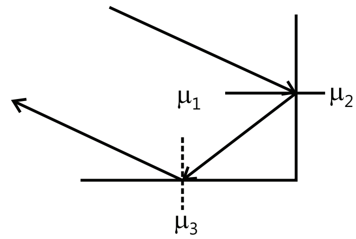
(A) The second image is real, inverted by 1/5th magnification w.r.t. AB.

(B) The second image is virtual and erect by 1/5th magnification w.r.t. AB.

(C) The second image moves toward the convex mirror.

(D) The second image moves away from the convex mirror.

Q.12 In the diagram shown, a ray of light is incident on the interface between 1 and 2 at an angle slightly greater than the critical angle. The light undergoes TIR at this interface. After that, the light ray falls at interfaces 1 and 3, and again it undergoes TIR. Which of the following relations should hold true?



(A) $\mu_1 < \mu_2 < \mu_3$ (B) $\mu_1^2 - \mu_2^2 > \mu_3^2$

(C) $\mu_1^2 - \mu_3^2 > \mu_2^2$ (D) $\mu_1^2 + \mu_2^2 > \mu_3^2$

Q.13 For refraction through a small angle prism, the angle of deviation:

(A) Increases with an increase in RI of prism.

(B) Will decrease with an increase in RI of prism.

(C) Is directly proportional to the angle of prism.

(D) Will be 2 D for a ray of RI = 2.4 if it is *d* for a ray of RI = 1.2.

Q.14 For the refraction of light through a prism

(A) For every angle of deviation, there are two angles of incidence.

(B) The light travelling inside an equilateral prism is necessarily parallel to the base when prism is set for a minimum deviation.

(C) There are two angles of incidence for a maximum deviation.

(D) The angle of minimum deviation will increase if refractive index of prism is increased keeping the outside medium unchanged if $\mu_p > \mu_s$.

Q.15 A man of height 170 cm wants to see his complete image in a plane mirror (while standing). His eyes are at a height of 160 cm from the ground.

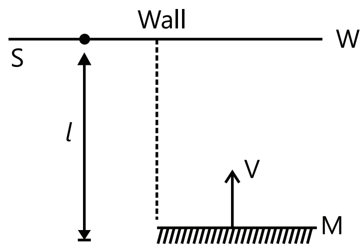
(A) Minimum length of the mirror=80 cm.

(B) Minimum length of the mirror=85 cm.

(C) Bottom of the mirror should be at a height 80 cm or less.

(D) Bottom of the mirror should be at a height 85 cm.

Q.16 A flat mirror M is arranged parallel to a wall W at a distance l from it. The light produced by a point sources S kept on the wall is reflected by the mirror and produces a light spot on the wall. The mirror moves with a velocity *v* toward the wall.



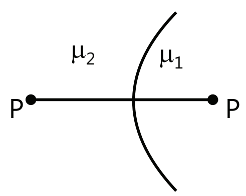
(A) The spot of light will move with the speed *v* on the wall.

(B) The spot of light will not move on the wall.

(C) As the mirror comes closer, the spot of light will become larger and shift away from the wall with a speed larger than *v*.

(D) The size of the light spot on the wall remains the same.

Q.17 Two reflecting media are separated by a spherical interface as shown in the figure. PP′ is the principal axis; μ_1 and μ_2 are the medium of refraction, respectively, then,



(A) If $\mu_2 > \mu_1$, then there cannot be a real image of a real project.

(B) If $\mu_2 > \mu_1$, then there cannot be a real image of a virtual object.

(C) If $\mu_1 > \mu_2$, then there cannot be a virtual image of a virtual object.

(D) If $\mu_1 > \mu_2$, then there cannot be a real image of a real object.

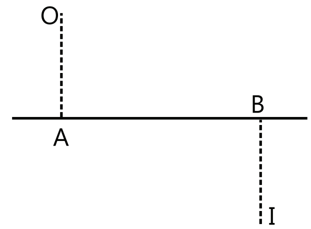
Q.18 A luminous point object is placed at O. whose image is formed at I as shown in the figure. AB is the optical axis. Which of the following statements are correct?

(A) If a lens is used to obtain an image, the lens must be converging.

(B) If a mirror is used to obtain an image, the mirror must be a convex mirror having a pole at the point of intersection of lines OI and AB.

(C) Position of the principal focus of mirror cannot be found.

(D) I is a real image.



Q.19 A lens is placed in the XYZ coordinate system such that its optical center is the origin and the principal axis is along the x-axis. The focal length of the lens is 20 cm. A point object has been placed at the point (-40 cm, +1 cm, −1 cm). Which of the following are correct about coordinates of the image?

(A) x = 40 cm (B) y =+1 cm

(C) z = +1 cm (D) z=−1 cm

Q.20 Which of the following can form a diminished, virtual and erect image of your face.

(A) Converging mirror (B) Diverging mirror

(C) Converging lens (D) Diverging lens

Assertion Reasoning Type

Q.21 Statement-I: If a source of light is placed in front of rough wall, its image is not seen.

Statement-II: The wall does not reflect light.

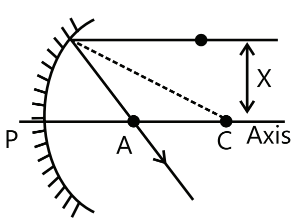
(A) Statement-I is true, and statement-II is true; statement-II is correct explanation for statement-I

(B) Statement-I is true, and statement-II is true; statement -II is NOT the correct explanation for statement-I.

(C) Statement-I is true, and statement-II is false.

(D) Statement-I is false, and statement-II is true.

Q.22 Statement-I: As the distance x of a parallel ray from axis increases, the focal length decreases



Statement-II: As x increases, the distance from the pole to the point of intersection of a reflected ray with the principal axis decreases.

(A) Statement-I is true, and statement-II is true; statement-II is correct explanation for statement-I.

(B) Statement-I is true, and statement-II is true; statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, and statement-II is false.

(D) Statement-I is false, and statement-II is true.

Q.23 Statement-I: When an object dipped in a liquid is viewed normally, the distance between the image and the object is independent of the height of the liquid above the object.

Statement-II: The normal shift is independent of the location of the slab between the object and the observer.

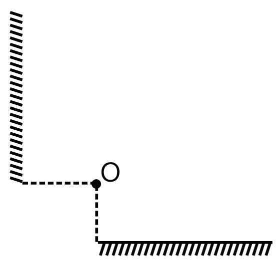
(A) Statement-I is true, statement-II is true, and statement-II is the correct explanation for statement-I.

(B) Statement-I is true, statement-II is true, and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, and statement-II is false.

(D) Statement-I is false, and statement-II is true.

Q.24 Statement-I: When two plane mirrors are kept perpendicular to each other as shown in the figure (O is the point object), three images will be formed.



Statement-II: In case of a multiple reflection, the image of one surface can act as an object for the next surface.

(A) Statement-I is true, statement-II is true, and statement-II is the correct explanation for statement-I.

(B) Statement-I is true, statement-II is true, and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, and statement-II is false.

(D) Statement-I is false, and statement-II is true.

Q.25 Statement-I: Keeping a point object fixed, if a plane mirror is moved, the image will definitely move.

Statement-II: In case of a plane mirror, the distance between a point object and its image from a given point on mirror is equal.

(A) Statement-I is true, statement-II is true, and statement-II is the correct explanation for statement-I.

(B) Statement-I is true, statement-II is true, and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, and statement-II is false.

(D) Statement-I is false, and statement-II is true.

Comprehension Type

Paragraph 1: Spherical aberration in spherical mirrors is a defect that is due to the dependence of focal length '*f*' on the angle of incidence 'θ' as shown in the figure is given by

$$f = R - \frac{R}{2}\sec\theta,$$

where *R* is radius of curvature of mirror and *q* is the angle of incidence. The rays that are close to the principal axis are called marginal rays. As a result, different rays focus at different points and the image of a point object is not a point.

Q.26 If f_p and f_m represent the focal length of paraxial and marginal rays, respectively, then the correct relationship is:

(A) $f_p = f_m$ (B) $f_p > f_m$

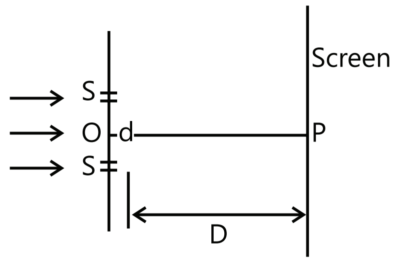
(C) $f_p < f_m$ (D) none

Q.27 If the angle of incidence is 60°, then the focal length of this ray is:

(A) *R* (B) *R*/2

(C) 2*R* (D) 0

Paragraph 2: A student is performing Young's double slit experiment. There are two slits S_1 and S_2. The distance between them is *d*. There is a large screen at a distance *D* (*D* >>*d*) from the slits. The setup is shown in the following figure. A parallel beam of light is incident on it. A monochromatic light of wavelength λ is used. The initial phase difference between the two slits behave as two coherent sources of light is zero. The intensities of light waves on the screen coming out of S_1 and S_2 are same, i.e. I_0. In this situation, the principal maximum is formed at point P. At the point on screen where the principal maximum is formed, the phase difference between two interfering waves is zero.



Q.28 The total deviation suffered by the ray falling on the mirror at an angle of incidence 60° is

(A) 180° (B) 90°

(C) Cannot be determined (D) None

Q.29 For paraxial rays, focal length approximately is

(A) *R* (B) *R*/2

(C) 2*R* (D) none

Q.30 Which of the following statements are correct regarding spherical aberration:

(A) It can be completely eliminated.

(B) It cannot be completely eliminated, but it can be minimized by allowing either paraxial or marginal rays to hit the mirror.

(C) It is reduced by taking mirrors with large aperture.

(D) None of these.

Q.31 Initially, the distance of third minima from principal maxima will be

(A) $\frac{3\lambda D}{2d}$ (B) $\frac{3\lambda D}{d}$

(C) $\frac{5\lambda D}{4d}$ (D) $\frac{5\lambda D}{2d}$

Q.32 A glass slab of thickness *t* and refractive index μ is introduced before S_2. Now, P does not remain the point of principal maximum. Suppose the principal maximum forms at a point P' on screen, then PP' is equal to

(A) $\frac{tD(\mu-1)}{d}$ (B) $\frac{tD(\mu-1)}{2d}$

(C) $\frac{D(\mu-1)}{t}$ (D) $\frac{D(\mu-1)}{d}$

Q.33 Use the statement given in previous question. Now, a parallel beam is incident at an angle α w.r.t. line OP, such that the principal maximum again comes at point P (see figure). The value of α is

(A) $\sin^{-1}\frac{t(\mu-1)}{d}$ (B) $\cos^{-1}\frac{t(\mu-1)}{d}$

(C) $\sin^{-1}\frac{t(\mu-1)D}{d}$ (D) $\sin^{-1}\frac{tD}{d}$

Q.34 Match the Column

Column I	Column II
(A) Conversing system	(p) Convex lens
(B) Concave lens	(q) Concave lens
(C) A virtual image is formed by	(r) Concave mirror
(D) Magnification < 1 is possible with	(s) Convex mirror

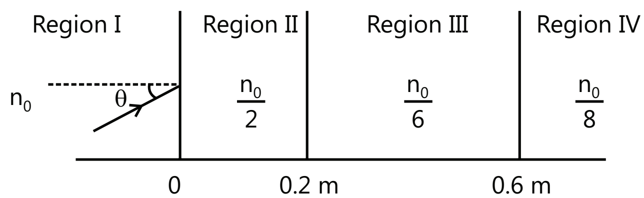
Previous Years' Questions

Q.1 A student performed the experiment of determination of the focal length of a concave mirror by *u-v* method using an optical bench of length 1.5 m. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The five sets of (*u*, *v*) values recorded by the student (in cm) are: (42, 56), (48, 48), (60, 40), (66, 33) and (78, 39). The data set (s) that cannot come from experiment and is (are) incorrectly recorded, is (are) ***(1999)***

(A) (42, 56) (B) (48, 48)

(C) (66, 33) (D) (78, 39)

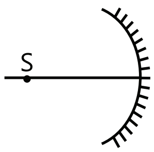
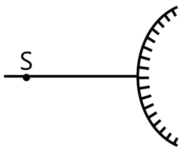
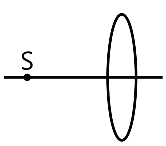
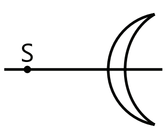
Q.2 A light beam travels from Region I to Region IV (See figure). The refractive index in Regions I, II, III and IV are $n_0, \frac{n_0}{6}$ and $\frac{n_0}{8}$, respectively. The angle of incidence θ for which the beam misses entering Region IV (as in the figure): ***(2004)***



(A) $\sin^{-1}\left(\frac{3}{4}\right)$ (B) $\sin^{-1}\left(\frac{1}{8}\right)$

(C) $\sin^{-1}\left(\frac{1}{4}\right)$ (D) $\sin^{-1}\left(\frac{1}{3}\right)$

Q.3 An optical component and an object s placed along its optical axis are given in column I. The distance between the object and the component can be varied. The properties of images are given in column II. Match all the properties of images from column II with the appropriate components given in column I. Indicate your answer by darkening the appropriate bubbles of the 4 x 4 matrix given in the ORS. ***(2006)***

Column I	**Column II**
(A) 	(p) Real image
(B) 	(q) Virtual image
(C) 	(r) Magnified image
(D) 	(s) Image at infinity

Q.4 Two beams of red and violet colors are pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be ***(2007)***

(A) 30° for both the colors

(B) Greater for the violet color

(C) Greater for the red color

(D) Equal but not 30° for both the colors

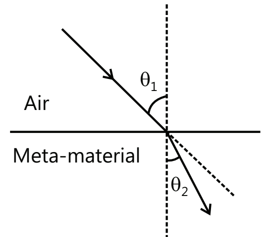
Q.5 Which one of the following statements is WRONG in the context of X-rays generated from an X-ray tube? ***(2001)***

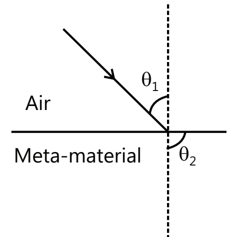
(A) The wavelength of the characteristics X-rays decreases when the atomic number of the target increases.

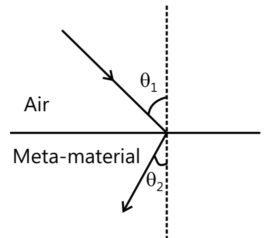
(B) The cutoff wavelength of the continuous X-rays depends on the atomic number of the target.

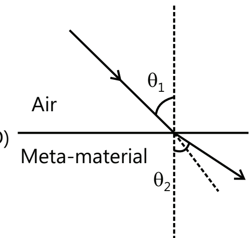
(C) The intensity of the characteristics X-rays depends on the electrical power given to the X-ray tube.

(D) Cutoff wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube.

(A) 

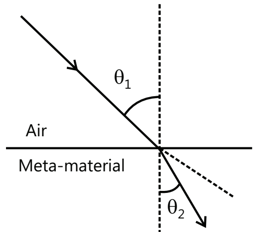
(B) 

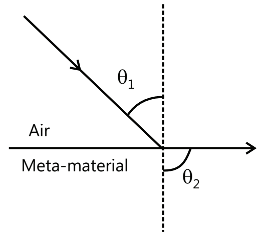
(C) 

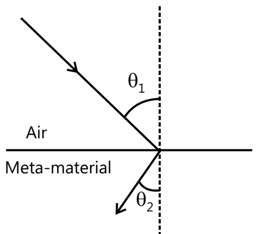
(D) 

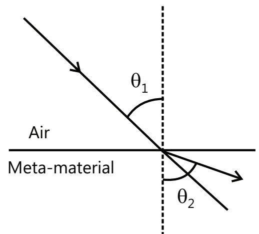
Paragraph: Most materials have a refractive index $n > 1$. Therefore, when a light ray from air enters a naturally occurring material, then by the Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_2}$, it is understood that the refracted ray bends toward the normal, but it never emerges on the same side of the normal as the incident ray. According to the electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_1\mu_1}$, where *c* is the speed of electromagnetic waves in vacuum, *v* is its speed in the medium, ε_1 and μ_1 are the relative permittivity and permeability of the medium, respectively. In a normal material, both ε_1 and μ_1 are positive, implying positive *n* for the medium. When both ε_1 and μ_1 are negative, one must choose the negative root of *n*. Such materials with negative refractive indices can now be artificially prepared and are called meta-materials. They exhibit a significantly different optical behavior, without violating any physical laws. Since *n* is negative, it results in a change in the direction of propagation of the refracted light. However, similar to the normal materials, the frequency of light remains unchanged upon refraction even in meta-materials. ***(2012)***

Q.6 For light incident from air on a meta-material, the appropriate ray diagram is

(A)


(B)


(C)


(D)


Q.7 Choose the correct statement.

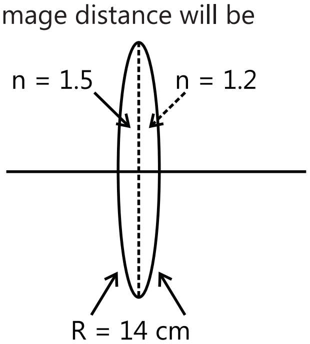
(A) The speed of light in the meta-material is $v = c|n|$.

(B) The speed of light in the meta-material is $v = \frac{c}{|n|}$.

(C) The speed of light in the meta-materials is v = c.

(D) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, where λ_{air} is the wavelength of the light in air.

Q.8 A biconvex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index *n* of the first lens is 1.5 and that of the second lens is 1.2. Both curved surfaces are of same radius of curvature *R* = 14 cm. For this biconvex lens, for an object distance of 40 cm, the image distance will be ***(2009)***



Q.9 A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is ***(2011)***

(A) Virtual and at a distance of 16 cm from the mirror

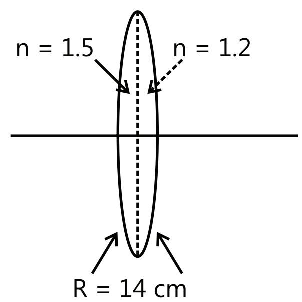
(B) Real and at a distance of 16 cm from the mirror

(C) Virtual and at a distance of 20 cm from the mirror

(D) Real and at a distance of 20 cm from the mirror

Q.10 The image of an object approaching a convex mirror of a radius of curvature 20 m along its optical axis moves from $\frac{25}{3}$m to $\frac{50}{7}$m in 30 s. What is the speed of the object in km/h? ***(2011)***

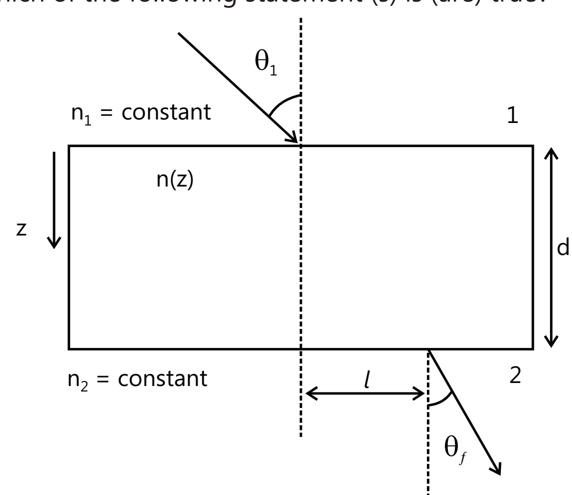
Q.11 A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature R = 14 cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be – ***(2012)***



(A) – 280.0 cm (B) 40.0 cm (C) 21.5 cm (D) 13.3 cm

Q.12 A transparent slab of thickness d has a refractive index n (z) that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and $n_2 (> n_1)$, as shown in the figure. A ray of light is incident with angle θ_1 from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement *l*. ***(2016)***

Which of the following statement (s) is (are) true?



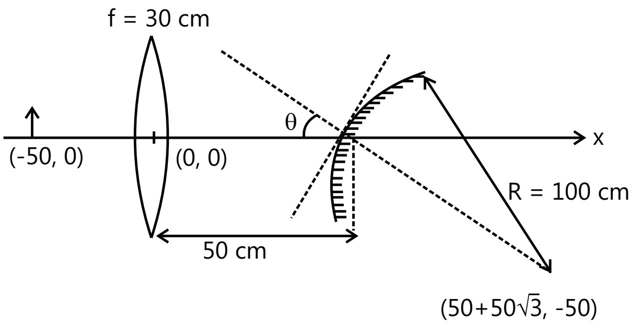
(A) $n_1 \sin\theta_i = n_2 \sin\theta_f$

(B) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$

(C) l is independent of n_2

(D) l is dependent on n (z)

Q.13 A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle $\theta = 30°$ to the axis of the lens, as shown in the figure. ***(2015)***



If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are (2016)

(A) $(25, 25\sqrt{3})$ (B) $(125/3, 25/\sqrt{3})$

(C) $(50 - 25\sqrt{3}, 25)$ (D) (0, 0)

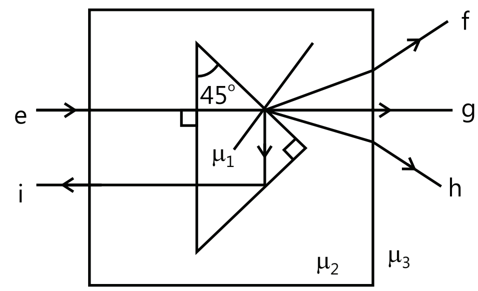
Q.14 A ray of light travelling in the direction $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$. The angle of incidence is ***(2013)***

(A) 30° (B) 45° (C) 60° (D) 75°

Q.15 The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is ***(2013)***

(A) 1 m (B) 2 m (C) 3 m (D) 4 m

Q.16 A right angled prism of refractive index μ_1, is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure. A ray of light 'e' enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2 and μ_3, it takes one of the four possible paths 'ef', 'eg', 'eh', or 'ei'.



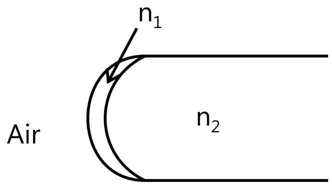
Match the paths in list I with conditions of refractive indices in list II and select the correct answer using the codes given below the lists: ***(2013)***

	List I		**List II**
P.	$e \to f$	1.	$\mu_1 > \sqrt{2}\,\mu_2$
Q.	$e \to g$	2.	$\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
R.	$e \to h$	3.	$\mu_1 = \mu_2$
S.	$e \to i$	4.	$\mu_2 < \mu_1 < \sqrt{2}\,\mu_2$ and $\mu_2 > \mu_3$

Codes:

	P	**Q**	**R**	**S**
A	2	3	1	4
B	1	2	4	3
C	4	1	2	3
D	2	3	4	1

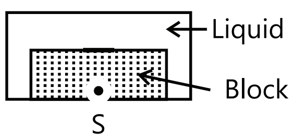
Q.17 A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then *(2014)*



(A) $|f_1| = 3R$ (B) $|f_1| = 2.8R$

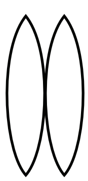
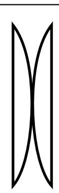
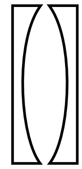
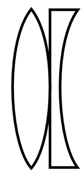
(C) $|f_2| = 2R$ (D) $|f_2| = 1.4R$

Q.18 A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is *(2014)*



(A) 1.21 (B) 1.30 (C) 1.36 (D) 1.42

Q.19 Four combinations of two thin lenses are given in list I. The radius of curvature of all curved surfaces is r and the refractive index of all the lenses is 1.5. Match lens combinations in List I with their focal length in list II and select the correct answer using the code given below the lists. *(2014)*

	List I		**List II**
P			2 r
Q			r/2
R			- r
S			r

Code:

(A) P-1, Q-2, R-3, S-4

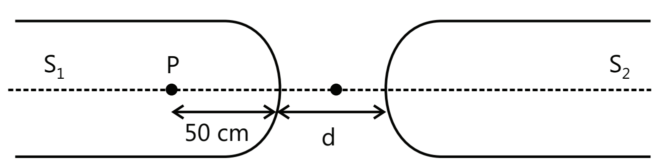
(B) P-2, Q-4, R-3, S-1

(C) P-4, Q-1,R-2, S-3

(D) P-2, Q-1, R-3, S-4

Q.20 Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index 7/6, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is *(2015)*

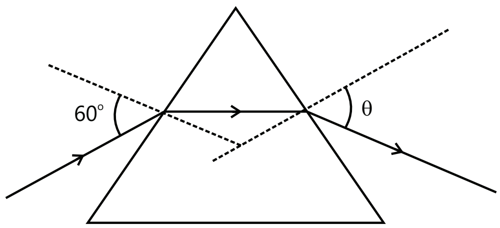
Q.21 Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is *(2015)*



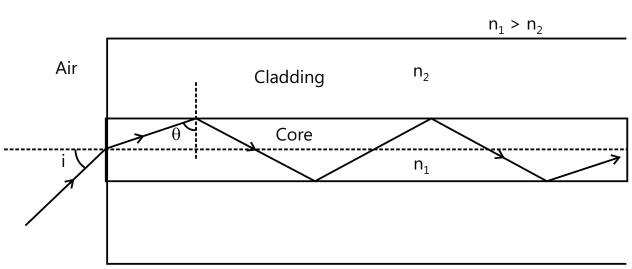
(A) 60 cm (B) 70 cm

(C) 80 cm (D) 90 cm

Q.22 A monochromatic beam of light is incident at $60°$ on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see the figure). For $n=\sqrt{3}$ the value of θ is $60°$ and $\frac{d\theta}{dn}=m$. The value of m is ***(2015)***



Paragraph 1: Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as sin i_m



Q.23 For two structures namely S_1 with $n_1=\sqrt{45}/4$ and $n_2=3/2$ and S_2 with $n_1=8/5$ and $n_2=7/5$ and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option(s) is(are) ***(2015)***

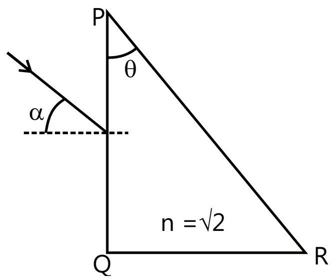
(A) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

(B) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same as that of S_2 immersed in water

(C) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$.

(D) NA of S_1 placed in air is the same as that of S_2 placed in water.

Q.24 A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n=\sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of $45°$. The angle θ of the prism is ***(2016)***



(A) $15°$ (B) $22.5°$ (C) $30°$ (D) $45°$

Q.25 A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true? ***(2016)***

(A) The refractive index of the lens is 2.5

(B) The radius of curvature of the convex surface is 45 cm

(C) The faint image is erect and real

(D) The focal length of the lens is 20 cm

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q.11 Q.12 Q.24

Exercise 2

Q. 3 Q.5 Q.7
Q.8 Q.10 Q.18
Q.23 Q.27 Q.37

JEE Advanced/Boards

Exercise 1

Q.3 Q.4 Q.8
Q.13 Q.22 Q.23

Exercise 2

Q.6 Q.9 Q.11
Q.24 Q.25 Q.30
Q.31 Q.33

Answer Key

JEE Main/ Boards

Exercise 1

Q.1 $2.12\times10^{8}\ \text{ms}^{-1}$

Q.2 $u=-30$ cm

Q.4 $\frac{3\times10^{8}}{\sqrt{2}}\text{m/s}$

Q.5 The image is formed on the same side of object.

Q.9 (b) (i) When the refractive index of the medium increases, the resolving power increases.

(ii) When the wavelength of the radiation increases, the resolving power decreases.

Q.11 For convex lens +7.5 cm, for concave lens +48 cm.

Q.12 A virtual image of height 10 cm is formed at a distance of 25 cm from the lens on the same side of the object.

Q.14 The angle of deviation is decreased.

Q.16 (i) +5 D; (ii) 10 cm

Q.17 25 cm

Q.20 $f_{eq}=\infty$

Q.21 (ii) -10

Q.23 (a) 5

Q.24 2.14

Q.25 $f_0=40\text{cm}; f_e=5\text{cm}$

Exercise 2

Single Correct Choice Type

Q.1 A **Q.2** B **Q.3** A **Q.4** C **Q.5** B **Q.6** A

Q.7 D **Q.8** D **Q.9** A **Q.10** A **Q.11** C **Q.12** D

Q.13 B **Q.14** B **Q.15** B **Q.16** C **Q.17** D **Q.18** D
Q.19 C **Q.20** B **Q.21** C **Q.22** A

Previous Years' Questions

Q.1 D **Q.2** A **Q.3** A $\rightarrow$ p, r; B $\rightarrow$ q, s, t; C $\rightarrow$ p, r, t; D $\rightarrow$ q, s
Q.4 A,B,C **Q.5** 6 **Q.6** D **Q.7** C **Q.8** B **Q.9** D
Q.10 D **Q.11** B **Q.12** B **Q.13** D **Q.14** D **Q.15** D
Q.16 A **Q.17** D

JEE Advanced/Boards

Exercise 1

Q.1 (i) 75° (ii) 165° (iii) 195° (iv) 285° (v) 315°
Q.2 160 cm; 320 cm **Q.3** 75 cm **Q.4** 2 cm **Q.5** 80 m/s **Q.7** 16 ft
Q.8 (i) $\tan\theta = \frac{dy}{dx} = \cot i$ (ii)1 (iii) $y = k^2(x/4)^4$ (iv) 4.0,1;
(v) It will become parallels to x-axis **Q.9** 5 cm **Q.10** h=5.95 m **Q.11** Same **Q.12** 1.625
Q.13 1000 m/s **Q.14** $\mu_1^2 + \mu_3^2 + \mu_5^2 = 2 + \mu_2^2 + \mu_4^2$ **Q.15** 9 m
Q.16 (a) $\sin^{-1}\left(\frac{1}{5}\right)$ (b) air **Q.17** $\left(\frac{d}{R}\right)_{max} = \frac{1}{2}$ **Q.18** 45° **Q.19** $\frac{\sqrt{43}}{5}$
Q.20 $\sqrt{3}$ **Q.21** 15 cm toward the combination
Q.22 (a) $5000\,W/m^2$ (b) $0.02\,W/m^2$ (c) 0.214 cm (d) $24.56\,W/m^2$
Q.23 +60, +4/5 **Q.24** 5.9 cm, 10.9 cm
Q.25 90 cm from the lens toward right

Exercise 2

Single Correct Choice Type

Q.1 B **Q.2** A **Q.3** C **Q.4** D **Q.5** B **Q.6** D
Q.7 B **Q.8** A **Q.9** A

Multiple Correct Choice Type

Q.10 B, D **Q.11** B, C **Q.12** B, C **Q.13** A, C **Q.14** A, B, D **Q.15** B, C
Q.16 B, D **Q.17** A, C **Q.18** A, D **Q.19** A, C **Q.20** B, D

Assertion Reasoning Type

Q.21 C **Q.22** A **Q.23** D **Q.24** D **Q.25** D

Comprehension Type

Q.26 B **Q.27** D **Q.28** D **Q.29** B **Q.30** B **Q.31** D
Q.32 A **Q.33** A

Match the Column

Q.34 A → p, r; B → q, s; C → p, q, r, s; D → p, q, r, s

Previous Years' Questions

Q.1 C, D	**Q.2** B	**Q.3** A → p, q, r, s; B → q; C → p, q, r, s; D → p, q, r, s			
Q.4 A	**Q.5** B	**Q.6** C	**Q.7** B	**Q.8** B	**Q.9** B
Q.10 3	**Q.11** B	**Q.12 A,** C, D	**Q.13** A	**Q.14** A	**Q.15** C
Q.16 D	**Q.17** A, C	**Q.18** C	**Q.19** B	**Q.20** 7	**Q.21** B
Q.22 2	**Q.23** A, C	**Q.24** A	**Q.25** A, D		

Solutions

JEE Main/Boards

Exercise 1

Sol 1: $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}} = \dfrac{\sin\left(\dfrac{90}{2}\right)}{\sin\left(\dfrac{60}{2}\right)} = \dfrac{\frac{1}{\sqrt{2}}}{1/2} = \sqrt{2}$

Here A = 60°, δ_m = 30°.

Now $\mu = \dfrac{c}{v} \Rightarrow v = \dfrac{c}{\mu} \Rightarrow v = \dfrac{3\times10^8}{\sqrt{2}} = 2.12\times10^8\ ms^{-1}$

Sol 2: Lens Formula: $\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$. Let object is placed at distance x from lens and image is found at distance y from lens. For real image v is positive $v=+y,\ u=-x,\ m=\dfrac{v}{u}=\dfrac{+y}{-x}=-2$ (For real image m is negative) $\Rightarrow y=2x$ .....(i) $\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{y}-\dfrac{1}{-x}=\dfrac{1}{20}$ (f is positive for convex lens)

$\Rightarrow \dfrac{1}{2x}+\dfrac{1}{x}=\dfrac{1}{20}$ (using (1)) $\Rightarrow \dfrac{3}{2x}=\dfrac{1}{20} \Rightarrow x=30\,\text{cm}$

u=-30 cm

Sol 3: (i) Focal length of a concave mirror is independent of the medium and wavelength of light. So there will not be any change.

(ii) Focal length of a concave lens depends on the refractive index μ of the medium which in-turn depends upon the wavelength of light. μ decreases with increasing wavelength. So for red light μ will decrease.

As per lens maker's formula $\left[\dfrac{1}{f}=(\mu-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)\right]$ as μ decrease, f increase.

Sol 4: Total internal reflection (TIR) takes place when light travels from denser medium towards rarer medium and at the interface the angle of incidence exceeds θ_c, the critical angle, and the incident beam is completely reflected at the boundary (interface). Critical angle $\theta_c=\sin^{-1}\left(\dfrac{\mu_{Rarer}}{\mu_{Denser}}\right)$

$45=\sin^{-1}\left(\dfrac{1}{\mu}\right) \Rightarrow \dfrac{1}{\mu}=\dfrac{1}{\sqrt{2}} \Rightarrow \mu=\dfrac{c}{v}=\sqrt{2}$

$\Rightarrow v=\dfrac{c}{\sqrt{2}}=\dfrac{3\times10^8}{\sqrt{2}}=2.12\times10^8\ ms^{-1}$

Sol 5: Lens formula.

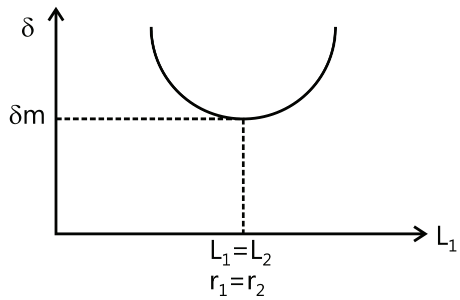
$\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$ (for concave lens f is (–) ve)

Now for $u=-f,\ \dfrac{1}{v}-\dfrac{1}{-f}=\dfrac{1}{-f}$

$\dfrac{1}{v}=-\dfrac{1}{f}-\dfrac{1}{f}=-\dfrac{2}{f} \Rightarrow v=-\dfrac{f}{2}$

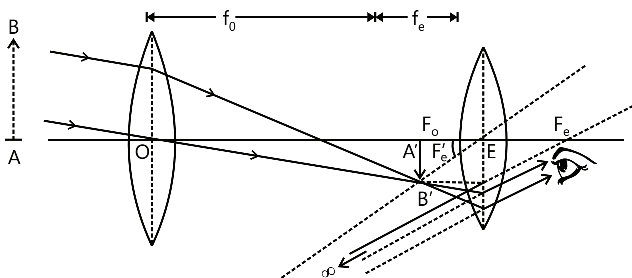
Image is virtual, diminished and on the same side as object.

Sol 6:



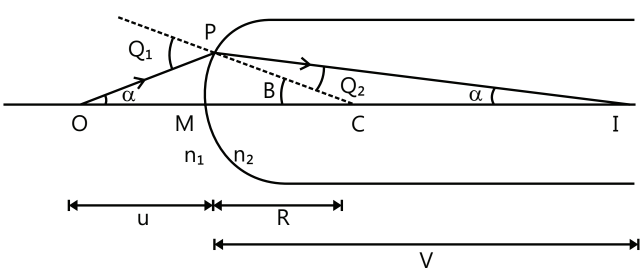
The variation of angle of deviation δ with the angle of incidence i of the ray incident on the first refracting surface of the prism is shown in figure. For one angle of incidence it has a minimum value δ_{min}. At this value the ray passes symmetrically through the prism.

Sol 7: For relaxed eye, intermediate image should lie at first focus of eye piece or $u_e = f_e$



Magnification $M_\infty = \frac{f_0}{f_e}$

Sol 8: Reflection from a spherical surface: Here $n_1 < n_2$. Ray leaves point O and focuses at point I. Snell's Law at point P $n_1 \sin\theta_1 = n_2 \sin\theta_2$ or $n_1\ \theta_1 = n_2\ \theta_2$ (For small angles)



From geometry of figure

$\theta_1 = \alpha + \beta,\ \beta = \theta_2 + \gamma$

Eliminating θ_1 and θ_2 we get $\beta = \frac{n_1}{n_2}(\alpha + \beta) + \gamma$

Or $n_1\alpha + n_2\gamma = (n_2 - n_1)\beta$ ....(A)

Now angle at C is $\beta = \frac{S}{R}(S = \text{arc}(PM))$

Also in paraxial approximation $\alpha = \frac{S}{u}$ and $\gamma = \frac{S}{v}$

Putting these expressions with proper signs, in eqn. A, we get $\frac{n_1}{-u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R_1}$

Lens Maker's Formula: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

As wavelength of light increases, the refractive index n decreases and from the lens maker formula we see that, as n decreases, f increases.

Sol 9: (i) Magnifying power:

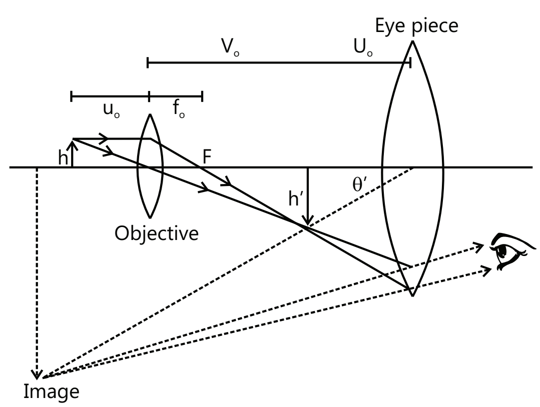
$m = \frac{v}{u}\left(\frac{D}{f_e}\right)$ for normal adjustment

$m = \frac{v}{u}\left(1 + \frac{D}{f_e}\right)$ for final image at D, least distance for clear vision.

(ii) Resolving Power: $R = \frac{1}{\Delta d} = \frac{2\mu\sin\theta}{\lambda}$

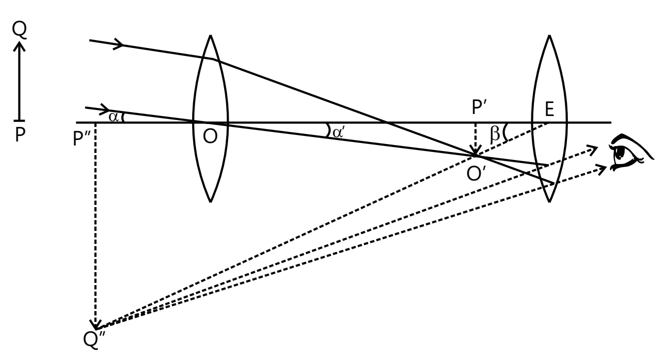
$\mu \rightarrow$ Refractive index of the medium between the object and the objective.

$\lambda \rightarrow$ Wavelength of light.



Here we see that

(i) as μ increases, R increases.



(ii) as λ increases, R decreases.

Sol 10: Refracting astronomical telescope: It consists of an objective lens of a large focal length (f_o) and large aperture, also an eye lens of small aperture and focal length.

(i) Magnification when final image is formed at D,

$\Rightarrow m = -\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$ and length of telescope,

$$L = |f_o| + \frac{f_e D}{f_e + D}$$

Sol 11: (i) Lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$u = +12$ cm, $f = +20$ cm

$$\Rightarrow \frac{1}{v}=\frac{1}{u}+\frac{1}{f}=\frac{1}{12}+\frac{1}{20}=\frac{1}{4}\left(\frac{1}{3}+\frac{1}{5}\right)=\frac{8}{4\times 15}$$

$\Rightarrow v = +7.5$ cm

(ii) $f = -16 \text{ cm} \Rightarrow \frac{1}{v}=\frac{1}{12}-\frac{1}{16}=\frac{4-3}{48}=\frac{1}{48}$

$\Rightarrow v = +48$ cm

Sol 12: $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

Here $R_1 = +30$ cm ; $R_2 = -30$ cm; $\mu = 1.6$

$$\Rightarrow \frac{1}{f}=(0.6)\left(\frac{1}{30}\right)\times 2=\frac{2}{50}=\frac{1}{25}\Rightarrow f + 25 \text{ cm}$$

$$u = 12.5 \text{ cm}, \frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{25}-\frac{1}{12.5}=-\frac{1}{25}$$

$$\Rightarrow v = -25 \text{ cm}, \ m=\frac{h_2}{h_1}=\frac{v}{u}=\frac{-25}{-12.5}=2$$

$\Rightarrow h_2 = 2h_1 = 2\times 5$ cm $= 10$ cm

Image is virtual and erect, on the same side as the object.

Sol 13: Predominance of bluish colour in a clear sky is due to the phenomena of scattering of light in the atmosphere around earth. If size of the air particles are smaller than the wavelength, the scattering is proportional to $1/\lambda^4$. This is the Rayleigh's law of scattering. The light of short wavelengths are strongly scattered by the air molecules and reach the observer.

Among the shorter wavelengths, the colour blue is present in large proportion in sunlight.

Sol 14: Angle of minimum deviation δ_m and angle of prism A are related as, $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

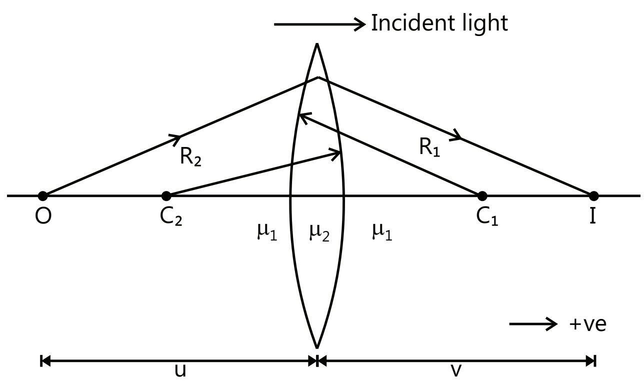
Glass prism of refractive index 1.5 is immersed in a liquid of refractive index 1.3 so the relative refractive index of the prism decreases. $\mu' = \frac{1.5}{1.3} = 1.15$

So as per above equation as A is constant for a prism, as μ decreases, δ_m also decreases.

Sol 15: [Refer question 7 solution]

Sol 16: Consider an object O placed at a distance u from a convex lens as shown in figure. Let its image I after two refractions from spherical surfaces of radii R_1 (positive) and R_2 (negative) be formed at a distance v from the lens. Let v_1 be the distance of image formed by refraction from the refracting surface of radius R_1. This image acts as an object for the second surface. Using,

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$



twice, we have $\frac{\mu_2}{v_1}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ ... (i)

and $\frac{\mu_1}{v}-\frac{\mu_2}{v_1}=\frac{\mu_1-\mu_2}{-R_2}$ ... (ii)

Adding Eqs. (i) and (ii) and then simplifying, we get

$$\frac{1}{v}-\frac{1}{u}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \text{... (iii)}$$

This expression relates the image distance v of the image formed by a thin lens to the object distance u and to the thin lens properties (index of refraction and radii of curvature). It is valid only for paraxial rays and only when the lens thickness is much less then R_1 and R_2. The focal length f of a thin lens is the image distance that corresponds to an object at infinity. So, putting $u=\infty$ and $v=f$ in the above equation, we have

$$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\text{....(iv)}$$

If the refractive index of the material of the lens is μ and it is placed in air, $\mu_2=\mu$ and $\mu_1=1$ so that Eq. (iv) becomes

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\text{....(v)}$$

This is called the lens maker's formula because it can be used to determine the values of R_1 and R_2 that are needed for a given refractive index and a desired focal length f.

Combining Eqs. (iii) and (v), we get

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ ....(vi) which is known as the lens formula.

(i) $P=P_1+P_2=10D-5D=5D$

(ii) $f=\frac{1}{P}=\frac{1}{5D}=0.2\text{m}=20\text{cm};\ m=+2=\frac{v}{u}$

$\Rightarrow v=2u$

For virtual image m is positive

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}\Rightarrow\frac{1}{2u}-\frac{1}{u}=\frac{1}{20}$$

$$\Rightarrow-\frac{1}{2u}=\frac{1}{20}\Rightarrow u=-10\text{ cm}$$

Sol 17:

$$P=P_1+P_2=6-2=4D\Rightarrow f=\frac{1}{P}=\frac{1}{4D}=0.25\text{m}$$

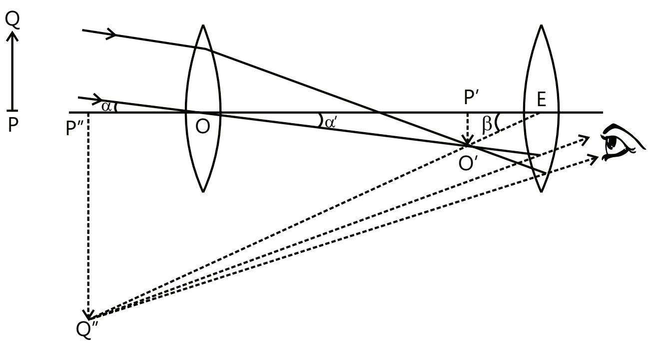
$\Rightarrow f=25\text{cm}$

Sol 18: The speed of light in vacuum is a universal constant denoted by c. When a light wave travels in a transparent material, the speed is decreased by a factor μ, called the refractive index of the material.

$$\mu=\frac{\text{speed of light in vacuum}}{\text{speed of light in the material}}$$

For graph refer figure of question 6.

Sol 19: (a) (i)



(ii) Reflector telescope advantages:

1. Reflector telescopes do not suffer from chromatic aberration because all wavelengths will reflect off the mirror in the same way.

2. Support for the objective mirror is all along the back side so they can be made very BIG.

3. Reflector telescopes are cheaper to make than refractors of the same size.

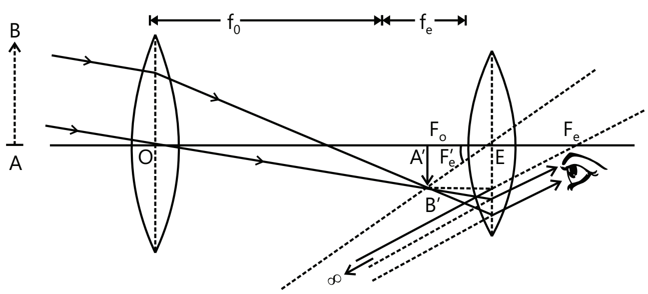
4. Because light is reflecting off the objective, rather than passing through it, only one side of the reflector telescope's objective needs to be perfect.

(b) f_1 = +10 cm, f_2 = -10 cm, u = -30 cm.

$$\frac{1}{v_1}=\frac{1}{f_1}+\frac{1}{u_1}=\frac{1}{10}-\frac{1}{30}=\frac{2}{30}\Rightarrow v_1=15\text{ cm}$$

So for the concave lens

$$u_2=+(15-5)\text{ cm}=+10\text{ cm}$$



$$\frac{1}{v_2}=\frac{1}{f_2}+\frac{1}{u_2}=\frac{1}{-10}+\frac{1}{10}=0\Rightarrow v_2=\infty$$

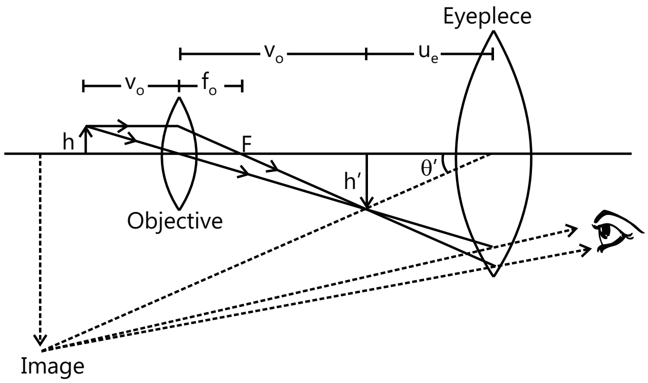
Final image will be formed at infinity.

Sol 20: $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f}-\frac{1}{f}=0\Rightarrow F=\infty$

Sol 21: (i) Astronomical Telescope for normal adjustment

It consists of two converging lenses placed coaxially.

The one facing the distant object is called the objective and has a large aperture and a large focal length. The other is called the eyepiece, as the eye is placed close to it. It has a smaller aperture and a smaller focal length. The lenses are fixed in tubes. The eyepiece tube can slide within the objective tube so that the separation between the objective and the eyepiece may be changed.



When the telescope is directed towards a distant object PQ, the objective forms a real image of the object in its focal plane. If the point P is on the principal axis, the image point P′ is at the second focus of the objective. The rays coming from Q are focused at Q′. The eyepiece forms a magnified virtual image P″Q″ of P′Q′. This image is finally seen by the eye. In normal adjustment, the position is so adjusted that the final image is formed at infinity. In such a case, the first image P′Q′ is formed in the first focal plane of the eyepiece. The eye is least strained to focus this final image. The image can be brought closer by pushing the eyepiece closer to the first image. Maximum angular magnification is produced when the final image is formed at the near point.

(ii) $m = -\frac{f_0}{f_e} = -\frac{p_e}{p_o} = -\frac{10D}{1D} = -10$

Here the objective has large focal length and smaller Power.

(b) (i) Figure shows a simplified version of a compound microscope and the ray diagram for image formation. It consists of two converging lenses arranged coaxially. The one facing the object is called the objective and the one close to the eye is called the eyepiece or ocular.

The object is placed at a distance u_0 from the objective which is slightly greater than its focal length f_0. A real and inverted image is formed at a distance v_0 on the other side of the objective. This image works as the object for the eyepiece. For normal adjustment, the position of the eyepiece is so adjusted that the image formed by the objective falls in the focal plane of the eyepiece. The final image is then formed at infinity. It is erect with respect to the first image and hence, inverted with respect to the object. The eye is least strained in this adjustment as it has to focus the parallel rays coming to it. The position of the eyepiece can also be adjusted in such a way that the final virtual image is formed at the near point. The angular magnification is increased in this case. The ray diagram in figure refers to this case.

(ii) Magnifying power of a Compound microscope is

$$m = -\frac{v_0}{u_0}\left(\frac{D}{f_e}\right) \rightarrow \text{ normal adjustment and}$$

$$m = -\frac{v_0}{u_0}\left(1+\frac{D}{f_e}\right) \rightarrow \text{final image at D.}$$

Now for large magnification, m is to be large, so f_e should be small and u_0 should be small. Now object is placed at a distance u_0 from the objective which is slightly greater than its focal length f_o. So for u_0 to be small, f_o should also be small.

Sol 22: Refer question 19.(a).(ii)

Sol 23: When two plane mirrors are placed at an angle θ to each other, the object is kept between them, then the numbers of images observed is $n = \frac{360^o}{\theta}$. If n is even then number of image is (n-1).

So here $\frac{360^o}{\theta} = \frac{360^o}{60} = 6 \Rightarrow n = 6-1 = 5$

(b) At each reflection some of the light energy is lost due to absorption at the mirror surface. So the intensity of the reflected ray goes on decreasing at multiple reflections due to parallel mirrors.

Sol 24: For compound microscope $m = -\frac{v_o}{u_o}\left(\frac{D}{f_e}\right)$ (normal adjustment)

$$-20 = \left(-\frac{v_o}{u_o}\right)\left(\frac{25\text{ cm}}{3\text{cm}}\right) \Rightarrow -\frac{v_o}{u_o} = \frac{-60}{25} = \frac{-12}{5} = -2.4$$

For final image at least distance:

$$m = -\frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right) \Rightarrow -20 = -\frac{v_o}{u_o}\left(1+\frac{25}{3}\right)$$

$$\Rightarrow -\frac{v_o}{u_o} = \frac{-20}{28/3} = \frac{-5\times 3}{7} \Rightarrow -\frac{v_o}{u_o} = -2.14$$

Sol 25: Astronomical telescope $m = \frac{-f_o}{f_e} \rightarrow$ normal adjustment

$$\Rightarrow -8 = -\frac{f_o}{f_e} \Rightarrow f_o = 8f_e$$

$L = f_o + f_e = 45$ cm $\Rightarrow 8f_e + f_e = 45$ cm

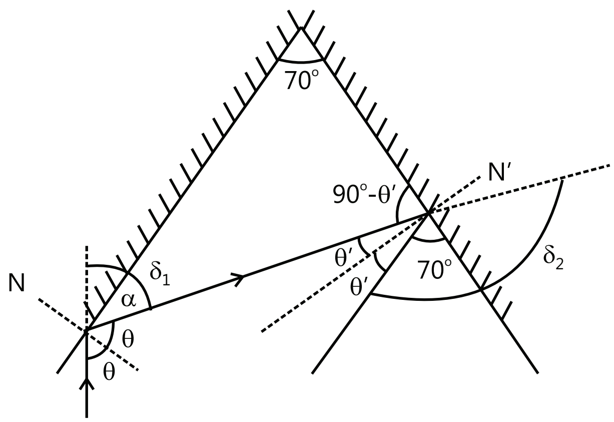
$\Rightarrow f_e = 5$ cm and $f_e = 40$ cm

Exercise 2

Single Correct Choice Type

Sol 1: (A)

$\delta_1 = 180^o - 2\theta;\ \delta_2 = 180^o - 2\theta' = 70^o + 90^o - \theta'$

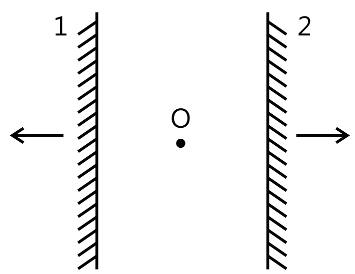


$\Rightarrow \theta' = 20^o$

$\alpha = 180^o - 70^o - (90^o - \theta') = 180^o - 70^o - 70^o$

$\alpha = 40^o \Rightarrow \theta = 90^o - \alpha = 50^o$

Sol 2: (B) With respect to mirror1 the object is going array from mirror. So first image will also more away w.r.t mirror 1 with same speed v. So with respect to object O the image speed is



$$v_{image1} = v_{image1,mirror1} + v_{mirror1,obj} = v + v = 2v$$

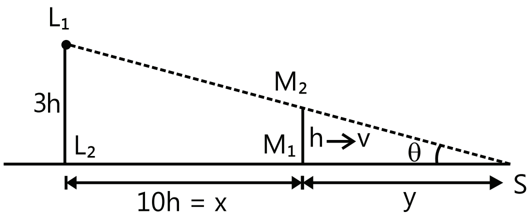
Now this image becomes object for mirror 2. With respect to mirror 2 the image is going away towards left with speed 3v. So its second image will more away towards right with speed 3v w.r.t mirror 2. Hence speed w.r.t O will be 3v+v=4v

Hence for nth image $v_{image} = 2nv$

Sol 3: (A) $\tan\theta = \frac{h}{y} = \frac{3h}{10\,h + y} \lim_{x\to\infty}$

$\Rightarrow 10h + y = 3y \Rightarrow y = 5h$

For general case

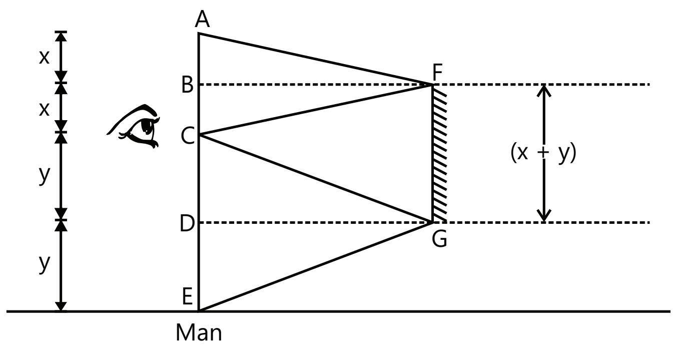


$$\tan\theta = \frac{3h}{x+y} = \frac{h}{y} \Rightarrow 3y = x + y \Rightarrow 2y = x$$

$$\Rightarrow 2.\frac{dy}{dt} = \frac{dx}{dt} \Rightarrow \frac{dy}{dt} = \frac{1}{2}.v$$

Sol 4: (C) A is head and E is feet of man. C is the eye. The mirror can be placed anywhere between the centre line BF (of AC) and DG (of CE) to get full image from head to feet.

So here CE=1.4 m. So DE should be 0.7 m. But mirror is 0.8 m from ground so feet will not be visible. The upper edge of mirror is at height (0.8+0.75) m equal to 1.55 m which is more than BE.



$$BE = BC + CE = \frac{AC}{2} + CE = \frac{0.1}{2} + 1.4$$
$$= 0.05 + 1.4 = 1.45\,m$$

So head will be visible.

Sol 5: (B) Rays from S going right from the normal will not reach the bottom horizontal mirror as they will hit the inclined mirror and get deviated. So C will see only the image formed by inclined mirror.

Sol 6: (A) Object O moves towards M_1 so image 1 due to M_1 will move towards left i.e. towards M_2.

For M_2 we have formula for speed of image as $\frac{dv}{dt} = -\left(\frac{v^2}{u^2}\right)\frac{du}{dt}$. So negative sign means final image 2 due to M_2 will move opposite to image 1 i.e. towards right.

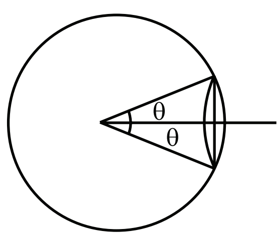
Sol 7: (D) Intensity incident

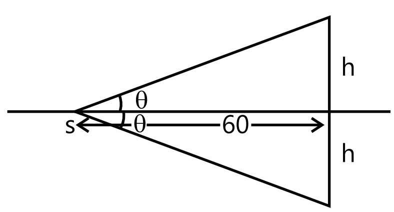
$$= \frac{P\theta^2}{4 \times (\text{area on which light is incident})}$$

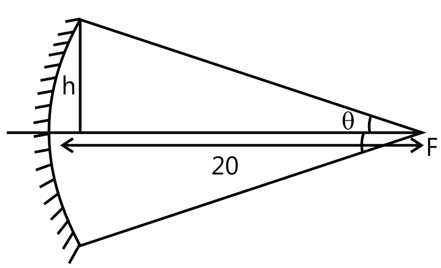
When the mirror is not present, light is reaching the screen up to height h. Maximum area on which light is incident $= \pi h^2$

$$\text{Intensity} = \frac{P\theta^2}{4\pi h^2} \text{ and}$$

$$\tan\theta = \frac{h}{60} \Rightarrow \frac{Ph^2}{4\pi h^2 \times (60)^2} = \frac{P}{4\pi \times (60)^2}$$







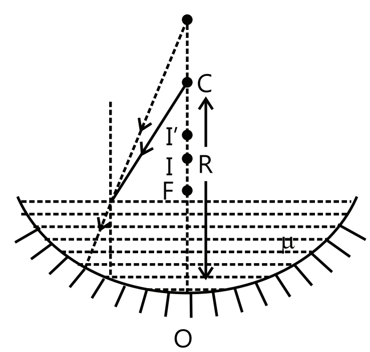
When the mirror is present

$$\text{Intensity} = \frac{P\theta_1^{\,2}}{4\pi h_1^{\,2}} \text{ and } \tan\theta = \frac{h}{20} \Rightarrow \frac{P}{4\pi \times (20)^2}$$

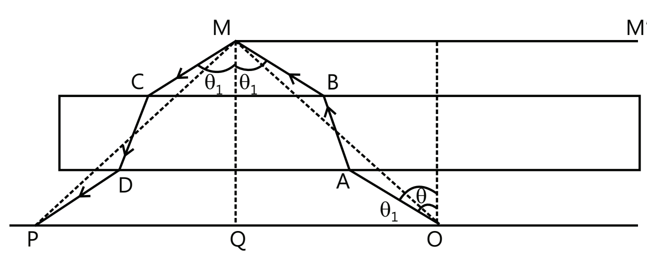
$$\text{Ratio} = \frac{\text{Intensity when mirror present}}{\text{Intensity when mirror object}}$$

$$= \frac{\dfrac{P}{4\pi \times (20)^2} + \dfrac{P}{4\pi \times (60)^2}}{\dfrac{P}{4\pi \times (20)^2}} = \frac{10}{1}$$

Sol 8: (D) As a result of water the apparent height of source will be beyond C, at C′. OC′ = Rμ. So its image I′ will not be formed at C but it will be formed between C and focus F of the mirror. But again in the return path of rays they will be again refracted at water to air boundary and final image I will be further shifted downwards towards O.



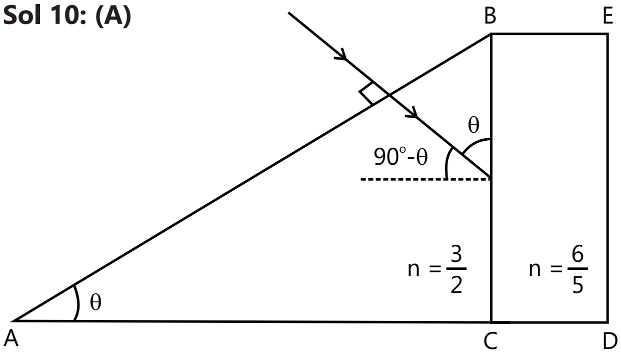
Sol 9: (A) The slab will cause a lateral shift in the incident rays as well as in the reflected rays from the circular mirror MM′. Now the angle of emergence θ_1 will be equal to the angle of incidence in case of a slab.



The rays reaching the edge M of the circular mirror after passing through the glass slab will be leaving the source O at a greater angle (θ_1) with the normal as compared to the angle (θ) when there is no slab. But due to symmetry of incident and reflected rays, the reflected rays from the edge M, after passing through the slab will reach the some point Q on the ground where they would have reached when there was no slab.

Here we have, OQ=QP, both without and with slab, between source O and mirror MM'.

Sol 10: (A)



$$90-\theta \ge \theta_c = \sin^{-1}\left(\frac{6/5}{3/2}\right);$$

$$90-\theta \ge \sin^{-1}\left(\frac{4}{5}\right);$$

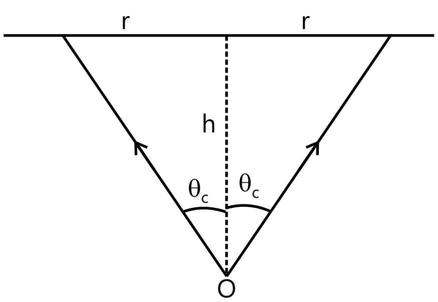
$$90-\theta \ge 53^{o} \Rightarrow \theta \le 37^{o}$$

Sol 11: (C)

$$h = 4m\ ;\ \theta_c = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$= \sin^{-1}\left(\frac{1}{5/3}\right) = \sin^{-1}\left(\frac{3}{5}\right) = 37^{o}$$

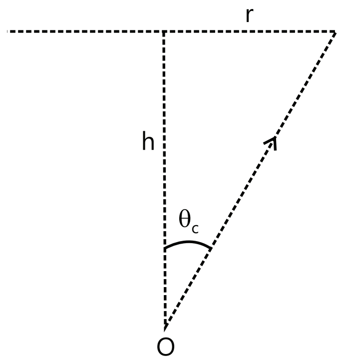
$$\tan\theta_c = \frac{r}{h} \Rightarrow r = h\tan 37^{o} = \frac{3h}{4} = \frac{3\times 4m}{4}$$



$$\Rightarrow \text{diameter} = 2r = 6m$$

Sol 12: (D) $\theta_c = \sin^{-1}\frac{1}{\mu} = \sin^{-1}\left(\frac{3}{4}\right) = 48.59^{o}$

Solid angle subtended at source of light O by the circular area of radius r is



$$\Omega = 2\pi(1-\cos\theta_c)$$

If total intensity is I then, intensity per unit solid angle is $\frac{I}{4\pi.}$.So intensity through the circular area is,

$$I = \frac{I}{4\pi}.2\pi(1-\cos\theta_c)$$

$$\Rightarrow \frac{I'}{I} = \frac{(1-\cos 48.59)}{2} \Rightarrow \frac{I}{I} = 16.9\%$$

Sol 13: (B) $\frac{1}{v_1} = \frac{1}{f} + \frac{1}{(-u_1)} = \frac{1}{f} - \frac{1}{u_1} = \frac{u_1 - f}{f u_1}$

$$v_1 = \frac{f\,u_1}{u_1 - f} \text{ and } v_2 = \frac{f\,u_2}{u_2 - f}$$

$$\text{Now } m_1 = \frac{v}{u} = \frac{v_1}{-u_1} = \frac{-f}{u_1 - f}$$

$$\text{and } m_2 = \frac{-f}{u_2 - f}.\text{also } |m_1| = |m_2|$$

Now m_1 is negative (real image) and m_2 is positive (virtual image). So we have,

$$\frac{f}{u_1 - f} = \frac{-f}{u_2 - f} \Rightarrow u_2 - f = -u_1 + f$$

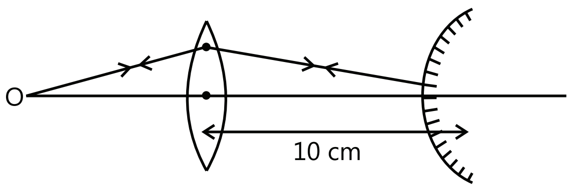
$$\Rightarrow \quad u_1 + u_2 = 2f \Rightarrow f = \frac{u_1 + u_2}{2}$$

Sol 14: (B) The image formed by first lens will lie at its second lens focus. This image will act as an object for the second lens. For the rays to become parallel after passing through the second lens, the object for second lens should lie on its first focus. Thus the distance between the two lenses will be equal to sum of their focal lengths.

$$D = f_1 + f_2 = 20\,cm + 10\,cm = 30\,cm$$

Sol 15: (B) Image formed by lens be at distance v_1 from lens. $\frac{1}{v_1}=\frac{1}{f_1}+\frac{1}{u_1}=\frac{1}{10}+\frac{1}{-15}=\frac{3-2}{30}=\frac{1}{30}$

v_1 = 30 cm from lens.



For convex lens, u_2 = +(30 - 10) cm = +20 cm

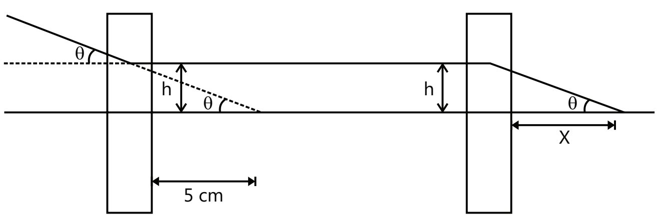
v_2= +20 cm, because rays retrace their path after reflection.

$$\frac{1}{v_2}+\frac{1}{u_2}=\frac{1}{f_2}\Rightarrow\frac{1}{f_2}=\frac{1}{20}+\frac{1}{20}=\frac{1}{10}\Rightarrow f_2=+10\text{ cm}$$

Sol 16: (C) For first lens, convergent ray becomes parallel to principal axis after refraction.

So f_1 = +5 cm.

For second lens, ray parallel to principal axis becomes convergent and parallel to incident ray.



So focal will length of second lens will be x as shown in figure.

$$\tan\theta=\frac{h}{5}=\frac{h}{x}\Rightarrow x=5\text{ cm}\Rightarrow f_2=x=5\text{ cm}$$

Sol 17: (D) Let as work in the frame of reference attached to the lens. Lens formula: $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

Differentiating w.r.t. time,

$$-v^{-2}\frac{dv}{dt}-(-u^{-2})\frac{du}{dt}=0\text{; (f is constant)}$$

$$\Rightarrow\frac{dv}{dt}=\left(\frac{v^2}{u^2}\right)\frac{du}{dt}$$

Initially when u=f, $v\to\infty$ so speed image is very large and finally when $u\to\infty, v\to f$ and the speed of image is very low (nearly zero). With respect to lens, as object moves left, the image also moves left.

Speed of image with respect to object is

$$v_{I,O}=v_{I,L}+v_{L,O}$$

$$v_{I,O}=\left(\frac{dv}{dt}\right)_{\text{(towards left)}}+(-v)_{\text{(towards right)}}$$

Sol 18: (D) At first refracting surface we have, $\sin i_1=\mu\sin r_1$. So as i_1 decreases, r_1 also decreases.

Now for prism r_1+r_2 =A (constant). So as r_1 decreases, r_2 increases. At the second refracting surface we have, $\mu\sin r_2=\sin i_2$. So as r_2 increases, i_2 also increases. So out of all choices D is most appropriate as amount of increase in i_2 should be less than amount of decrease in i_1.

Sol 19: (C) For prism with refracting angle A, we have

$$\mu=\frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}\Rightarrow\sqrt{\frac{3}{2}}=\frac{\sin(90^o+\delta_m)/2}{\sin 45^o}$$

$$\Rightarrow\sqrt{\frac{3}{2}}=\sin\left(\frac{90^o+\delta_m}{2}\right)\Rightarrow\frac{90^o+\delta_m}{2}=60^o$$

$$\Rightarrow\delta_m=120^o-90^o=30^o$$

Sol 20: (B) At min deviation $i_m=\frac{A+\delta_m}{2}=\frac{A+38^o}{2}$

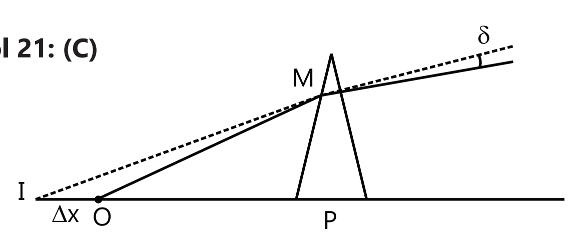
Now $\delta=(i_1+i_2)-A$; Here $i_1=42^o$

and $i_2=62^o$, $\delta=44^o$

$$\Rightarrow 44^o=(42^o+62^o)-A\Rightarrow A=104^o-44^o=60^o$$

$$\Rightarrow i_m=\frac{60^o+38^o}{2}=49^o$$

Sol 21: (C)



$$\delta=(\mu-1)A;\ \mu=1.5$$

$$\delta=0.5\times5^o=2.5^o\Rightarrow\delta=\frac{2.5}{180}\pi=\frac{5}{360}\pi=\frac{\pi}{72}$$

$$OP\approx OM\approx IM=10\text{ cm};\quad\Delta x=\delta\times(OM)=10\ \delta\text{ cm}$$

$$\Delta x = \frac{\pi}{72}\times 10\text{ cm} = \frac{5\pi}{36}\text{cm}$$

Sol 22: (A) At second refracting surface

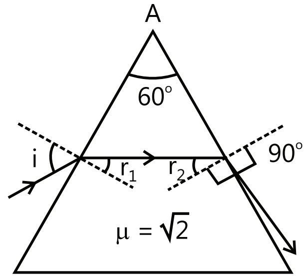
$\mu \sin r_2 = 1 \sin 90$

$\Rightarrow \sin r_2 = \frac{1}{\mu} = \frac{1}{\sqrt{2}}$

$\Rightarrow r_2 = 45^o$

$\Rightarrow r_1 = A - r_2$

$= 60^o - 45^o = 15^o$



$\Rightarrow$ At first refracting surface , $\sin i_1 = \mu \sin r_1$

$\Rightarrow \sin i_1 = \sqrt{2}.\sin 15^o = \sqrt{2}.\frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2}$

$\Rightarrow i_1 = \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right).$

Previous Years' Questions

Sol 1: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ = constant

Sol 2: (A) An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment, distances are measured by a vernier scale provided on the microscope.

Sol 3: (A) $\rightarrow$ since $\mu_1 < \mu_2$, the ray of light will bend towards normal after first refraction.

(B) $\rightarrow \mu_1 > \mu_2$, the ray of light will bend away from the normal after first refraction.

(C) $\rightarrow$ $\mu_2 = \mu_3$ means in second refraction there will be no change in the path of ray of light.

(D) $\rightarrow$ Since $\mu_2 > \mu_3$, ray of light will bend away from the normal after second refraction.

Therefore the correct options are as under.

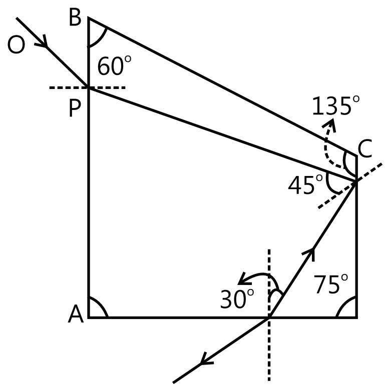
(A) $\rightarrow$ p, r

(B) $\rightarrow$ q, s, t

(C) $\rightarrow$ p, r, t

(D) $\rightarrow$ q, s

Sol 4: (A, B, C) Using Snell's law



$\sin^{-1}\frac{1}{\sqrt{3}} < \sin^{-1}\frac{1}{\sqrt{2}}$

Net deviations is 90°

Sol 5:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or $\quad \frac{u}{v} - 1 = \frac{u}{f}$ or $\quad \frac{u}{v} = \left(\frac{u+f}{f}\right)$

$\therefore \quad m = \frac{v}{u} = \left(\frac{f}{u+f}\right)$

$$\frac{m_{25}}{m_{30}} = \frac{\left(\frac{20}{-25+20}\right)}{\left(\frac{20}{-50+20}\right)} = 6$$

$\therefore$ Answer is 6.

Sol 6: (D) Case I: u = – 240 cm, v = 12, by lens formula

$\frac{1}{f} = \frac{7}{80}$

Case II: v = 12 – $\frac{1}{3}$ = $\frac{35}{3}$

(normal shift = 1 – $\frac{2}{3}$ = $\frac{1}{3}$)

f = $\frac{7}{80}$

u = 5.6

Sol 7: (C) L.C = $\frac{1}{60}$

Total Reading = 585 + $\frac{9}{60}$ = 58.65

Sol 8: (B) As intensity is maximum at axis.

$\therefore$ μ will be maximum and speed will be minimum on the axis of the beam.

$\therefore$ Beam will converge.

Sol 9: (D) For a parallel cylindrical beam, wave front will be planar.

Sol 10: (D) Case I: u = -240 cm, v = 12, by Lens formula

$$\frac{1}{f} = \frac{7}{80}$$

Case II: $v = 12 - \frac{1}{3} = \frac{35}{3}$

$$\left(\text{Normal shift} = 1 - \frac{2}{3} = \frac{1}{3}\right)$$

$$f = \frac{7}{80}$$

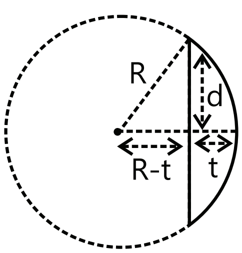
u = 5.6

Sol 11: (B) Self-explanatory

Sol 12: (B)

$$R^2 = d^2 + (R-t)^2$$

$$R^2 - d^2 = R^2\left\{1 - \frac{t}{R}\right\}^2$$



$$1 - \frac{d^2}{R^2} = 1 - \frac{2t}{R}$$

$$R = \frac{(3)^2}{2\times(0.3)} = \frac{90}{6} = 15 \text{ cm}$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right)$$

f = 30 cm

Sol 13: (D) $\frac{f_m}{f} = \frac{(\mu - 1)}{\left(\frac{\mu}{\mu_m} - 1\right)}$

$$\Rightarrow \frac{f_1}{f} = \frac{\left(\frac{3}{2} - 1\right)}{\left(\frac{3/2}{4/3} - 1\right)} = 4$$

$$\Rightarrow f_1 = 4f$$

$$\frac{f_2}{f} = \frac{\left(\frac{3}{2} - 1\right)}{\left(\frac{3/2}{5/3} - 1\right)} = -5$$

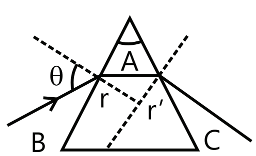
$$\Rightarrow f_2 < 0$$

Sol 14: (D) As frequency of visible light increases refractive index increases. With the increase of refractive index critical angle decreases. So that light having frequency greater than green will get total internal reflection and the light having frequency less than green will pass to air.

Sol 15: (D) At face AB,

$$\sin\theta = \mu \sin r$$

At face AC $r' < \theta_c$



$$A - r < \sin^{-1}\frac{1}{\mu}$$

$$\therefore r > A - \sin^{-1}\frac{1}{\mu}$$

$$\therefore \sin r > \sin\left(A - \sin^{-1}\frac{1}{\mu}\right)$$

$$\frac{\sin\theta}{\mu} > \sin\left(A - \sin^{-1}\frac{1}{\mu}\right)$$

$$\theta > \sin^{-1}\left[\mu\sin\left(A - \sin^{-1}\frac{1}{\mu}\right)\right]$$

Sol 16: (A) $\theta = 1.22\dfrac{\lambda}{D}$

Minimum separation = $\left(25\times10^{-2}\right)\theta = 30\ \mu m$

Sol 17: (D) $\delta = i + e - A \Rightarrow A = 74°$

$$\mu = \frac{\sin\left(\dfrac{A+\delta_{min}}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} = \frac{5}{3}\sin\left(37° + \frac{\delta_{min}}{2}\right)$$

μ_{max} can be $\dfrac{5}{3}$, so μ will be less than $\dfrac{5}{3}$

Since δ_{min} will be less than $40°$, so

$$\mu < \frac{5}{3}\sin 57° < \frac{5}{3}\sin 60° \Rightarrow \mu < 1.446$$

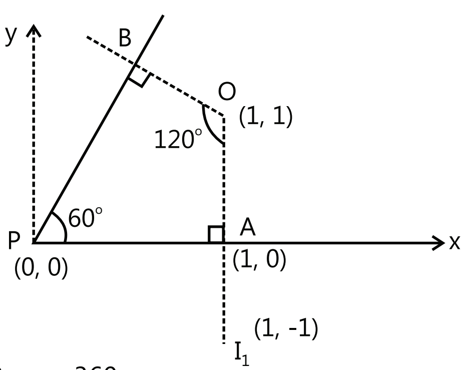
So the nearest possible value of μ should be 1.5

JEE Advanced/Boards

Exercise 1

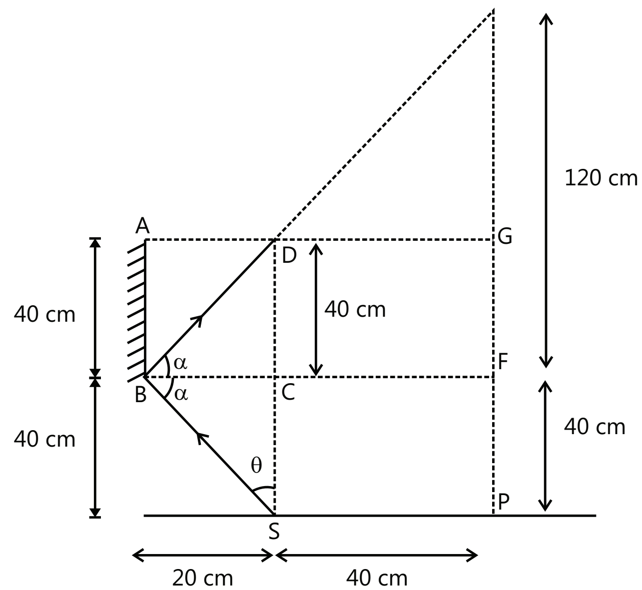
Sol 1: $OB = OP\sin15^{o} = \sqrt{2}\left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right) = \dfrac{\sqrt{3}-1}{2}$

Number of image



$$n = \frac{360}{\theta} - 1 = \frac{360}{\theta} - 1 = 5$$

Sol 2:



By similar triangles

Δ BCD ~ Δ BFE; so $EF = 3\times CD$

Because $BF = 3\times BC$

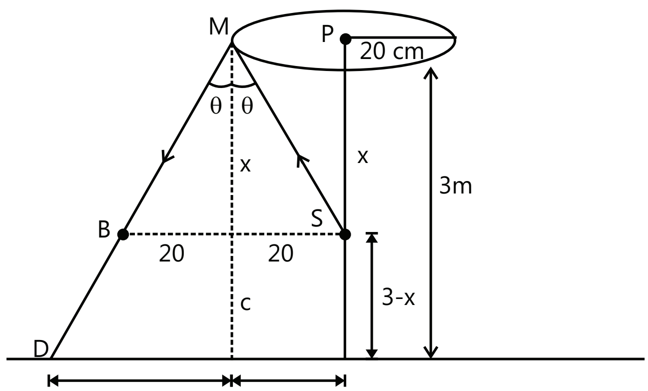
$\Rightarrow EF = 120$ cm $\Rightarrow EP = 160$ cm

$\therefore$ Minimum height of eye is 160 cm.

And similarly maximum height will be

$E'P = 80 + 3\times80 = 320$cm

Sol 3: By property of similar triangles,

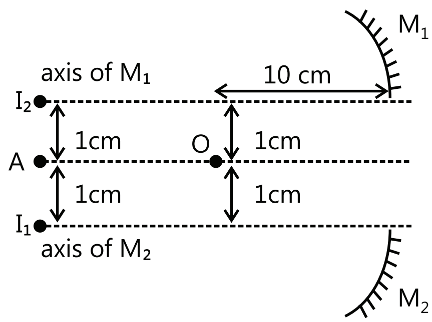


$\Delta MAB \sim \Delta MCD$; $\dfrac{x}{20\text{ cm}} = \dfrac{300\text{ cm}}{y}$

$y + 20 = 100$ cm; $\Rightarrow y = 80$ cm

$$\Rightarrow x = \frac{20\times300\text{ cm}}{80} = 75\text{cm}$$

Sol 4:



$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$; $f=-20$ cm, $u=-10$ cm

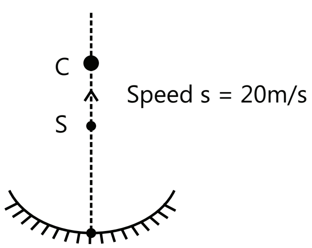
$\frac{1}{v}-\frac{1}{10}=-\frac{1}{20}\Rightarrow\frac{1}{v}=\frac{1}{10}-\frac{1}{20}=\frac{2-1}{20}=\frac{1}{20}\Rightarrow v=20$

$M=\frac{v}{u}=-\frac{20}{(-10)}=2$

Image will be erect with respect to the axis of each mirror. Distance between images is 2 cm.

Sol 5: OS=15 m; OC =20 m

Height of ball after 4 s is,



$H=15+20\times4-\frac{1}{2}\times10\times16=15+80-80=15\text{m}$

Speed of ball after 4 s is, $s_{ball}=20-10\times4$

$\Rightarrow s_{ball}=20-40=-20\ \text{ms}^{-1}$

So ball is moving downward with speed of 20 ms^{-1} and is at height 15 m above the mirror.

So u=-15 m, and $f=\frac{20}{-2}\text{m}=-10$ m

Mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ gives

$\frac{1}{v}=\frac{1}{-10}-\frac{1}{(-15)}=\frac{1}{15}-\frac{1}{-10}=\frac{2-3}{30}=\frac{-1}{30}$

$\Rightarrow v=-30$ cm

Relation between speed of image and speed of object for lens is

$$\frac{dv}{dt}=\left(\frac{v^2}{u^2}\right).\frac{du}{dt}\Rightarrow\frac{dv}{dt}=\left(\frac{(-30)^2}{(-15)^2}\right).\ 20\text{m s}^{-1}\ \text{(downward)}$$

$$\frac{dv}{dt}=(4)\times20\ \text{ms}^{-1}=80\text{m s}^{-1}\ \text{(downward)}$$

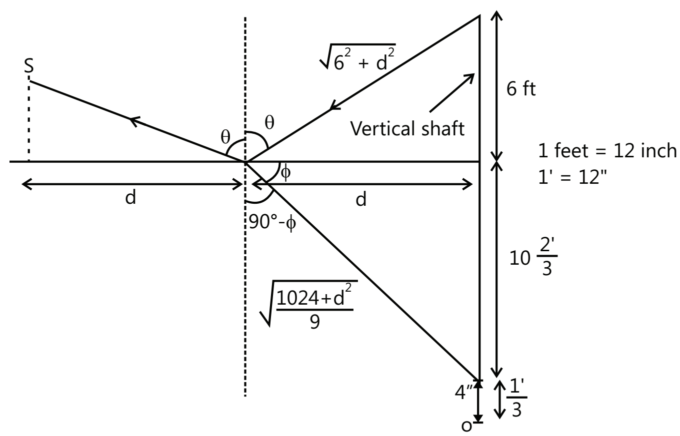
Sol 6: Refer theory

Sol 7: Apply Snell's law:

$1\times\sin\theta=\frac{4}{3}\sin(90-\varphi)$

$1\times\frac{d}{\sqrt{30+d^2}}=\frac{4}{3}\times\frac{d\times3}{\sqrt{1024+d^2}}$

$\sqrt{1024+d^2}=4\sqrt{36+d^2}$



On squaring both sides,

$1024+9d^2=16(36+d^2)$; $1024+9d^2=576+16d^2$

7d2=448; $d^2=\frac{448}{7}=64$

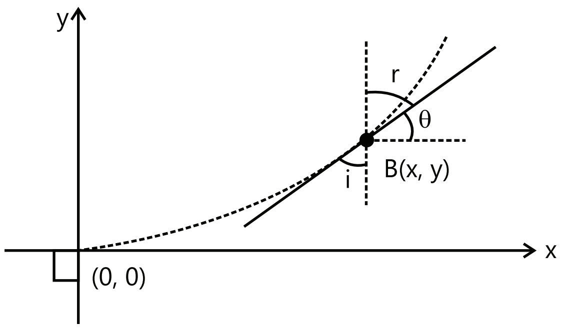
d= 8 feet, width=2d=16 feet

Sol 8: At any point by Snell's law

$1\sin 90=n(y).\ \sin(90^o-\theta)$;

$1=(kg^{3/2}+1)^{1/2}\ \cos\theta$

Here angle of incidence at B(x, y) is i = 90° – θ



$$\cos\theta = \frac{1}{(ky^{3/2}+1)^{1/2}}$$

$$\Rightarrow \tan\theta = \frac{dy}{dx} = \frac{1}{2} = \frac{\sqrt{ky^{3/2}+1-1}}{1} = \sqrt{ky^{3/2}}$$

(i) Now as

$$i = 90^o - \theta \Rightarrow \theta = 90^o - i \text{ So } \frac{dy}{dx} = \tan\theta = \cot i$$

(ii) Initial angle at air is 90° and n=1. At point B(x, y) angle of incidence is i.

So we have by Snell's law

$$1.\sin 90^o = n \sin i$$

$$\Rightarrow n \sin i = 1$$

(iii) Now, $\frac{dy}{dx} = \sqrt{k}\ y^{3/4}$ [From (i)]

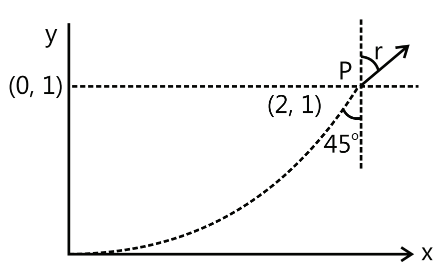
$$\Rightarrow \int_0^y \frac{dy}{y^{3/4}} = \int_0^x \sqrt{k}\ dx \Rightarrow \frac{y^{+1/4}}{+1/4} = \sqrt{k}\,x$$

$$\Rightarrow y^{+1/4} = \frac{\sqrt{k}\,x}{4} \Rightarrow y = k^2\left(\frac{x}{4}\right)^4 \quad \text{...(ii)}$$

(iv) For the point P, we have y=1.0 m k=1

$$\Rightarrow y = k^2\left(\frac{x}{4}\right)^4 \text{ gives } 1 = \left(\frac{x}{4}\right)^4 \Rightarrow x = 4.0 \text{ m}$$

(v) At P, we have



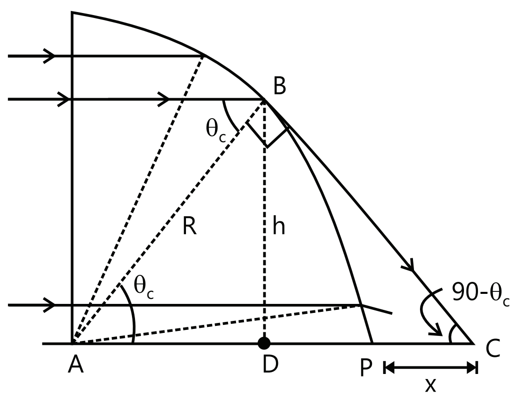
$$\frac{dy}{dx} = \tan\theta = 1 \Rightarrow \theta = 45^o \text{ [From (i) put y = 1]}$$

$$n = (1+1)^{1/2} = \sqrt{2} \Rightarrow \frac{\sin 45^o}{\sin r} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin r = 1 \Rightarrow r = 90^o$$

⇒ Ray of right becomes parallel to x axis.

Sol 9: At first refracting surface the rays will pass undeviated. At the second refracting surface the rays are refracting from denser to rarer medium and hence suffer Total Internal Refraction if $i > \theta_c$



$$\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right) \Rightarrow \theta_c = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^o$$

From right Δ BCD, $\angle BCD = 90^o - \theta_c$

From ΔABD, $h = R\sin\theta_c = R\ \sin 60^o = R\frac{\sqrt{3}}{2}$

$$DC = h\cot(90^o - \theta_c) = h\ \cot 30^o = R\frac{\sqrt{3}}{2}.\sqrt{3} = \frac{3R}{2}$$

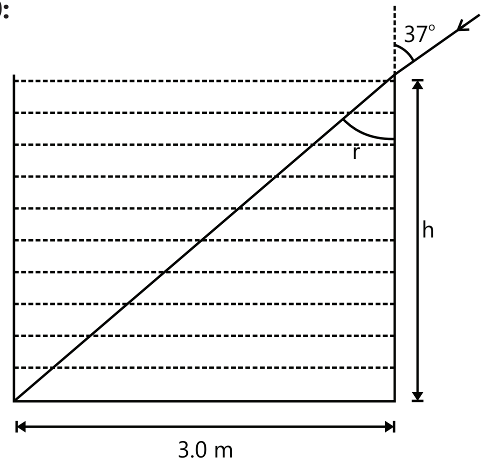
$$AP = R,\ AD = R\cos\theta_c = \frac{R}{2}$$

$$AC = AD + DC = \frac{R}{2} + \frac{3R}{2} = 2R;$$

$$x = pc = AC - AP = 2R - R = R = 5cm$$

$$\Rightarrow x = 5 \text{ m}$$

Sol 10:

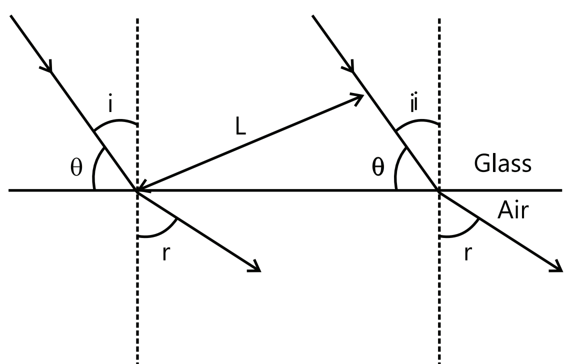


$$\frac{\sin i}{\sin r}=\mu \Rightarrow \frac{\sin\ 37^{o}}{\sin r}=\mu=\frac{4}{3}\Rightarrow \sin r=\frac{3}{\sqrt{h^2+9}}=\frac{3/5}{4/3}$$

$$\Rightarrow \frac{1}{\sqrt{h^2+9}}=\frac{3}{30}\Rightarrow 400=9\,(h^2+9)\Rightarrow h^2=\frac{400}{9}-9$$

$$\Rightarrow h=\sqrt{\frac{319}{9}}=5.95\text{ m}$$

Sol 11: $\frac{\sin i}{\sin r}=\mu$ All the ray in the bean are deviated by same angle so width of beam will not change after it goes over to air.



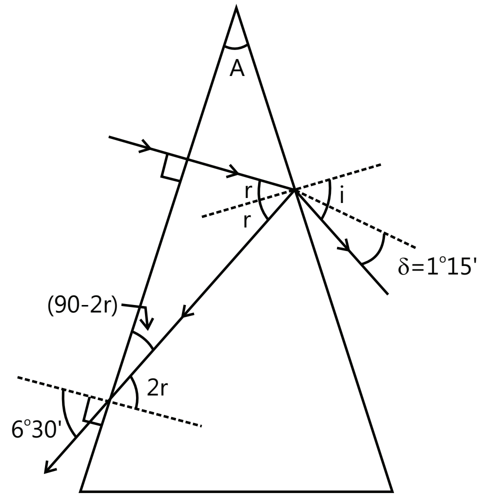
Sol 12: $\delta=(\mu-1)A=1^{o}15'=1.25^{o}=(\mu-1)\ A$ ... (i)

Now for prism $r_1+r_2=A$. Here $r_1=0$,

$r_2=r$

$\Rightarrow r=A$ ... (ii)

For reflection of reflected ray at first face of prism



$$\mu\sin 2r=\sin 6.5=\frac{6.5}{180}\pi \quad 1$$

$$\Rightarrow 2r\mu=\frac{6.5\pi}{180}\Rightarrow r\mu=\frac{6.5\pi}{360} \qquad \text{... (iii)}$$

From (i), (ii) & (iii) we get

$$\frac{1.25\ \pi}{180}=(\mu-1)A=\mu A-A \qquad \text{... (iv)}$$

$$\text{and } \frac{6.5\ \pi}{360}=\mu A \qquad \text{... (v)}$$

Subtract (iv) from (v) to get

$$\frac{(6.5-2.5)\pi}{360}=A\Rightarrow A=\frac{\pi}{90}\ \text{rad}=2^{o}$$

and put value of A in (v) to get

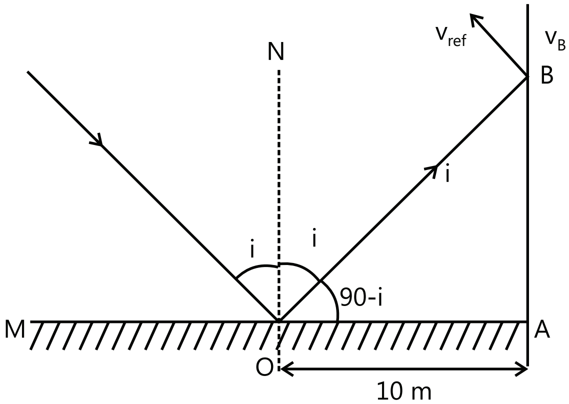
$$\mu=\frac{6.5\ \pi}{360}\times\frac{90}{\pi}\Rightarrow\mu=1.625$$

Sol 13: For a given incident ray, if the mirror is rotated through an angle θ, then the reflected ray turns through an angle of 2 θ. So if angular speed of mirror is ω then the angular speed by which the reflected ray is rotated is 2 ω.

$$\omega_{refl}=2\times\omega=2\times\frac{9\ rev}{\pi\ sec}=\frac{18}{\pi}\times 2\pi\frac{rad}{sec}$$

$$\omega_{refl}=36\text{ rad s}^{-1}$$

When



$i = 37^o$, we have

$OB = \dfrac{OA}{\cos(90-i)}$

$= \dfrac{OA}{\sin i} = \dfrac{10}{\sin 37} = \dfrac{10}{3/5} m$

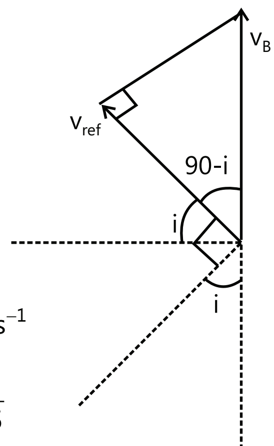
$\Rightarrow OB = \dfrac{50}{3} m$

$\Rightarrow$ speed $v_{refl} = \omega_{refl} \times [OB]$

$\Rightarrow v_{refl} = 36 \times \dfrac{50}{3} m\ s^{-1} = 600 m\ s^{-1}$

$v_B = \dfrac{v_{ref}}{\cos(90-i)} = \dfrac{v_{ref}}{\sin i} = \dfrac{600}{3/5}$

$= 200 \times 5 = 1000\ m/s.$



Sol 14: $1 \sin 90 = \mu_1 \sin r_1$........(i)

$\mu_1 \sin r_2 = \mu_2 \sin r_3$........(ii)

$\mu_2 \sin r_4 = \mu_3 \sin r_5$........(iii)

$\mu_3 \sin r_6 = \mu_4 \sin r_7$........(iv)

$\mu_4 \sin r_8 = \mu_5 \sin r_9$........(v)

$\mu_5 \sin r_{10} = 1 \sin 90$........(vi)

Also as all prisms are isosceles right angled prisms, we have

$r_1 + r_2 = r_3 + r_4 = r_5 + r_6 = r_7 + r_8 = r_9 + r_{10} = 90^o$.....(viii)

From equations (i), (ii) and (viii) we get,

$1 = \mu_1 \sin r_1$; $\mu_2 \sin r_3 = \mu_1 \cos r_1$

$\Rightarrow \mu_1^2 = 1 + \mu_2^2 \sin^2 r_3$ .....(A)

From (iii) $\mu_3 \sin r_5 = \mu_2 \cos r_3$ ......(B)

(A) and (B) gives

$\mu_1^2 + \mu_3^2 \sin^2 r_5 = 1 + \mu_2^2$ ....(C)

From (iv)

$\mu_3 \cos r_5 = \mu_4 \sin r_7$ ...(D)

(C) and (D) gives

$\mu_1^2 + \mu_3^2 = 1 + \mu_2^2 + \mu_4^2 \sin^2 r_7$ ....(E)

From (v) $\mu_5 \sin r_9 = \mu_4 \cos r_7$ ......(F)

(E) and (F) gives

$\mu_1^2 + \mu_3^2 + \mu_5^2 \sin^2 r_9 = 1 + \mu_2^2 + \mu_4^2$ .....(G)

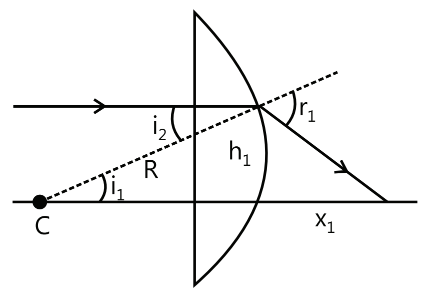
From (vi) $\mu_5 \cos r_9 = 1$ ....(H)

(G) and (H) gives

$\Rightarrow \mu_1^2 + \mu_3^2 + \mu_5^2 = 2 + \mu_2^2 + \mu_4^2$

Sol 15: Snell's law at spherical surface for the first ray

$\mu \sin i_1 = \sin r_1$



$\Rightarrow 1.6\left(\dfrac{h_1}{R}\right) = \dfrac{h_1}{\sqrt{h_1^2 + x_1^2}} \Rightarrow h_1^2 + x_1^2 = \dfrac{R^2}{2.56}$

$\Rightarrow 0.5^2 + x_2^2 = \dfrac{20^2}{2.56} \Rightarrow x_2 = 12.49\ m$

Similarly for second ray

$\Rightarrow 1.6\left(\dfrac{h_2}{R}\right) = \dfrac{h_2}{\sqrt{h_2^2 + x_2^2}} \Rightarrow h_2^2 + x_2^2 = \dfrac{R^2}{2.56}$

$\Rightarrow x_2^2 = \dfrac{20^2}{2.56} - 12^2 \Rightarrow x_2 = 3.5\ m$

$\Rightarrow \Delta x_2 = x_1 - x_2 = 9\ m$

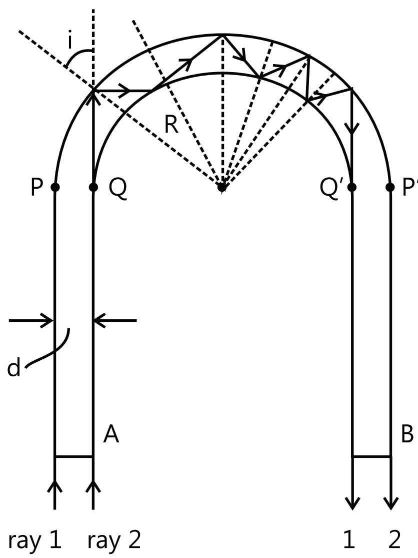
Sol 16: (a) For total internal reflection at the concrete-air interface we have critical angle

$\dfrac{\sin\theta_c}{\sin 90} = \dfrac{\mu_2}{\mu_1} = \dfrac{v_1}{v_2} = \dfrac{340 m/s}{1700 m/s}$

$\Rightarrow \sin\theta_c = \dfrac{1}{5} \Rightarrow \theta_c = \sin^{-1}\left(\dfrac{1}{5}\right)$

(b) The concrete is a rarer medium for sound because the speed of sound is higher in concrete, while air will be denser medium for sound as the speed of sound is lower in air. So for TIR, sound must travel in air which is denser medium in this case.

Sol 17: The outer most ray of the beam , ray 1, will be tangential to the circular surface of rod at point P and hence angle of incidence is 90°, hence greater than critical angle, and hence will travel tangentially at all points of the circular portion from P to P′.



The inner most ray of the beam, ray 2, will be incident on the inner circular surface at angle i.

From the geometry of figure we see that,

CQ = R; CA= R +d

R=(R+d) cos (90 - i) $\Rightarrow$ R = (R + d) sin i

Here angle i will be the least of all angles of incidence of ray 2 during its path inside the critical rod. So, if i is greater than critical angle then ray 2 will surfer TIR at all point in circular rod.

$$\sin i > \frac{1}{\mu} \Rightarrow \frac{R}{R+d} > \frac{1}{\mu} \Rightarrow \frac{1}{1+\frac{d}{R}} > \frac{1}{\mu} \Rightarrow 1+\frac{d}{R} < \mu$$

$$\Rightarrow \frac{d}{R} < (\mu-1) \Rightarrow \left(\frac{d}{R}\right)_{max} = (\mu-1) = 1.5-1$$

$$\Rightarrow \left(\frac{d}{R}\right)_{max} = \frac{1}{2}$$

Sol 18: In a prime $r_1+r_2=A$;

Here $r_1=r$, $r_2=0^o$, $A=30^o \Rightarrow r = 30^o$ .....(i)

At first refracting surface

$$\frac{\sin i}{\sin r} = \mu = \sqrt{2} \Rightarrow \sin i = (\sin 30^o)\sqrt{2} = \sqrt{2}\left(\frac{1}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow i = 45^\circ$$

Sol 19: Angle of deviation is gives as $\delta = (i_1 + i_2) - A$

Hence $i_2 - i_1$ = 23°, A =60°, δ =23°

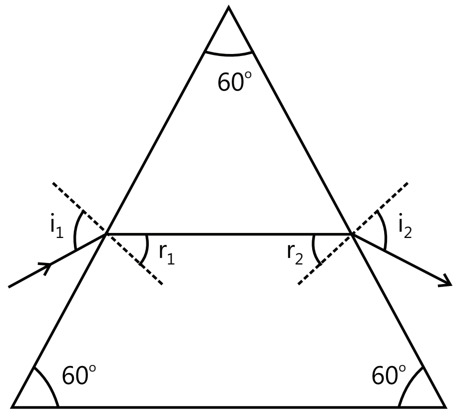
$\Rightarrow 23° = i_1 + i_2 - 60$

$\Rightarrow i_1 + i_2 = 83°$ ... (i)

$\Rightarrow i_2 - i_1 = 23°$ ... (ii)

From (i) and (ii) we get i_1 = 30°, i_2=53°

Snell's law at first refracting surface.

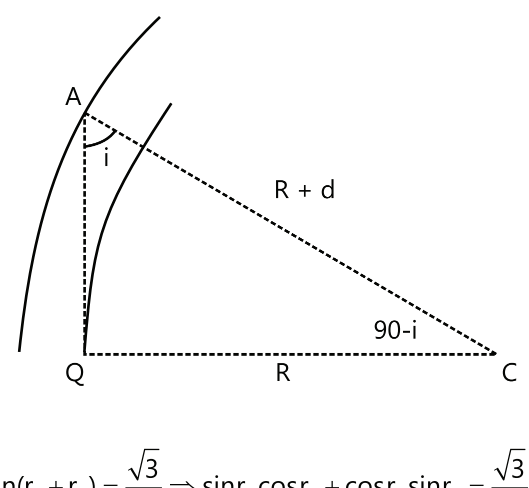


$$\sin i_1 = \mu \sin r_1 \Rightarrow \frac{1}{2} = \mu \sin r_1 \Rightarrow \sin r_1 = \frac{1}{2\mu}$$

Snell's law at second refracting surface.

$$\mu \sin r_2 = \sin i_2 \Rightarrow \sin r_2 = \frac{1}{\mu}.\frac{4}{5} = \frac{4}{5\mu}$$

Now $r_1+r_2=60°$



$$\Rightarrow \sin(r_1+r_2) = \frac{\sqrt{3}}{2} \Rightarrow \sin r_1 \cos r_2 + \cos r_1 \sin r_2 = \frac{\sqrt{3}}{2} \Rightarrow \mu$$

$$\Rightarrow \frac{1}{2\mu}.\sqrt{1-\frac{16}{25\mu^2}} + \sqrt{1-\frac{1}{4\mu^2}}.\frac{4}{5\mu} = \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{43}}{5}$$

Sol 20: Deviation suffered by the transmitted ray is

$\delta_1 = (60^o - r_1) + (60^o - r_1)$

$\delta_1 = 120 - 2r_1$ ... (i)

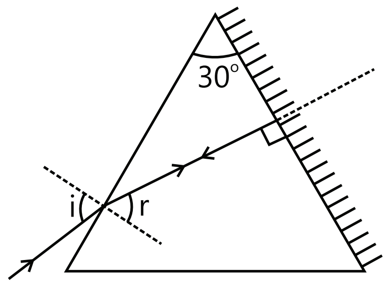
Deviation suffered by the reflected ray after emerging out of sphere.

$\delta_2 = (60^o - r_1) + (180^o - 2r_1) + (60^o - r_1)$

$\delta_2 = 300^o - 4r_1$ ...(ii)

Now $\delta_1 = \frac{1}{3}\delta_2 \Rightarrow 3\delta_1 = \delta_2$ ...(iii)

From (i), (ii) and (iii) we get

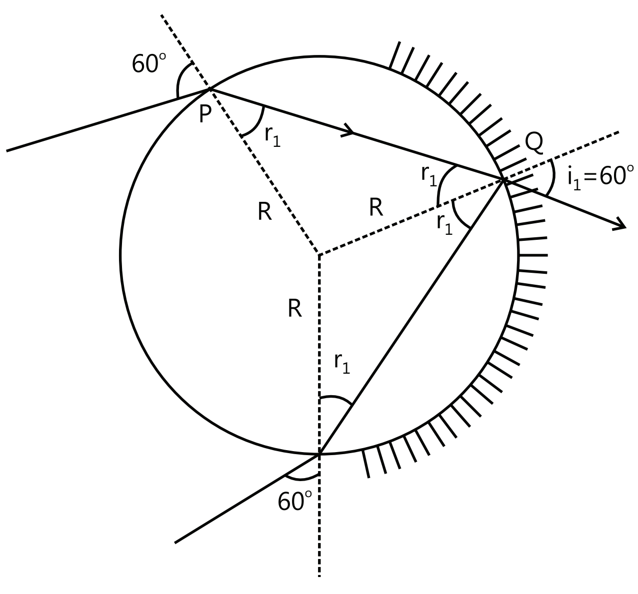


$3(120° - 2r_1) = 300° - 4r_1$

$\Rightarrow 360° - 6r_1 = 300° - 4r_1$

$\Rightarrow +2r_1 = +60° \Rightarrow r_1 = 30^o$ ...(iv)

Snell's law of point P



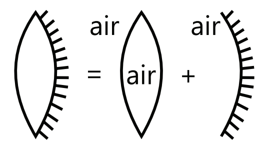
$\frac{\sin 60}{\sin r_1} = \mu \Rightarrow \mu = \frac{\sqrt{3}/2}{1/2} \Rightarrow \mu = \sqrt{3}$

Sol 21: When air is filled between two similar glass pieces.

$P_e = 2P_L + P_M$

$P_L = \frac{1}{f_L}$ where $\frac{1}{f_L} = \left(\frac{\mu_L - \mu_m}{\mu_m}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\Rightarrow \frac{1}{f_L}\left(\frac{1-1}{1}\right)\left(\frac{1}{R} - \frac{1}{-R}\right) \Rightarrow \frac{1}{f_L} = 0$



Then P_L=0 $P_M = -\frac{1}{f_M}$ where $f_M = = -\frac{R}{2}$

(since mirror is concave)

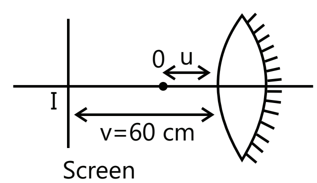
Given that screen is placed at a distance 60 cm from the combination

$m = -\frac{v}{u} \Rightarrow$ Two times magnification means

m=-2, then v=2u$\Rightarrow$ 60=2u$\Rightarrow$ v=30cm

For equivalent combination $P_e = -\frac{1}{f_e} \Rightarrow f_e = -\frac{1}{P_e} = -\frac{R}{2}$

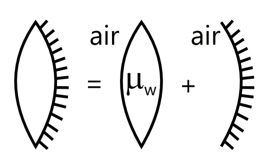
Apply mirror formula to the equivalent combination of mirror.



$\frac{1}{v} + \frac{1}{u} = \frac{1}{f_e}$

$\Rightarrow \frac{1}{-60} + \frac{1}{-30} = \frac{1}{f_e} \Rightarrow -\frac{1}{60} - \frac{1}{30} = -\frac{2}{R}\left(\text{since } f_e = -\frac{R}{2}\right)$

R=40cm



Again if air between the glass pieces is replaced by water.

$P_e = 2P_L + P_M$

$P_L = \frac{1}{f_L}$ where $\frac{1}{f_L} = \left(\frac{\mu_1 - 1}{1}\right)\left(\frac{1}{40} - \frac{1}{-40}\right)$

$\Rightarrow (\mu_w = 4/3) \Rightarrow P_L = \frac{1}{60} \Rightarrow P_M = -\frac{1}{f_M}$

Where $\frac{1}{f_M} = -\frac{R}{2}$(for concave mirror)

$\frac{1}{f_M} = -\frac{R}{2}$(for concave mirror)

$P_M = \frac{2}{R} = \frac{2}{40}$

$\Rightarrow P_e = 2P_L + P_M$

$2 \times \frac{1}{60} + \frac{2}{40} = \frac{5}{60} = \frac{1}{12}$

$-\frac{1}{f_e} \Rightarrow f_e = -12$ cm

Again apply $\frac{1}{v} + \frac{1}{u} = \frac{1}{f_e}$

$\Rightarrow \frac{1}{-60} + \frac{1}{u} = \frac{1}{-12} \Rightarrow u = -15$ cm

When air is filled between the gap, object distance=30 when water is filled between the gas, object distance=15 cm, Then, object is displaced by 15 cm towards the combination.

Sol 22: (a) At surface light intensity

$$I = \frac{4.5\pi W}{4\pi.(1.5)^2 \times 10^{-4} m^2} \Rightarrow I = \frac{1.125 \times 10^4}{2.25} Wm^{-2}$$

$$= 5000\, Wm^{-2}$$

(b) $I_P = \frac{4.5\pi W}{4\pi.(7.5)^2 m^2} = 0.02 Wm^{-2}$

(c) $u = -7.5m = -750cm$

$f = +30cm$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{30} - \frac{1}{750} = \frac{14}{750} = \frac{7}{375}$$

$\Rightarrow v = \frac{375}{7}$cm,

Magnification, $m = \frac{v}{u} = \frac{375/7}{-750} \Rightarrow m = -\frac{1}{14}$

$\Rightarrow$ diameter of image of bulb, $d_1 = \frac{3.0\,cm}{14} = 0.214\,cm$

(d) Light intensity at image is the intensity focused by the lens.

$$I_{image} = \frac{0.02 \times \pi\,(7.5)^2 \times 10^{-4} W}{4\pi\left(\frac{0.214}{2}\right)^2 \times 10^{-4} m^2} s$$

$I_{image} = 24.56\, Wm^{-2}$

Sol 23: Focal length of Plano - concave lens

$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{R}\right)$

$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(-\frac{1}{30}\right) = -\frac{1}{60} \Rightarrow f_1 = -60$ cm

Focal length of plano-convex lens

$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$

$\frac{1}{f_1} = \left(\frac{5}{4} - 1\right)\left(\frac{1}{30}\right) = \frac{1}{120} \Rightarrow f_2 = 120$ cm

(i) Plane surface of Plano-convex lens is silvered. So the equivalent focal length of the system,

Power $= \frac{1}{-F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{-f_m} + \frac{1}{f_2} + \frac{1}{f_1}$

f_m = focal length of plane mirror $= \infty$

$\Rightarrow \frac{1}{-F} = \frac{2}{f_1} + \frac{2}{f_2} = -\frac{2}{60} + \frac{2}{120} = \frac{-1}{60}$

$\Rightarrow +F = +60\,cm$

Focal length = 60 cm

The equivalent system behaves as a convex mirror.

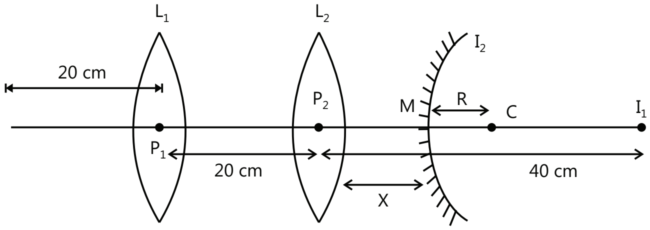
(ii) Mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$. Hence u = -15 cm, f = +60 cm

$\Rightarrow \frac{1}{v} = \frac{1}{60} - \frac{1}{-15} = \frac{1}{60} + \frac{4}{60} = \frac{1}{2}$

$\Rightarrow v = +12cm$

Magnification, $m = -\frac{v}{u} = \frac{-12}{-15} \Rightarrow m = \frac{4}{5}$

Sol 24: Image from L_1:



$$\frac{1}{v_1}=\frac{1}{f}+\frac{1}{u}=\frac{1}{15}-\frac{1}{20}=\frac{4-3}{60}=\frac{1}{60}$$

$$\Rightarrow v_1=60\,\text{cm from } L_1$$

For lens L_2:

$$u_2=+40\,\text{cm},\ \frac{1}{v_2}=\frac{1}{15}+\frac{1}{40}=\frac{8+3}{120}$$

$$\Rightarrow v_2=+\frac{120}{11}\text{cm from } L_2$$

This final image should lie at the centre of curvature of convex mirror, so MC = R = 5 cm

$$\text{So}, P_2M=P_2C_2-MC=\frac{120}{11}-5=\frac{120-55}{11}\text{cm}$$

$$\Rightarrow x=\frac{65}{11}\text{cm}=5.91\,\text{cm}$$

Sol 25: For equi-convex lens radius R= f= 30 cm

Refraction at surface I: Air to glass

$$\frac{3/2}{v_1}-\frac{1}{(-90)}=\frac{\frac{3}{2}-1}{+30\text{ cm}} \quad \text{... (i)}$$

Refraction at surface II: Glass to water

$$\frac{4/3}{v_2}-\frac{3/2}{v_1}=\frac{\frac{4}{3}-\frac{3}{2}}{-30\text{ cm}} \quad \text{... (ii)}$$

Add (i) and (ii)

$$\frac{4}{3v_2}+\frac{1}{90}=\frac{\frac{3}{2}-1-\frac{4}{3}+\frac{3}{2}}{30}$$

$$\Rightarrow \frac{4}{3v_2}=\frac{2}{90}-\frac{1}{90}\Rightarrow v_2=+120\,\text{cm}$$

For mirror, image of lens acts as object.

For mirror u_3=+ (120-80) = 40 cm (right from mirror)

So v_3 = –40 cm (left from mirror)

Refraction from surface II after reflection from mirror.

u_4=–40 cm (left from surface II)

$$\frac{3/2}{v_5}-\frac{4/3}{-40}=\frac{\frac{3}{2}-\frac{4}{3}}{+30} \quad \text{... (iii)}$$

Refraction at surface I: Glass to air

$$\frac{1}{v_6}-\frac{3/2}{v_5}=\frac{1-\frac{3}{2}}{-30} \quad \text{... (iv)}$$

Add (iii) and (iv)

$$\frac{1}{v_6}+\frac{1}{30}=\frac{\frac{3}{2}-\frac{4}{3}-1+\frac{3}{2}}{30}$$

$$\Rightarrow \frac{1}{v_6}=\frac{2}{90}-\frac{1}{30}=\frac{-1}{90}\Rightarrow v_6=-90\,\text{cm}$$

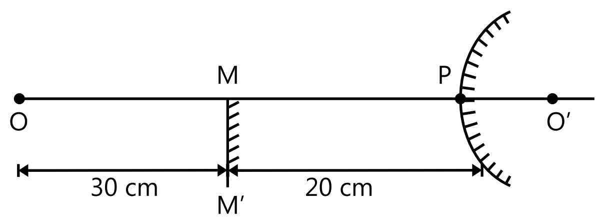
from lens (right from lens)

So final image is 90 cm right of lens.

Exercise 2

Single Correct Choice Type

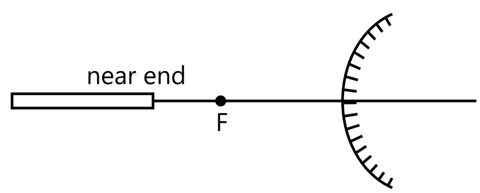
Sol 1: (B) Distance of image due to plane mirror from object will be 60 cm. So OO′ =60 cm. So distance of image from convex mirror PO′ =[OO′-OP]=10 cm



$\Rightarrow$Mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ will give focal length of mirror by putting u=-50 cm, v=+10 cm,

$$\text{So, } \frac{1}{f}=\frac{1}{10}-\frac{1}{50}=\frac{4}{50}=\frac{2}{25}\Rightarrow f=\frac{25}{2}$$

Radius of curvature R=2f =25 cm



Sol 2: (A) Mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{u-f}{fu}\Rightarrow v=\frac{fu}{u-f}.$$

At near end |u|> |f|. u and f both are negative .

So v is negative. At far end we have $u=-\infty$. So mirror formula gives $\frac{1}{v_\infty}=\frac{1}{f}-\frac{1}{-\infty}=\frac{1}{f}\Rightarrow v_\infty=f$.

For near point $|v_{near}|>|f|$.

Image length

$$|v_{near}| - |v_\infty| = \frac{fu}{u-f} - f = \frac{fu - uf + f^2}{u-f}$$

$$\Rightarrow \Delta I = \frac{f^2}{u-f}$$

Sol 3: (C) Velocity of image

$$\frac{dv}{dt} = -\left(\frac{v^2}{u^2}\right)\frac{du}{dt};\ \frac{dv}{dt} = -\left(\frac{v^2}{20^2}\right).4\text{ cm s}^{-1}.$$

From mirror formula

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{-12} - \frac{1}{-20} = \frac{1}{20} - \frac{1}{12} = \frac{3-5}{60} = -\frac{1}{30}$$

$$\Rightarrow v = -30\text{ cm} \Rightarrow \frac{dv}{dt} = -\left(\frac{900}{400}\right)4\text{ cm s}^{-1} = -9\text{ cm s}^{-1}$$

Sol 4: (D) Area of mirror, $A_1 = \pi\frac{d^2}{4}$.

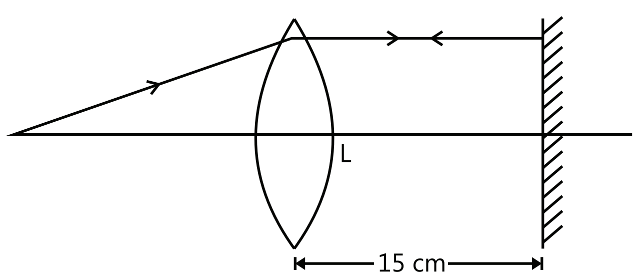
Area left after putting opaque,

$$A_2 = A_1 - \pi\frac{d^2}{16} = \pi\frac{d^2}{4} - \pi\frac{d^2}{16}$$

$$\Rightarrow A_2 = \pi\frac{d^2}{4}\left(1 - \frac{1}{4}\right) = \frac{3}{4}\frac{\pi d^2}{4} = \frac{3}{4}A_1$$

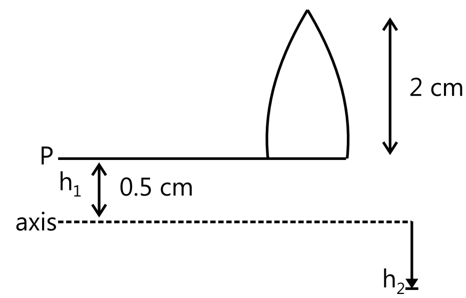
Focal length will not change and intensity become $\frac{3}{4}I$.

Sol 5: (B) Rays should fall normally on plane mirror. This will happen if rays become parallel to principal axis after passing through lens. So OL = f=30 cm.



Sol 6: (D) Lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$; u = -30 cm,

f = 20 cm



$$\frac{1}{v} = \frac{1}{20} - \frac{1}{30} = \frac{3-2}{60} = \frac{1}{60} \Rightarrow v = 60$$

Magnification $m = \frac{v}{u} \Rightarrow m = \frac{60}{-30} = -2 \Rightarrow h_2 = -2h_1$

So $h_2 = -2 \times 0.5\text{ cm} = -1\text{ cm}$ (below axis)

So Image of P is 1.5 cm below XY.

Sol 7: (B) Distance between object and screen is D, displacement of lens is d, and so focal length of lens is

$$f = \frac{D^2 - d^2}{4D} = \frac{90^2 - 20^2}{4 \times 90} = 21.4\text{ cm}$$

Sol 8: (A) Object size

$$O = \sqrt{I_1 \times I_2} = \sqrt{6\text{ cm} \times 3\text{ cm}}$$

$$O = 4.24\text{ cm}$$

Sol 9: (A) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Let u = + x (virtual) and |f| = -f (concave mirror)

$$\Rightarrow \frac{1}{v} = -\frac{1}{|f|} - \frac{1}{x} = \frac{-x - |f|}{|f|x} \Rightarrow v = \frac{|f|x}{-x - |f|} \rightarrow (-)\text{ve}$$

So v is always negative when u is positive (+x).

Multiple Correct Choice Type

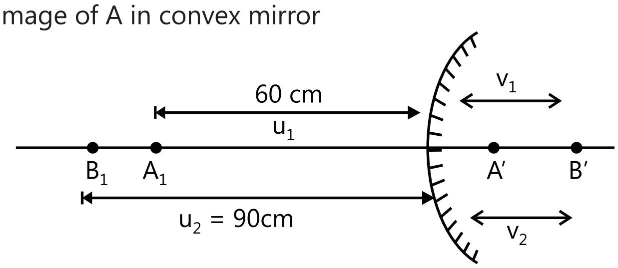
Sol 10: (B, D) Slope of reflecting surface at the desired point will be $\tan 45^\circ = 1$

$$\frac{dy}{dx} = 2\cos\left(\frac{\pi x}{L}\right) = 1 \Rightarrow \frac{\pi x}{L} = \frac{\pi}{3} \Rightarrow x = \frac{L}{3}$$

$$\Rightarrow y\left(\frac{L}{3}\right) = \frac{2L}{\pi}\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}L}{\pi}$$

Sol 11: (B, C) Length of object is AB = (50-20) cm =30 cm. After first reflection from plane mirror AB is inverted to B_1A_1 with distance from convex mirror as shown in figure.

Image of A in convex mirror



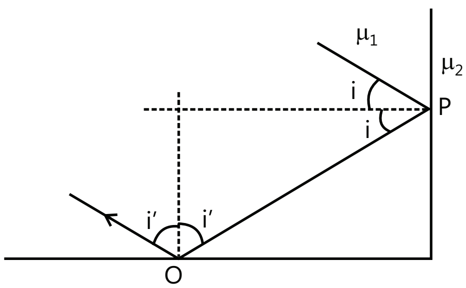
$$\frac{1}{v_1}=\frac{1}{f}-\frac{1}{u_1}\Rightarrow\frac{1}{v_1}=\frac{1}{60}-\frac{1}{-60}=\frac{1}{30}\Rightarrow v_1=+30\text{cm}$$

Image of B in convex mirror

$$\frac{1}{v_2}=\frac{1}{f}-\frac{1}{u_2}=\frac{1}{60}-\frac{1}{-90}=\frac{3+2}{180}=\frac{1}{36}$$

$$\Rightarrow v_2=+36\,\text{cm}\Rightarrow A'B'=(36-30)\,\text{cm}=6\,\text{cm}$$

Second image A′B′ is virtual and $\left(\frac{1}{5}\right)^{th}$ of magnification w. r. t. AB and erect.



Now formula for speed of image for convex mirror is, $\frac{dv}{dt}=\frac{v^2}{u^2}\frac{du}{dt}$. As object moves towards mirror, the image also moves towards the mirror.

Sol 12: (B, C) i + i′ =90° from figure

At point P, $\mu_1 \sin i > \mu_2$ ... (i)

At Q, $\mu_1 \sin i' > \mu_3$ or $\mu_1 \cos i > \mu_3$ ... (ii)

Squaring and adding (1) and (2) to get

$$\mu_1^2 > \mu_2^2+\mu_3^2 \Rightarrow \mu_1^2-\mu_2^2 > \mu_3^2 \Rightarrow \mu_1^2-\mu_3^2 > \mu_2^2$$

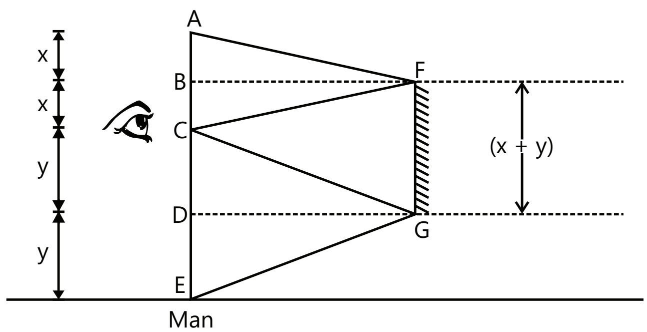
Sol 13: (A, C) Angle of deviation $\delta = (\mu - 1)A$.

Sol 14: (A, B, D) Angle of deviation $\delta=(i_1+i_2)-A$. i_1 is angle of incidence and i_2 is angle of emergence and angle of incidence and emergence are interchangeable.

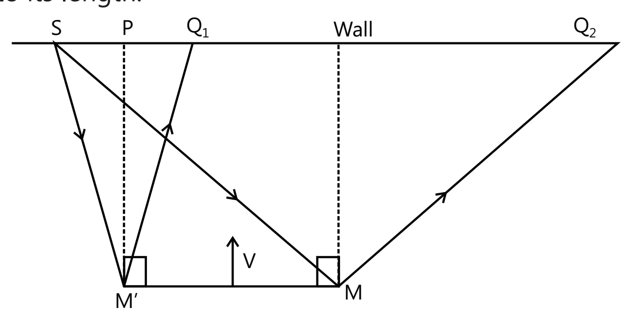
If δ_m is minimum deviation, $\frac{\mu_p}{\mu_s}=\frac{\sin\left(\frac{\delta_m+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

So as μ_p increases , δ_m also increases.

Sol 15: (B, C) The minimum length of a plane mirror to see one's full height in it is $\frac{H}{2}$, where H is the height of man. The mirror can be placed anywhere between the centre line BF (of AC) and DG (of CE). Eye is at C.



Sol 16: (B, D) The distance PQ_1 and PQ_2 will not change as the mirror MM′ moves with speed v perpendicular to its length.



Sol 17: (A, C) $\mu_2 > \mu_1$ Rays from real object will be deviated away from radius of curvature and hence will becomes more diverging. For virtual object the deviated rays may converge on the principle axis.

$\mu_1 > \mu_2$: For virtual object the deviated rays will converge on principle axis.

For real object the refracted ray will deviate towards radius of curvature and may coverage on principle axis.

Sol 18: (A, D) The image formed by a convex mirror is always, virtual and erect. So convex mirror cannot form inverted image of OA. Option B and C are ruled out.

Sol 19: (A, C) u = -40 cm; f = +20 cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{20}-\frac{1}{40}=\frac{1}{40}$$

$\Rightarrow v = +40$ cm, and magnification $m=\frac{v}{u}=\frac{40}{-40}$

$\Rightarrow m=-1\Rightarrow$ y and z coordinate's values will change their sign but magnitude will remain the same.

$\Rightarrow y_{image}=-1, z_{image}=+1$

Sol 20: (B, D) Convex mirror and concave lens for diminished, virtual, correct image of object.

Assertion Reasoning Type

Sol 21: (C) The wall does reflect light, but the reflection is irregular. Reflected rays are deviated in different directions.

Sol 22: (A) This phenomena is called spherical aberration. The rays close to the principal axis are focused at the geometrical focus F of the mirror as given by mirror formula. The rays farthest from the principal axis are focused at a point somewhat closer to mirror.

Sol 23: (D) For object in liquid. $d_{apparent}=\frac{d_{actual}}{\mu}$

For a slab: normal shift $\Delta x=t\left(1-\frac{1}{\mu}\right)$ where t is thickness of slab.

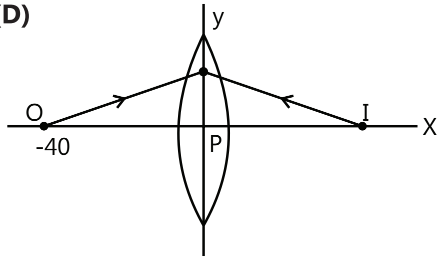
Sol 24: (D) Two image will be formed one for each mirror.

Sol 25: (D) If a plane mirror is moved such that its perpendicular distance from the point object does not change, then the image will not move.

Comprehension Type

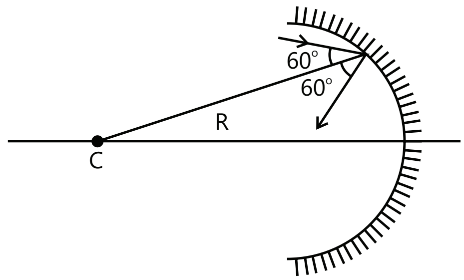
Sol 26: (B) Paraxial rays are focused at the geometrical focus F of the mirror. The marginal rays are focused at a point F' somewhat closer to the mirror.

Sol 27: (D)



$$f=R-\frac{R}{2}\sec 60^\circ=R-R=0$$

Sol 28: (D) Deviation $=180^o-2\times60^o=60^o$. Ans (D)



Sol 29: (B) $f=R-\frac{R}{2}\sec 0^\circ=\frac{R}{2}$

Sol 30: (B) Spherical aberration cannot be completely eliminated, but it can be minimized by allowing either paraxial or marginal rays to hit the mirror.

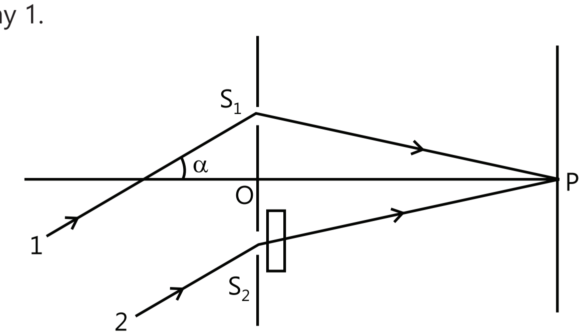
Sol 31: (D)

$$y_{min}=\frac{(2n-1)\lambda D}{2d} \qquad (n=1,2,..........)$$

For $n=+3$, $y_{min}=\frac{5\lambda D}{2d}$

Sol 32: (A) Shift in the fringe due to the glass slab is $\Delta y=(\mu-1)t\frac{D}{d}$ where t is thickness of glass slab. Due to glass slab path of ray from S_2 gets increased by $(\mu-1)t$.

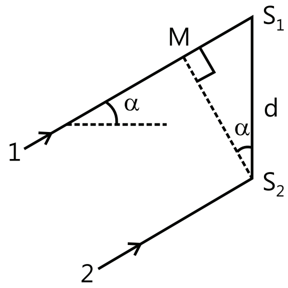
Sol 33: (A) Path of rays 1 is more than path of ray 2 by a distance $d\sin\alpha$. Draw perpendicular S_2M from S_2 to ray 1.



$\angle MS_2S_1=\alpha$ and $MS_1=d\sin\alpha$

This path difference is suffered before passing the slits S_1 & S_2. After passing through the slits, path of ray from

S_2 is increased by $(\mu-1)t$. For net path difference to be zero at point P we have,



$d \sin\alpha = (\mu-1)t$

$\Rightarrow \alpha = \sin^{-1}\left(\frac{(\mu-1)t}{d}\right)$

Match the Column

Sol 34: A → p, r; B → q, s; C → p, q, r, s; D → p, q, r, s

Previous Years' Questions

Sol 1: (C, D) $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ (mirror formula)

F = – 24 cm

Sol 2: (B) Critical angle from region III to region IV

$$\sin\theta_c = \frac{n_0/8}{n_0/6} = \frac{3}{4}$$

Now applying Snell's law in region I and region III

$$n_0 \sin\theta = \frac{n_0}{6}\sin\theta_C$$

or $$\sin\theta = \frac{1}{6}\sin\theta_C = \frac{1}{6}\left(\frac{3}{4}\right) = \frac{1}{8}$$

∴ $$\theta = \sin^{-1}\left(\frac{1}{8}\right)$$

Sol 3: (A, C, D) In case of concave mirror or convex lens image can be real, virtual, diminished, magnified or of same size.

(B) In case of convex mirror image is always virtual (for real object).

Sol 4: (A) At minimum deviation $(\delta = \delta_m)$:

$r_1 = r_2 = \frac{A}{2} = \frac{60^o}{2} = 30^o$ (For both colours)

Sol 5: (B) $\lambda_{cutoff} = \frac{hc}{eV}$ (independent of atomic number)

Sol 6: (C) The refractive index n for meta-material is negative.

Hence $\frac{\sin\theta_1}{\sin\theta_2}$ is negative.

Thus if θ_1 is negative, θ_2 will be negative. So the current choice is C.

Sol 7: (B) $N = \frac{c}{v} = v = \frac{c}{|n|}$ which is choice B

Also frequency $\nu = \frac{v}{\lambda}$ since v remains unchanged

$$\frac{v_{air}}{\lambda_{air}} = \frac{v_m}{\lambda_m}$$

$$\Rightarrow \lambda_m = \lambda_{air} \times \frac{v_m}{v_{air}} = \lambda_{air} \times \frac{v_m}{c} \times \frac{c}{v_{air}}$$

$$= \lambda_{air} \times \frac{n_{air}}{n_m} \qquad \left(\because v = \frac{c}{n}\right)$$

$$\frac{\lambda_{air}}{|n|} \qquad \left(\because n_m = |n| \text{ and } n_{air} = 1\right)$$

So choice D is wrong

Sol 8: (B) $P_T = (1.5 - 1)\left(\frac{1}{14} - 0\right) + (1.2 - 1)\left(0 - \frac{1}{-14}\right)$

$= \frac{0.5}{14} + \frac{0.2}{14} = \frac{1}{20}$

f = + 20 cm

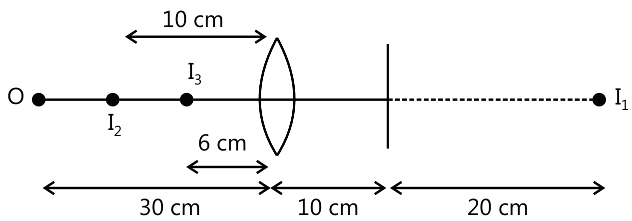
$\frac{1}{v} - \frac{1}{-40} = \frac{1}{20}$

$\frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{1}{40}$

∴ v = 40 cm

Sol 9: (B) Object is placed at distance 2*f* from the lens. So first image I_1 will be formed at distance *2f* on other side. This image I_1 will behave like a virtual object for mirror. The second image I_2 will be formed at distance 20 cm in front of the mirror, or at distance 10 cm to the left hand side of the lens.

Now applying lens formula



$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$\therefore \qquad \frac{1}{v}-\frac{1}{+10}=\frac{1}{+15}$

or $\quad v = 6$ cm

Therefore, the final image is at distance 16 cm from the mirror. But, this image will be real.

This is because ray of light is travelling from right to left.

Sol 10: (3) For $v_1 = \frac{50}{7}$ m, $u_1 = -25$ m

$v_2 = \frac{25}{3}$ m, $u_2 = -50$ m

Speed of object $= \frac{25}{30} \times \frac{18}{5} = 3$ km/h.

Sol 11: (B)

For the combination $\frac{1}{f_{eq}} = \frac{(\mu_1 - 1)}{R} + \frac{(\mu_2 - 1)}{R}$

$\boxed{f_{eq} = 20}$

Here u = - 40, f = 20

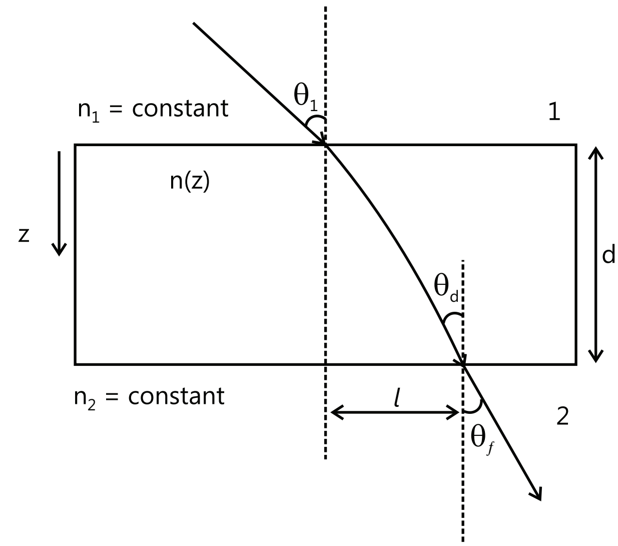
$\boxed{v = 40}$

Sol 12: (A, C, D) From Snell's Law

$n_1 \sin\theta_i = n(d)\sin\theta_d = n_2 \sin\theta_f$

The deviation of ray in the slab will depend on n (z)

Hence, l will depend on n (z) but not on n_2.



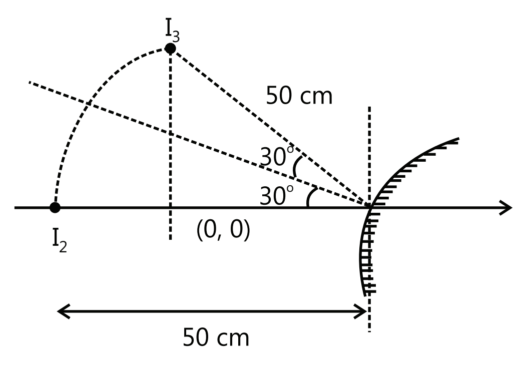
Sol 13: (A) First Image I_1 from the lens will be formed at 75 cm to the right of the lens.

Taking the mirror to be straight, the image I_1 after reflection will be formed at 50 cm to the left of the mirror.

On rotation of mirror by $30°$ the final image is I_3.

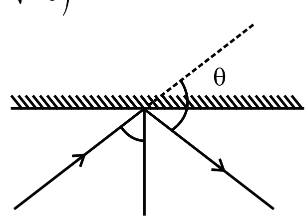
So x = 50 – 50 cos $60° = 25$ cm.

and y = 50 sin $60° = 25\sqrt{3}$ cm.



Sol 14: (A) Let angle between the directions of incident ray and reflected ray be θ

$$\cos\theta = \frac{1}{2}\left(\hat{i} + \sqrt{3}\,\hat{j}\right)$$

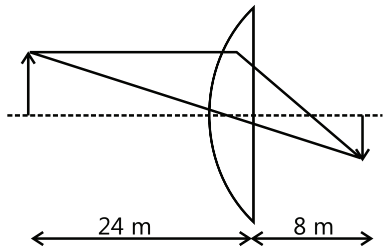


$$\cos\theta = \frac{1}{2}\left(\hat{i} + \sqrt{3}\,\hat{j}\right).\frac{1}{2}\left(\hat{i} + \sqrt{3}\,\hat{j}\right)$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = 120°$$

Sol 15: (C)



$$\mu = \frac{\lambda_a}{\lambda_m} = \frac{3}{2}$$

$$\Rightarrow \frac{1}{f} = \frac{\mu-1}{R} = \frac{1}{2R}$$

$$\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{8} - \frac{1}{-24} = \frac{1}{2R}$$

$$\Rightarrow \frac{3+1}{24} = \frac{1}{2R}$$

$$\Rightarrow R = 3\text{ m}$$

Sol 16: (D) P $\to$ (2); Q $\to$ (3); R $\to$ (4) ; S $\to$ (1)

P. $\mu_2 > \mu_1$.... (towards normal)

$\mu_2 > \mu_3$.... (away from normal)

Q. $\mu_1 = \mu_2$.... (No change in path)

$\angle i = 0 \Rightarrow \angle r = 0$ on the block.

R. $\mu_1 > \mu_2$.... (Away from the normal)

$\mu_2 > \mu_3$.... (Away from the normal)

$$\mu_1 \times \frac{1}{\sqrt{2}} = \mu_2 \sin r \Rightarrow \sin r = \frac{\mu_1}{\sqrt{2}\,\mu_2}.$$

Since $\sin r < 1 \Rightarrow \mu_1 < \sqrt{2}\,\mu_2$

S. For TIR:

$$45° > C \Rightarrow \sin 45° > \sin C \Rightarrow \frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1} \Rightarrow \mu_1 > \sqrt{2}\,\mu_2$$

Sol 17: (A, C) For air to glass

$$\frac{1.5}{f_1} = \frac{1.4-1}{R} + \frac{1.5-1.4}{R}$$

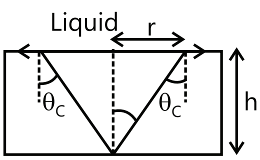
$$\therefore f_1 = 3R$$

For glass to air.

$$\frac{1}{f_2} = \frac{1.4-1.5}{-R} + \frac{1-1.4}{-R}$$

$$\therefore f_2 = 2R$$

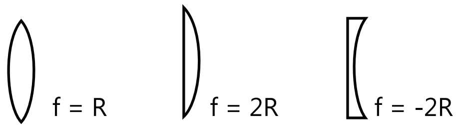
Sol 18: (C) $\tan\theta_c = \frac{r}{h} = \frac{5.77}{10} \approx \sqrt{3}$



$$\Rightarrow \sin\theta_c = \frac{\mu_\ell}{\mu_b}$$

$$\Rightarrow \mu_\ell = 2.72 \times \frac{1}{2} = 1.36$$

Sol 19: (B) $\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$



Use $\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$

(P) $\frac{1}{f_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R};\ f_{eq} = \frac{R}{2}$

(Q) $\frac{1}{f_{eq}} = \frac{1}{2R} + \frac{1}{2R} = \frac{1}{R};\ f_{eq} = R$

(R) $\frac{1}{f_{eq}} = -\frac{1}{2R} - \frac{1}{2R} = -\frac{1}{R};\ f_{eq} = -R$

(S) $\frac{1}{f_{eq}} = \frac{1}{R} - \frac{1}{2R} = \frac{1}{2R};\ f_{eq} = 2R$

Sol 20: Image by mirror is formed at 30 cm from mirror at its right and finally by the combination it is formed at 20 cm on right of the lens. So in air medium, magnification by lens is unity. In second medium, $\mu = \frac{7}{6}$. focal length of the lens is given by,

$$\frac{\frac{1}{10}}{\frac{1}{f}} = \frac{(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}{\left(\frac{1.5}{7/6}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)} \Rightarrow f = \frac{35}{2}\text{ cm}$$

So in second medium, final image is formed at 140 cm to the right of the lens. Second medium does not change the magnification by mirror. So $\left|\frac{M_2}{M_1}\right| = \left|\frac{M_{m_2}M_{\ell_2}}{M_{m_1}M_{\ell_1}}\right| = 7$

Sol 21: (B) For Ist refraction

$$\frac{1}{v} - \frac{1.5}{-50} = \frac{1-1.5}{-10}$$

$\Rightarrow v = 50\text{cm}$

For IInd refraction

$$\frac{1.5}{\infty} - \frac{1}{-x} = \frac{1.5-1}{+10}$$

$\Rightarrow x = 20$ cm

$\Rightarrow d = 70$ cm

Sol 22: Snell's Law on 1st surface: $\frac{\sqrt{3}}{2} = n \sin r_1$

$$\sin r_1 = \frac{\sqrt{3}}{2n} \quad \text{...(i)}$$

$$\Rightarrow \cos r_1 = \sqrt{1-\frac{3}{4n^2}} = \frac{\sqrt{4n^2-3}}{2n}$$

$$r_1 + r_2 = 60° \quad \text{.... (ii)}$$

Snell's Law on 2nd surface:

$n \sin r_2 = \sin\theta$

Using equation (i) and (ii)

$n\sin(60° - r_1) = \sin\theta$

$$n\left[\frac{\sqrt{3}}{2}\cos r_1 - \frac{1}{2}\sin r_1\right] = \sin\theta$$

$$\frac{d}{dn}\left[\frac{\sqrt{3}}{4}\left(\sqrt{4n^2-3}-1\right)\right] = \cos\theta\frac{d\theta}{dn}$$

For $\theta = 60°$ and $n = \sqrt{3}$

$$\Rightarrow \frac{d\theta}{dn} = 2$$

Sol 23: (A, C) $\theta \ge c$

$\Rightarrow 90° - r \ge c$

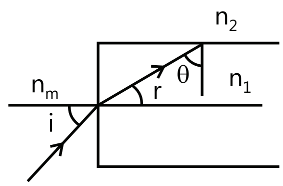
$\Rightarrow \sin(90° - r) \ge c$

$\Rightarrow \cos r \ge \sin c$

Using $\frac{\sin i}{\sin r} = \frac{n_1}{n_m}$ and $\sin c = \frac{n_2}{n_1}$

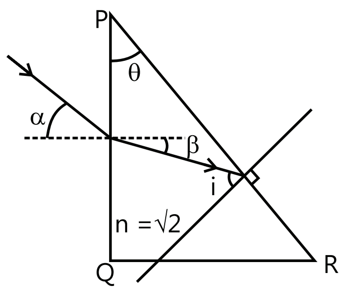
We get, $\sin^2 i_m = \frac{n_1^2 - n_2^2}{n_m^2}$

Putting values, we get, correct options as A & C



Sol 24: (A) $i = \beta + \theta$

For $\alpha = 45°$; by Snell's law,



$1 \times \sin 45° = \sqrt{2}\sin\beta$

$\Rightarrow \beta = 30°$

For TIR on face PR,

$$\beta + \theta = \theta_c = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

$\Rightarrow \theta = 45° - \beta = 15°$.

Sol 25: (A, D) For refraction through lens,

$\frac{1}{v} - \frac{1}{-30} = \frac{1}{f}$ and $-2 = \frac{v}{u}$

$\therefore v = -2u = 60$ cm

$\therefore f = +20\text{cm}$

For reflection

$$\frac{1}{10}+\frac{1}{-30}=\frac{2}{R}\Rightarrow R=30\text{ cm}$$

$$\left(n-1\right)\left(\frac{1}{R}\right)=\frac{1}{f}=\frac{1}{20}$$

$$\therefore n=\frac{5}{2}$$

The faint image is erect and virtual.