

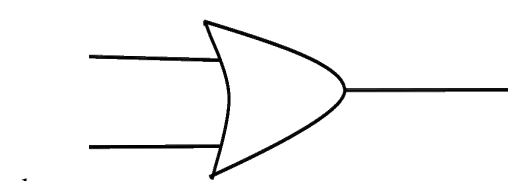
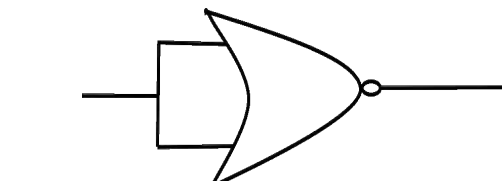
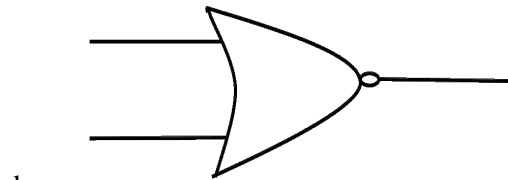
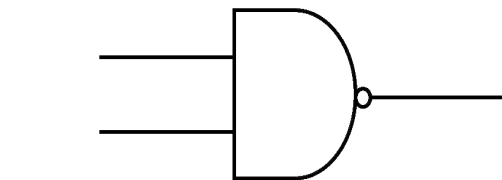
JEE Main 2020 Paper

Date of Exam: 7th January 2020 (Shift 1)

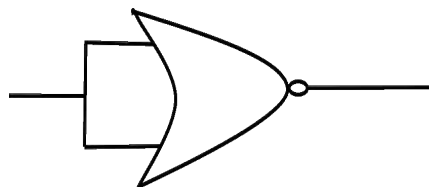
Time: 9:30 am- 12:30 pm

Subject: Physics

1. Which of the following gives reversible operation?



Solution: (c)



Since, there is only one input hence the operation is reversible.

2. A 60 HP electric motor lifts an elevator with a maximum total load capacity of 2000 kg. If the frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to (Given 1 HP = 746 W, $g = 10 \text{ m/s}^2$)

a. 1.9 m/s

b. 1.7 m/s

c. 2 m/s

d. 1.5 m/s

Solution:(a)

Friction will oppose the motion

$$\text{Net force} = 2000g + 4000 = 24000 \text{ N}$$

$$\text{Power of lift} = 60 \text{ HP}$$

$$\text{Power} = \text{Force} \times \text{Velocity}$$

$$v = \frac{P}{F} = \frac{60 \times 746}{24000}$$

$$v = 1.86 \text{ m/s}$$

3. A LCR circuit behaves like a damped harmonic oscillator. Comparing it with a physical spring-mass damped oscillator having damping constant b , mass m and oscillating with a force constant k , the correct equivalence will be

a. $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

b. $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

c. $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$

d. $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$

Solution:(a)

For damped oscillator by Newton's second law

$$-kx - bv = ma$$

$$kx + bv + ma = 0$$

$$kx + b\frac{dx}{dt} + m\frac{d^2x}{dt^2} = 0$$

For LCR circuit by KVL

$$-IR - L\frac{dI}{dt} - \frac{q}{c} = 0$$

$$\implies IR + L\frac{dI}{dt} + \frac{q}{c} = 0$$

$$\implies \frac{q}{c} + R\frac{dq}{dt} + L\frac{d^2q}{dt^2} = 0$$

By comparing

$$R \implies b$$

$$c \implies \frac{1}{k}$$

$$m \implies L$$

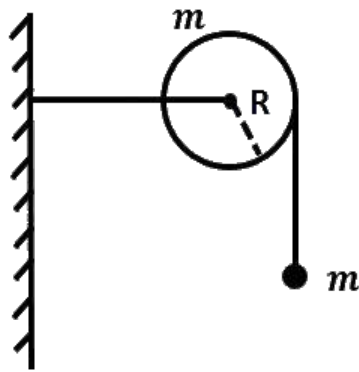
4. As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a flywheel (disc) of radius R and mass m . When released from the rest, the bob starts falling vertically. When it has covered a distance h , the angular speed of the wheel will be (there is no slipping between string and wheel)

a. $\frac{1}{R}\sqrt{\frac{4gh}{3}}$

b. $\frac{1}{R}\sqrt{\frac{2gh}{3}}$

c. $R\sqrt{\frac{2gh}{3}}$

d. $R\sqrt{\frac{4gh}{3}}$



Solution:(a)

By energy conservation,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow gh = \frac{v^2}{2} + \frac{\omega^2 R^2}{4} \quad (1)$$

Since the rope is inextensible and also it is not slipping,

$$\therefore v = R\omega \quad (2)$$

from eq. (1) and (2)

$$gh = \frac{\omega^2 R^2}{2} + \frac{\omega^2 R^2}{4}$$

$$\Rightarrow gh = \frac{3}{4}R^2\omega^2$$

$$\Rightarrow \omega^2 = \frac{4gh}{3R^2}$$

$$\Rightarrow \omega = \frac{1}{R}\sqrt{\frac{4gh}{3}}$$

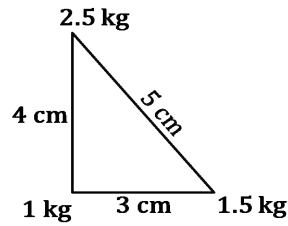
5. Three point particles of mass 1 kg, 1.5 kg and 2.5 kg are placed at corners of a right triangle of sides 4 cm, 3 cm and 5 cm as shown. The centre of mass of the system with respect to 1 kg mass is at the point

- 0.6 cm to the right of 1 kg and 2 cm above 1 kg mass
- 0.9 cm to the right of 1 kg and 2 cm above 1 kg mass
- 0.9 cm to the right of 1 kg and 2 cm below 1 kg mass
- 0.9 cm to the right of 1 kg and 1.5 cm above 1 kg mass

Solution: (b)

Taking 1 kg as the origin

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$



$$x_{com} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{5}$$

$$x_{com} = 0.9$$

$$y_{com} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$y_{com} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{5}$$

$$y_{com} = 2$$

Centre of mass is at (0.9, 2)

6. A parallel plate capacitor has plates of area A separated by distance 'd'. It is filled with a dielectric which has a dielectric constant varies as $k(x) = k(1 + \alpha x)$, where 'x' is the distance measured from one of the plates. If ($\alpha d \ll 1$), the capacitance of the system is

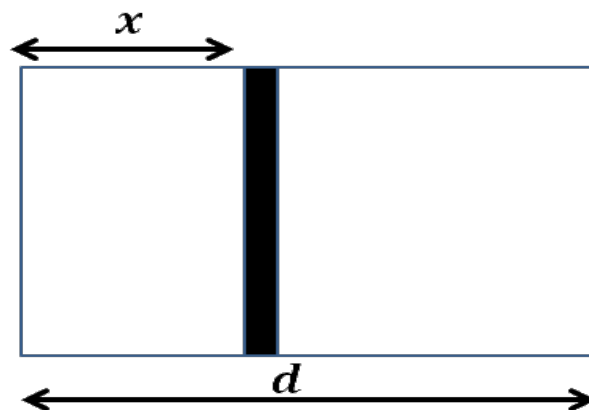
a. $\frac{A\epsilon_0 k}{d} \left[1 + \left(\frac{\alpha d}{2} \right)^2 \right]$

b. $\frac{A\epsilon_0 k}{d} \left[1 + \left(\frac{\alpha^2 d^2}{2} \right) \right]$

c. $\frac{A\epsilon_0 k}{d} [1 + \alpha d]$

d. $\frac{A\epsilon_0 k}{d} \left[1 + \left(\frac{\alpha d}{2} \right) \right]$

Solution:(d)



Given, $k(x) = k(1 + \alpha x)$

$$dC = \frac{A\epsilon_0 k}{dx}$$

Since all capacitance are in series, we can apply

$$\frac{1}{C_{eq}} = \int \frac{1}{dC} = \int_0^d \frac{dx}{k(1 + \alpha x) \epsilon_0 A}$$

$$\frac{1}{C_{eq}} = \left[\frac{\ln(1 + \alpha x)}{k\epsilon_0 A \alpha} \right]_0^d$$

On putting the limits from 0 to d

$$= \frac{\ln(1 + \alpha d)}{k\epsilon_0 A \alpha}$$

Using expression $\ln(1 + x) = x - \frac{x^2}{2} + \dots$

And putting $x = \alpha d$ where, x approaches to 0.

$$\frac{1}{C} = \frac{d}{k\epsilon_0 A d \alpha} \left[\alpha d - \frac{(\alpha d)^2}{2} \right]$$

$$\frac{1}{C} = \frac{d}{k\epsilon_0 A} \left[1 - \frac{\alpha d}{2} \right]$$

$$C = \frac{k\epsilon_0 A}{d} \left[1 + \frac{\alpha d}{2} \right]$$

7. The time period of revolution of an electron in its ground state orbit in a hydrogen atom is $1.6 \times 10^{-16} s$. The frequency of the electron in its first excited state (in s^{-1}) is :

a. 7.8×10^{14}

b. 7.8×10^{16}

c. 3.7×10^{14}

d. 3.7×10^{16}

Solution:(a)

Time period is proportional to $\frac{n^3}{Z^2}$.

Let T_1 be the time period in ground state and T_2 be the time period in it's first excited state.

$$T_1 = \frac{n^3}{2^2}$$

(Where, n = excitation level and 2 is atomic no.)

$$\frac{T_1}{T_2} = \left(\frac{n_1}{n_2} \right)^3$$

Given,

$$T_1 = 1.6 \times 10^{-16} s$$

So,

$$\frac{1.6 \times 10^{-16}}{T_2} = \left(\frac{1}{2} \right)^3$$

$$T_2 = 12.8 \times 10^{-16} s$$

Frequency is given by $f = \frac{1}{T}$

$$f = \frac{1}{12.8} \times 10^{16} Hz$$

Solution:(c)

As we know,

$$\begin{aligned}T.E_{ground} &= T.E_R \\ \frac{1}{2}mu^2 + \left(\frac{-GMm}{R}\right) &= \frac{1}{2}mv^2 + \left(\frac{-GMm}{2R}\right) \\ \frac{1}{2}mv^2 &= \frac{1}{2}mu^2 + \left(\frac{-GMm}{2R}\right) \\ v^2 &= u^2 + \left(\frac{-GMm}{R}\right) \\ \Rightarrow v &= \sqrt{u^2 + \left(\frac{-GMm}{R}\right)}\end{aligned}\tag{1}$$

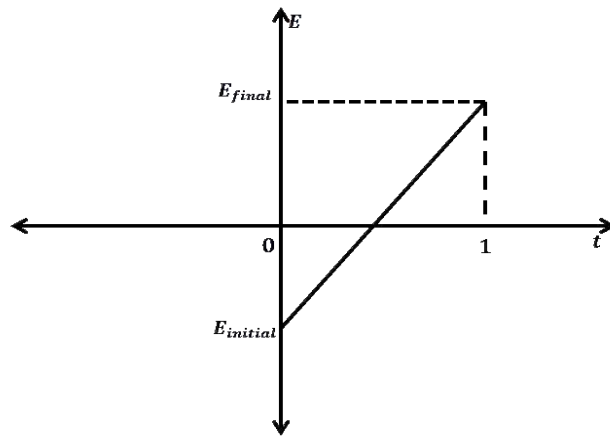
The rocket splits at height R . Since, separation of rocket is impulsive therefore conservation of momentum in both radial and tangential direction can be applied.

$$\begin{aligned}\frac{m}{10}V_T &= \frac{9m}{10}\sqrt{\frac{GM}{2R}} \\ \frac{m}{10}V_r &= m\sqrt{u^2 - \frac{GM}{R}}\end{aligned}$$

$$\begin{aligned}\text{Kinetic energy of satellite} &= \frac{1}{2} \times \frac{m}{10}(V_T^2 + V_R^2) = \frac{m}{20} \left(81\frac{GM}{2R} + 100u^2 - 100\frac{GM}{R}\right) \\ &= \frac{m}{20} \left(100u^2 - \frac{119GM}{2R}\right) \\ &= 5m \left(u^2 - \frac{119GM}{200R}\right)\end{aligned}$$

10. A long solenoid of radius R carries a time (t) dependent current $I(t) = I_0t(1 - t)$. A ring of radius $2R$ is placed coaxially to the middle. During the time instant $0 \leq t \leq 1$, the induced current (I_R) and the induced EMF (V_R) in the ring changes as
- Current will change its direction and its emf will be zero at $t = 0.25$ sec.
 - Current will not change its direction and its emf will be maximum at $t = 0.5$ sec.
 - Current will not change direction and emf will be zero at $t = 0.25$ sec.
 - Current will change its direction and its emf will be zero at $t = 0.5$ sec.

Solution:(d)



Field due to solenoid near the middle = $\mu_o N I$

$$\text{Flux, } \phi = BA \quad \text{where } (A = \pi(R)^2)$$

$$= \mu_o N I_o t (1 - t) \pi R^2$$

$$E = -\frac{d\phi}{dt} \quad [\text{By Lenz's law}]$$

$$E = -\frac{d}{dt} (\mu_o N I_o t (1 - t)^2)$$

$$E = -\mu_o N I_o \pi R^2 \frac{d}{dt} [t(1 - t)]$$

$$E = -\pi \mu_o I_o N R^2 (1 - 2t)$$

Current will change its direction when EMF will be zero

$$\implies (1 - 2t) = 0$$

$$\text{So, } t = 0.5 \text{ sec}$$

11. The radius of gyration of a uniform rod of length l about an axis passing through a point $l/4$ away from the center of the rod and perpendicular to its axis is

a. $\sqrt{\frac{7}{48}} l$

b. $\sqrt{\frac{5}{48}} l$

c. $\sqrt{\frac{7}{24}} l$

d. $\sqrt{\frac{19}{24}} l$

Solution:(a)

We know that,

$$\left| \frac{E_0}{B_0} \right| = c$$

$$B_0 = 3 \times 10^{-8}$$

$$\implies E_0 = B_0 \times c = 3 \times 10^{-8} \times 3 \times 10^8$$

$$= 9 \text{ N/C}$$

$$\therefore E = E_0 \sin(\omega t - kx + \phi) \hat{k} = 9 \sin(\omega t - kx + \phi) \hat{k}$$

15. A polarizer analyzer set is adjusted such that the intensity of light coming out of the analyzer is just 10 % of the original intensity. Assuming that the polarizer analyzer set does not absorb any light, the angle by which the analyzer needs to be rotated further to reduce the output intensity to be zero is

a. 45°

b. 71.6°

c. 90°

d. 18.4°

Solution:(d)

$$\text{Intensity after polarisation through polaroid} = I_o \cos^2 \phi$$

$$\text{So, } 0.1I_o = I_o \cos^2 \phi$$

$$\Rightarrow \cos \phi = \sqrt{0.1}$$

$$\Rightarrow \cos \phi = 0.316$$

Since, $\cos \phi < \cos 45^\circ$ therefore, $\phi > 45^\circ$ If the light is passing at 90° from the plane of polaroid, than its intensity will be zero.

Then, $\theta = 90^\circ - \phi$ therefore, θ will be less than 45° . So, the only option matching is option d which is 18.4°

16. Speed of transverse wave of a straight wire having mass 6.0 g, length 60 cm and area of cross-section 1.0 mm^2 is 90 m/s. If the Young's modulus of wire is $1.6 \times 10^{11} \text{ Nm}^{-2}$, the extension of wire over its natural length is

a. 0.3 mm

b. 0.2 mm

c. 0.1 mm

d. 0.4 mm

Solution:(a)

$$\text{Given, } M = 6 \text{ grams} = 6 \times 10^{-3} \text{ kg}$$

$$L = 60 \text{ cm} = 0.6 \text{ m}$$

$$A = 1 \text{ mm}^2 = 1 \times 10^{-6} \text{ m}^2$$

$$\text{Using the relation, } v^2 = \frac{T}{\mu}$$

$$\Rightarrow T = \mu v^2 = V^2 \times \frac{M}{L}$$

As Young's modulus, $Y = \frac{\text{stress}}{\text{strain}}$

$$\text{Strain} = \frac{\text{Stress}}{Y} = \frac{T}{AY}$$

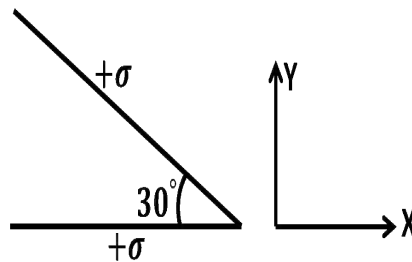
$$\text{Strain} = \frac{\Delta L}{L} = \frac{V^2 \frac{M}{L}}{AY} = V^2 \frac{M}{AYL}$$

$$\Rightarrow \Delta L = \frac{V^2 M}{AY}$$

$$\Delta L = \frac{8100 \times 6 \times 10^{-3}}{1 \times 10^{-6} \times 1.6 \times 10^{11}}$$

$$\Delta L = 0.3 \text{ mm}$$

17. Two infinite planes each with uniform surface charge density $+\sigma \text{ C/m}^2$ are kept in such a way that the angle between them is 30° . The electric field in the region shown between them is given by:



- a. $\frac{\sigma}{2\epsilon_0} \left[\left(1 - \frac{\sqrt{3}}{2}\right)\hat{y} - \frac{1}{2}\hat{x} \right]$
 b. $\frac{\sigma}{2\epsilon_0} \left[\left(1 + \frac{\sqrt{3}}{2}\right)\hat{y} - \frac{1}{2}\hat{x} \right]$
 c. $\frac{\sigma}{2\epsilon_0} \left[\left(1 - \frac{\sqrt{3}}{2}\right)\hat{y} + \frac{1}{2}\hat{x} \right]$
 d. $\frac{\sigma}{2\epsilon_0} \left[\left(1 + \frac{\sqrt{3}}{2}\right)\hat{y} + \frac{1}{2}\hat{x} \right]$

Solution:(a)

$$\text{Field due to single plate} = \frac{\sigma}{2\epsilon_0} = \left[\vec{E}_1 \right] = \left[\vec{E}_2 \right]$$

$$\text{Net electric field } \vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{\sigma}{2\epsilon_0} \cos 30^\circ (-\hat{j}) + \frac{\sigma}{2\epsilon_0} \sin 30^\circ (-\hat{i}) + \frac{\sigma}{2\epsilon_0} (\hat{j})$$

$$= \frac{\sigma}{2\epsilon_0} \left(1 - \frac{\sqrt{3}}{2} \right) (\hat{j}) - \frac{\sigma}{4\epsilon_0} (\hat{i})$$

$$= \frac{\sigma}{2\epsilon_0} \left[\left(1 - \frac{\sqrt{3}}{2} \right) \hat{y} - \frac{1}{2} \hat{x} \right]$$

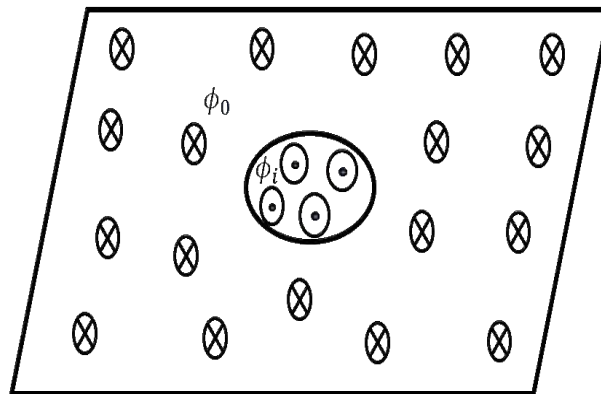
$$\implies \frac{\sqrt{3}}{2\sin\theta_1} = 2 \implies \sin\theta_1 = \frac{\sqrt{3}}{4} = 0.43$$

As, the value is coming less than 30° the only available option are 20° and 25° but by using approximation we get $\theta_1 = 25^\circ$

20. Consider a coil of wire carrying current I , forming a magnetic dipole placed in an infinite plane. If ϕ_i is the magnitude of magnetic flux through the inner region and ϕ_o is magnitude of magnetic flux through outer region then which of the following is correct?

- | | |
|-----------------------|----------------------|
| a. $\phi_i = -\phi_o$ | b. $\phi_i > \phi_o$ |
| c. $\phi_i < \phi_o$ | d. $\phi_i = \phi_o$ |

Solution:(a)

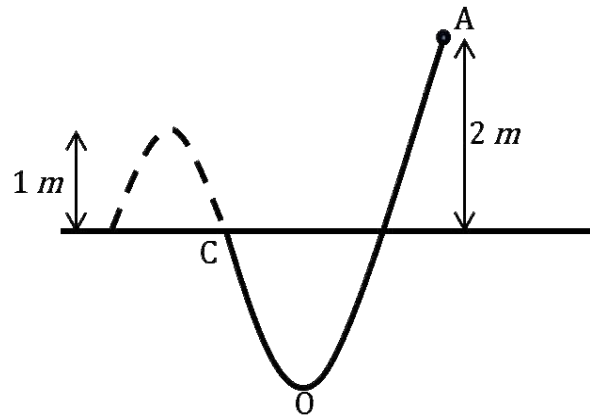


As magnetic field line of ring will form close loop therefore all outgoing from circular hole passing through the infinite plate.

$\therefore \phi_1 = -\phi_0$ (because the magnetic field lines going inside is equal to the magnetic field lines coming out.)

21. A particle of mass 1 kg slides down a frictionless track AOC starting from rest at A (height 2 m). After reaching C particle continues to move freely in air as a projectile. When it reaches its highest point P ($h=1 \text{ m}$) the kinetic energy of the particle (in J) is..... (take $g=10 \text{ m/s}^2$)

Solution:(10)



As the particle starts from rest the total energy at point A = $mgh = T.E_A$ (where $h = 2\text{ m}$)
 After reaching point P

$$T.E_c = K.E. + mgh$$

By conservation of energy

$$T.E_A = T.E_p$$

$$\implies K.E. = mgh = 10\text{ J}$$

22. A carnot engine operates between two reservoirs of temperature 900 K and 300 K . The engine performs 1200 J of work per cycle. The heat energy in(J) delivered by the engine to the low temperature reservoir in a cycle is

Solution:(600 J)

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{900} = \frac{2}{3}$$

$$\text{Given, } W = 1200\text{ J}$$

From conservation of energy

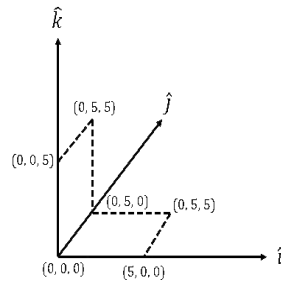
$$Q_1 - Q_2 = W$$

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{W}{Q_1} \implies Q_1 = 1800\text{ J}$$

$$\implies Q_2 = Q_1 - W = 600\text{ J}$$

23. A loop $ABCDEF A$ of straight edges has a six corner points $A(0,0,0)$, $B(5,0,0)$, $C(5,5,0)$, $D(0,5,0)$, $E(0,5,5)$, $F(0,0,5)$. The magnetic field in this region is $\vec{B} = (3\hat{i} + 4\hat{k})\text{ T}$. The quantity of the flux through the loop $ABCDEF A$ (in Wb) is -----

Solution:(175)



As we know, magnetic flux = $\vec{B} \cdot \vec{A}$

$$\implies (B_x + B_z) \cdot (A_x + A_z)$$

$$\implies (3\hat{i} + 4\hat{k}) \cdot (25\hat{i} + 25\hat{k})$$

$$\implies (75 + 100) \text{ Wb}$$

$$\implies 175 \text{ Wb}$$

24. A beam of electromagnetic radiation of intensity $6.4 \times 10^{-5} \text{ W/cm}^2$ is comprised of wavelength, $\lambda = 310 \text{ nm}$. It falls normally on a metal (work function 2 eV) of surface area 1 cm^2 . If one in 10^3 photons ejects an electron, total number of electrons ejected in 1 s is 10^x ($hc = 1240 \text{ eV} - \text{nm}$, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$), then the value of x is _____

Solution:(11)

$$\begin{aligned} P &= \text{Intensity} \times \text{Area} \\ &= 6.4 \times 10^{-5} \text{ W} - \text{cm}^{-2} \times 1 \text{ cm}^2 \\ &= 6.4 \times 10^{-5} \text{ W} \end{aligned}$$

For photoelectric effect to take place, energy should be greater than work function

Now,

$$E = \frac{1240}{310} = 4 \text{ eV} > 2 \text{ eV}$$

Therefore, photoelectric effect takes place

Here n is the number of photons emitted.

$$\begin{aligned} n \times E &= I \times A \\ \implies n &= \frac{IA}{E} = \frac{6.4 \times 10^{-5}}{6.4 \times 10^{-19}} = 10^{14} \end{aligned}$$

Where, n is number of incident photon

Since, 1 out of every 1000 photons are successful in ejecting 1 photoelectron

Therefore, the number of photoelectrons emitted is

$$= \frac{10^{14}}{10^3}$$

$$\therefore x = 11$$

25. A non- isotropic solid metal cube has coefficient of linear expansion as $5 \times 10^{-5}/^{\circ}C$ along the x-axis and $5 \times 10^{-6}/^{\circ}C$ along y-axis and z-axis. If the coefficient of volumetric expansion of the solid is $n \times 10^{-6}/^{\circ}C$ then the value of n is -----

Solution:(60)

We know that, $V = xyz$

$$\frac{\Delta v}{v} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

$$\frac{1}{T} \frac{\Delta v}{v} = \frac{1}{T} \frac{\Delta x}{x} + \frac{1}{T} \frac{\Delta y}{y} + \frac{1}{T} \frac{\Delta z}{z}$$

$$y = \alpha_x + \alpha_y + \alpha_z$$

$$y = 50 \times 10^{-6}/^{\circ}C + 5 \times 10^{-6}/^{\circ}C + 5 \times 10^{-6}/^{\circ}C$$

$$y = 60 \times 10^{-6}/^{\circ}C$$

$$\therefore n = 60$$